Computer Algebra
Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.3-Tangent/98-4.3.0-a-trg-^m-b-tan-^n

Nasser M. Abbasi

September 5, 2023 Compiled on September 5, 2023 at 8:58pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [387]. This is test number [98].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (387)	0.00 (0)
Mathematica	100.00 (387)	0.00 (0)
Maple	68.99 (267)	31.01 (120)
Fricas	62.27 (241)	37.73 (146)
Maxima	35.40 (137)	64.60 (250)
Mupad	31.52 (122)	68.48 (265)
Giac	21.19 (82)	78.81 (305)
Sympy	4.65 (18)	95.35 (369)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

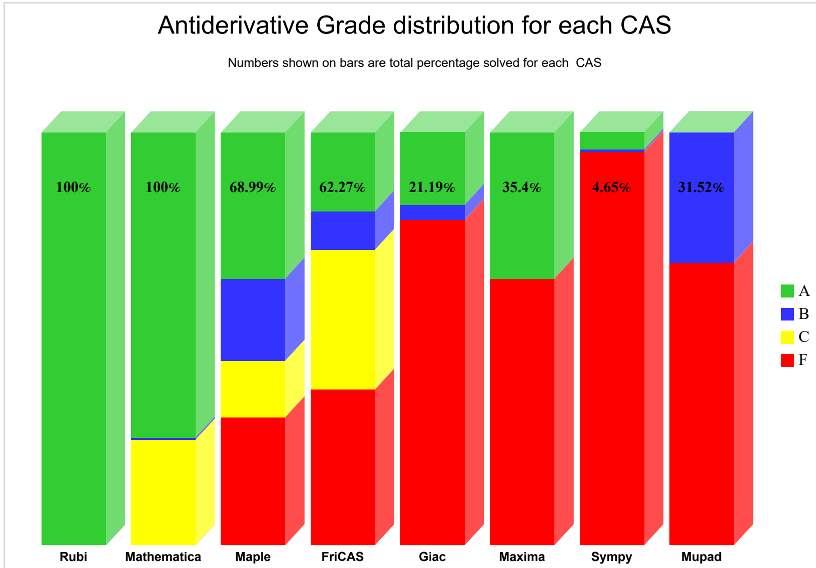
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

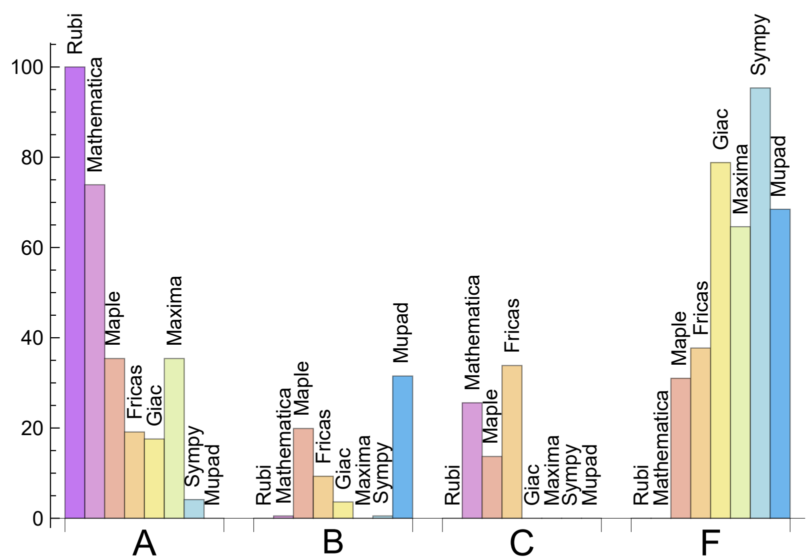
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	73.902	0.517	25.581	0.000
Maple	35.401	19.897	13.695	31.008
Maxima	35.401	0.000	0.000	64.599
Fricas	19.121	9.302	33.850	37.726
Giac	17.571	3.618	0.000	78.811
Sympy	4.134	0.517	0.000	95.349
Mupad	0.000	31.525	0.000	68.475

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	120	100.00	0.00	0.00
Fricas	146	95.89	0.00	4.11
Maxima	250	99.60	0.40	0.00
Mupad	265	0.00	100.00	0.00
Giac	305	86.23	6.23	7.54
Sympy	369	71.27	28.73	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.12
Fricas	0.23
Maxima	0.35
Giac	0.67
Mathematica	1.32
Mupad	4.55
Maple	6.11
Sympy	13.47

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	50.72	1.43	52.00	1.29
Mathematica	90.38	1.04	71.00	0.91
Rubi	103.99	1.00	78.00	1.00
Maxima	115.38	0.86	133.00	0.85
Mupad	146.28	2.64	77.50	1.10
Giac	206.54	2.66	176.00	1.01
Fricas	231.66	1.70	129.00	1.28
Maple	459.74	5.73	203.00	1.52

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

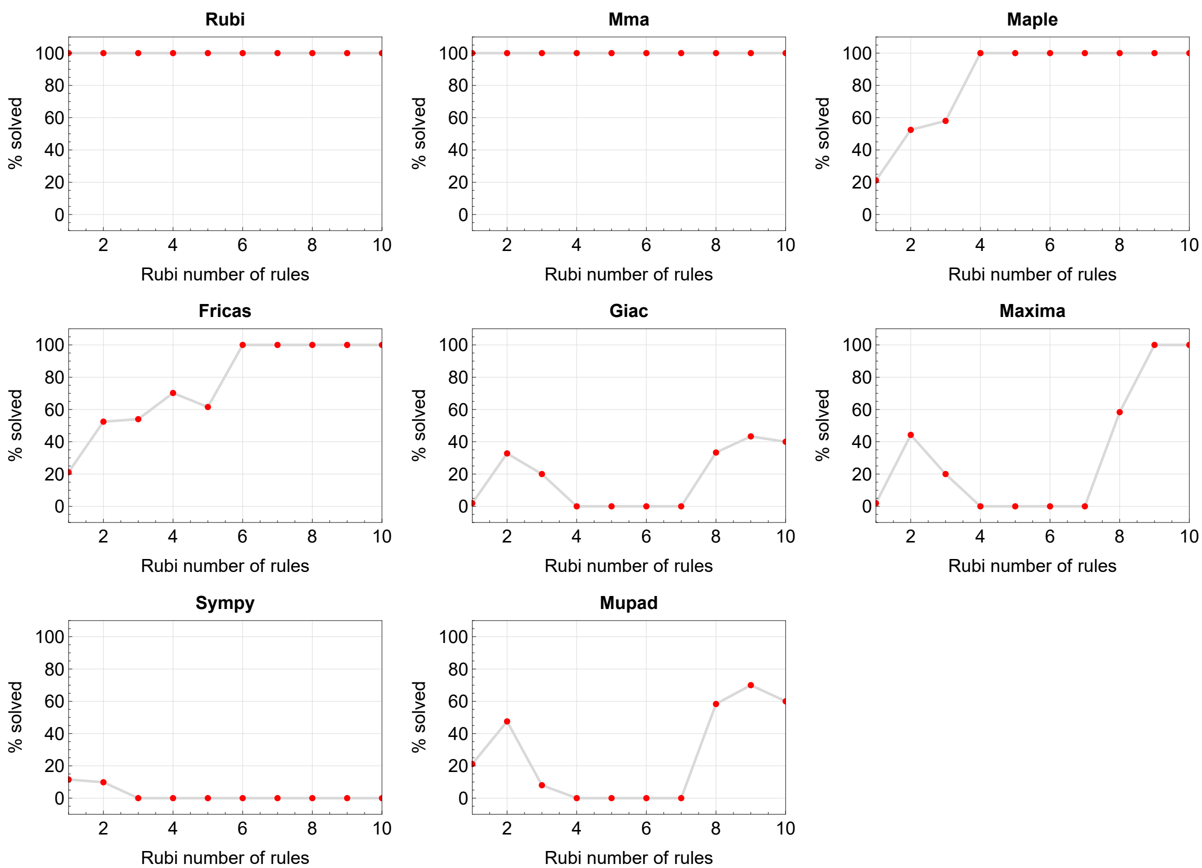


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

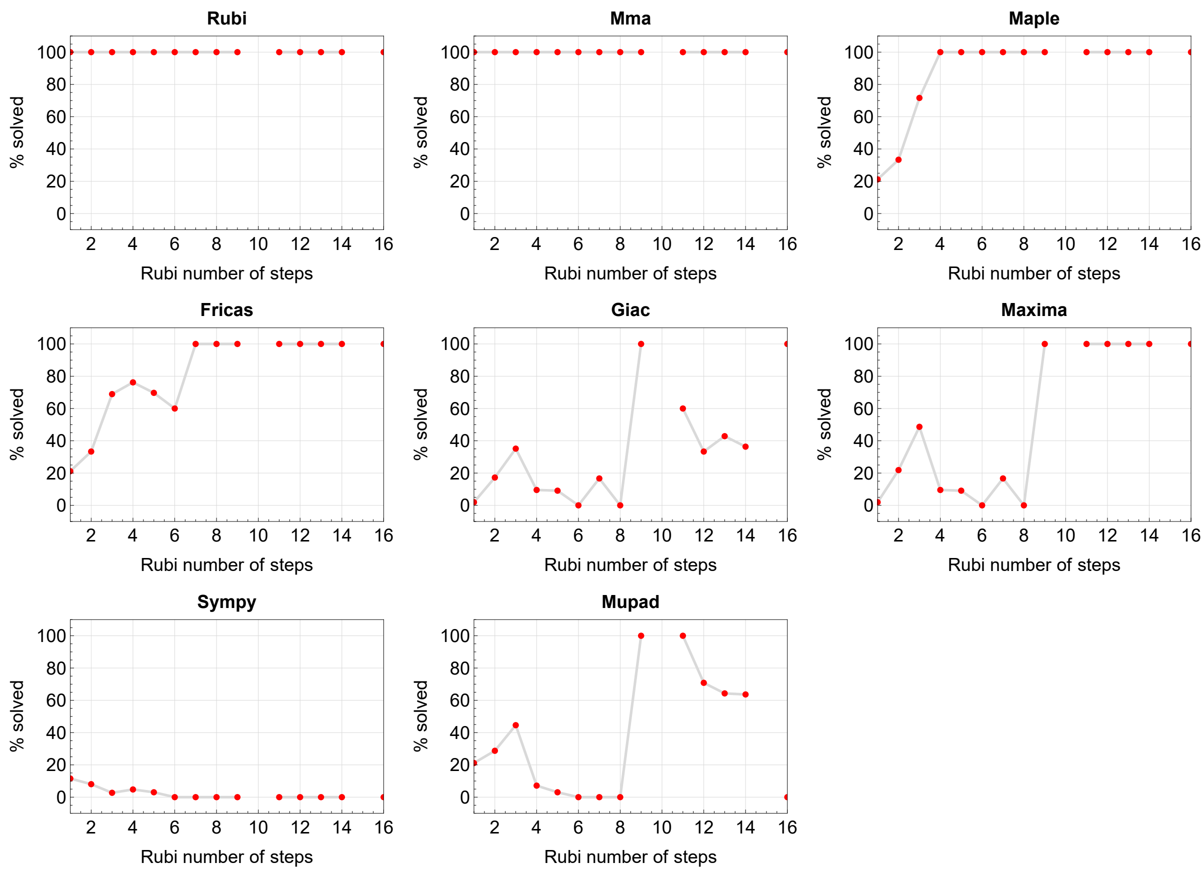


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

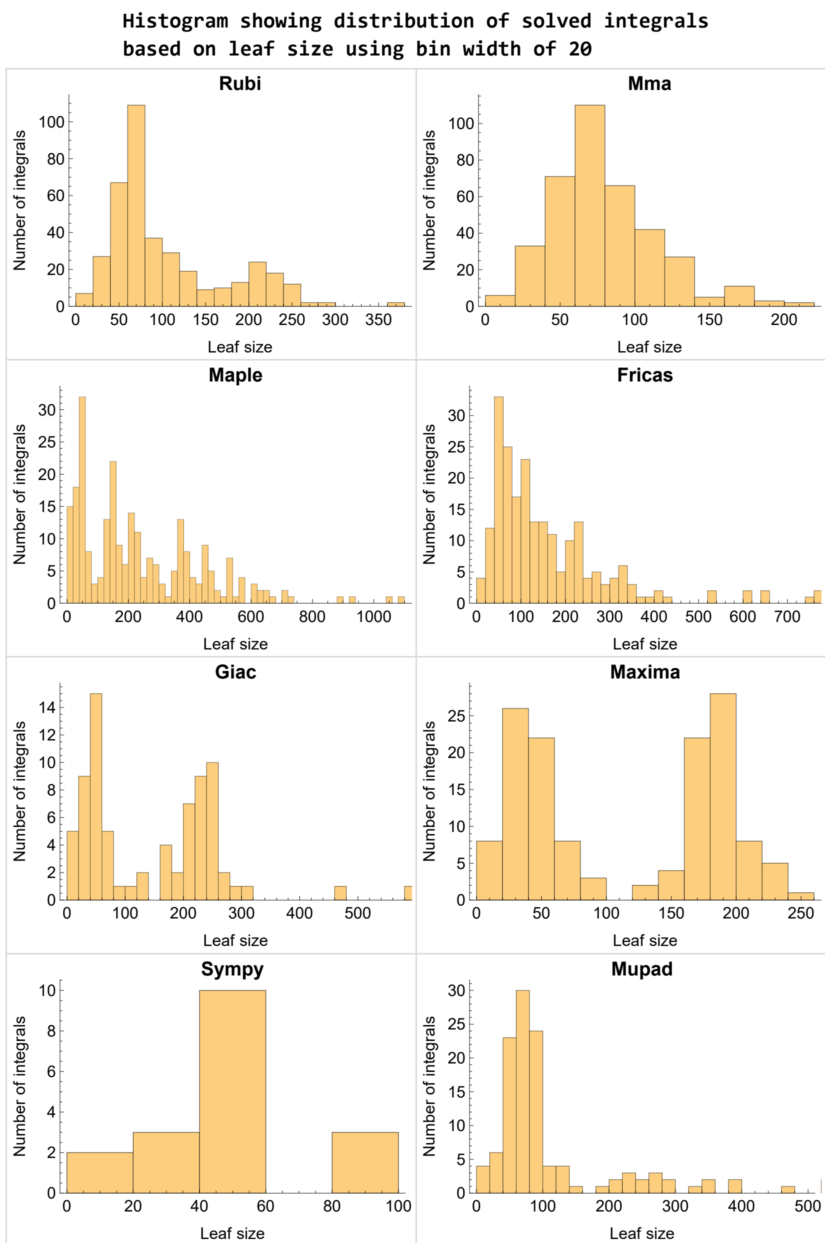


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

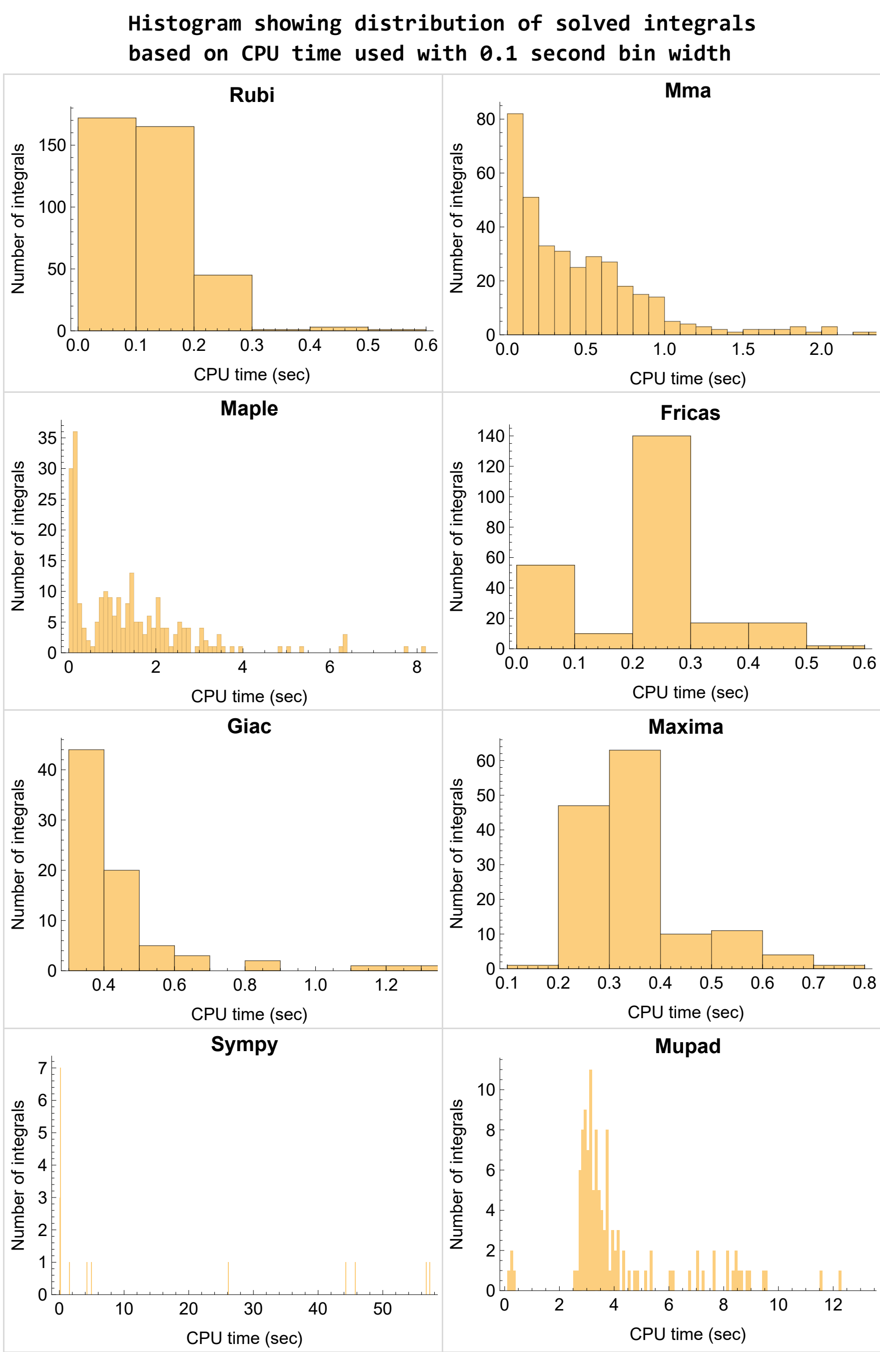


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

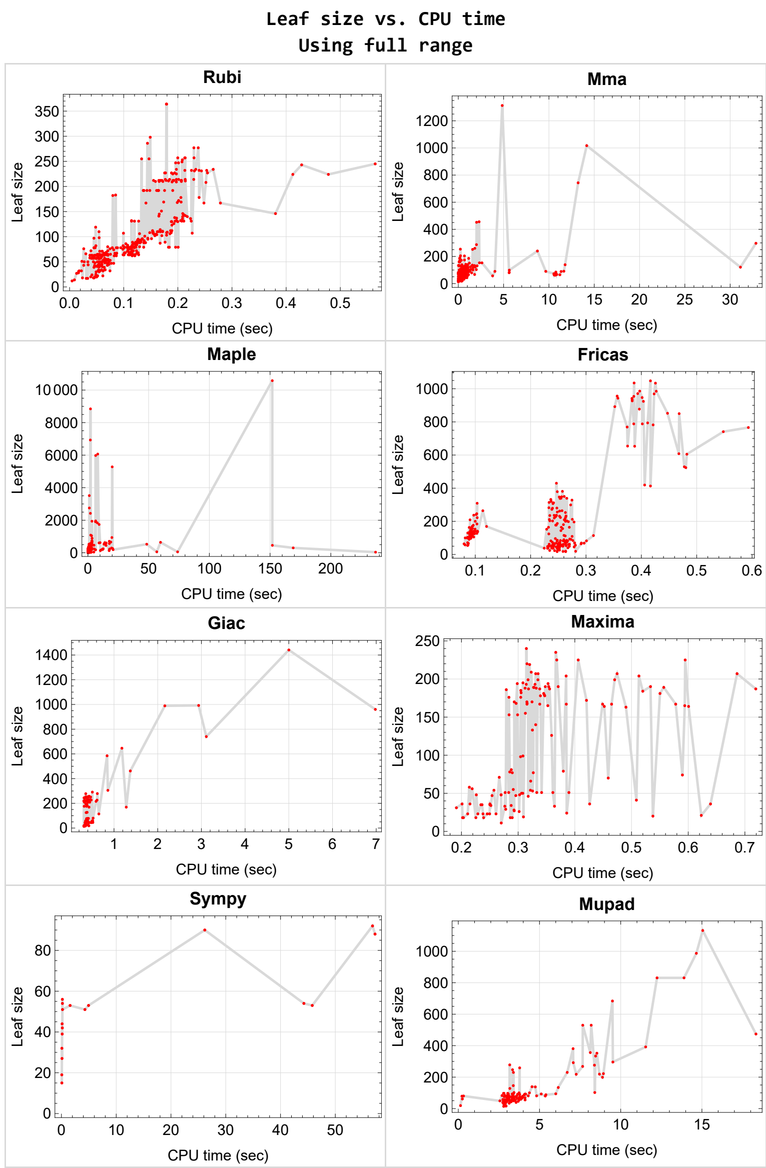


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {174, 180, 181, 183, 184, 185, 372, 373, 379, 387}

Maple {52, 54, 55, 59, 64, 65, 73, 74, 78, 84, 85, 94, 95, 99, 103, 104, 164, 165, 178, 179, 191, 192, 193, 194, 199, 200, 201, 202, 208, 214, 215, 230, 240, 245, 246, 251, 252, 257, 262, 323, 351, 352, 377, 378}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

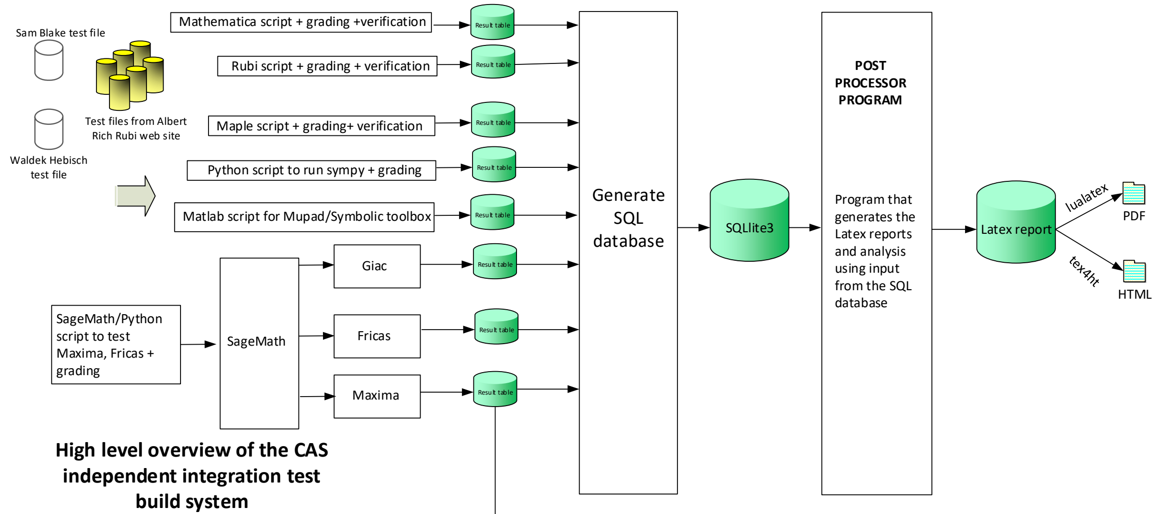
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 114, 115, 116, 117, 118, 119, 121, 123, 125, 127, 129, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 182, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 258, 259, 260, 261, 262, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 295, 297, 300, 304, 306, 307, 309, 311, 313, 315, 317, 319, 321, 322, 324, 326, 328, 330, 332, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 374, 375, 376, 377, 378, 380, 382, 383, 384, 385, 386 }

B grade { 379, 381 }

C grade { 17, 18, 21, 22, 39, 40, 41, 59, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 82, 83, 89, 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 120, 122, 124, 126, 128, 130, 132, 174, 180, 181, 183, 184, 185, 231, 232, 233, 234, 235, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 263, 264, 265, 266, 267, 268, 269, 270, 292, 294, 296, 298, 299, 301, 302, 303, 305, 308, 310, 312, 314, 316, 318, 320, 323, 325, 327, 329, 331, 333, 372, 373, 387 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 56, 57, 58, 61, 66, 67, 68, 75, 76, 77, 79, 86, 87, 88, 96, 97, 98, 100, 101, 105, 106, 107, 118, 125, 127, 129, 131, 134, 135, 136, 137, 163, 177, 195, 196, 197, 198, 203, 204, 205, 206, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 236, 237, 238, 239, 241, 247, 248, 249, 250, 253, 255, 258, 259, 260, 261, 291, 293, 295, 297, 300, 304, 306, 307, 309, 311, 313, 315, 317, 319, 321, 322, 324, 326, 328, 330, 332, 334, 353, 363, 364, 365, 376 }

B grade { 54, 55, 60, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 102, 103, 104, 108, 109, 110, 111, 112, 113, 114, 121, 123, 133, 138, 191, 192, 193, 194, 199, 200, 201, 202, 207, 208, 209, 214, 215, 230, 231, 232, 233, 234, 235, 240, 242, 243, 244, 251, 252, 254, 256, 262, 263, 264, 265, 266, 267, 268, 269, 270, 302 }

C grade { 52, 59, 78, 99, 115, 116, 117, 119, 120, 122, 124, 126, 128, 130, 132, 139, 140, 141, 142, 143, 144, 164, 165, 178, 179, 245, 246, 257, 292, 294, 296, 298, 299, 301, 303, 305, 308, 310, 312, 314, 316, 318, 320, 323, 325, 327, 329, 331, 333, 351, 352, 377, 378 }

F normal fail { 23, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 29, 36, 37, 38, 39, 40, 41, 52, 57, 58, 66, 67, 68, 76, 77, 87, 88, 114, 116, 121, 123, 127, 134, 135, 163, 164, 165, 177, 178, 179, 226, 227, 236, 237, 247, 248, 258, 259, 295, 297, 306, 319, 321, 324, 326, 332, 334, 351, 352, 353, 363, 364, 365, 376, 377, 378 }

B grade { 22, 56, 75, 86, 96, 97, 98, 105, 106, 107, 118, 125, 129, 131, 133, 136, 137, 138, 228, 238, 249, 260, 291, 293, 300, 302, 304, 307, 309, 311, 313, 315, 317, 322, 328, 330 }

C grade { 9, 10, 11, 12, 13, 14, 15, 16, 30, 31, 32, 33, 34, 35, 54, 55, 61, 62, 63, 64, 65, 71, 72, 73, 74, 80, 81, 82, 83, 84, 85, 92, 93, 94, 95, 101, 102, 103, 104, 112, 113, 115, 117, 119, 120, 122, 124, 126, 128, 130, 132, 139, 140, 141, 142, 143, 144, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 229, 230, 231, 232, 239, 240, 241, 242, 243, 250, 251, 252, 253, 254, 255, 261, 262, 263, 264, 265, 269, 270, 292, 294, 296, 298, 299, 301, 303, 305, 308, 310, 312, 314, 316, 318, 320, 323, 325, 327, 329, 331, 333 }

F normal fail { 23, 42, 43, 44, 45, 53, 59, 60, 69, 70, 78, 79, 89, 90, 91, 99, 100, 108, 109, 110, 111, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 233, 234, 235, 244, 245, 246, 256, 257, 266, 267, 268, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

F(-1) timedout fail { }

F(-2) exception fail { 46, 47, 48, 49, 50, 51 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 54, 55, 56, 57, 58, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 163, 164, 165, 177, 178, 179, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 230, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 258, 259, 260, 261, 262, 351, 352, 353, 363, 364, 365, 376, 377, 378 }

B grade { }

C grade { }

F normal fail { 23, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 59, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 82, 83, 89, 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 231, 232, 233, 234, 235, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

F(-1) timedout fail { 286 }

F(-2) exception fail { }

Giac

A grade { 1, 12, 13, 17, 18, 19, 20, 22, 26, 30, 31, 32, 33, 34, 35, 39, 40, 41, 54, 55, 56, 57, 58, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 163, 226, 227, 228, 229, 230, 236, 237, 238, 240, 247, 248, 249, 251, 252, 258, 259, 260, 262, 376 }

B grade { 2, 3, 4, 5, 6, 7, 8, 24, 25, 28, 29, 36, 37, 38 }

C grade { }

F normal fail { 11, 15, 21, 23, 27, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 60, 61, 62, 63, 71, 72, 80, 81, 82, 83, 89, 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 117, 118, 119, 120, 121, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 137, 138, 142, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 231, 232, 233, 235, 239, 241, 242, 243, 253, 254, 255, 256, 257, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

F(-1) timedout fail { 9, 10, 14, 16, 124, 134, 135, 136, 139, 140, 141, 143, 144, 149, 250, 261, 307, 308, 309 }

F(-2) exception fail { 59, 69, 70, 78, 79, 114, 115, 116, 122, 145, 146, 147, 148, 150, 151, 152, 234, 244, 245, 246, 363, 364, 365 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 27, 56, 57, 58, 66, 67, 68, 75, 76, 77, 86, 87, 88, 96, 97, 98, 105, 106, 107, 114, 116, 121, 123, 127, 129, 134, 135, 136, 163, 164, 165, 177, 178, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 236, 237, 238, 239, 247, 248, 249, 250, 258, 259, 260, 261, 295, 297, 304, 306, 313, 319, 321, 324, 326, 330, 332, 334, 351, 352, 353, 364, 365, 376, 377, 378 }

C grade { }

F normal fail { }

F(-1) timedout fail { 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 115, 117, 118, 119, 120, 122, 124, 125, 126, 128, 130, 131, 132, 133, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, 246, 251, 252, 253, 254, 255, 256, 257, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 298, 299, 300, 301, 302, 303, 305, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 320, 322, 323, 325, 327, 328, 329, 331, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 295, 304, 319, 321, 324, 326, 330, 332 }

B grade { 353, 376 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 65, 70, 71, 84, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 111, 112, 113, 116, 117, 118, 130, 131, 137, 146, 147, 148, 154, 155, 156, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 236, 237, 238, 239, 241, 242, 243, 244, 249, 250, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 296, 301, 302, 303, 317, 318, 320, 323, 325, 331, 336, 337, 338, 340, 341, 342, 344, 345, 346, 348, 349, 350, 351, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 374, 375, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387 }

F(-1) timedout fail { 58, 59, 64, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 89, 90, 99, 107, 108, 109, 110, 114, 115, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 157, 158, 159, 160, 170, 179, 180, 185, 234, 235, 240, 245, 246, 247, 248, 251, 252, 257, 267, 268, 291, 297, 298, 299, 300, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 322, 327, 328, 329, 333, 334, 335, 339, 343, 347, 373, 383 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	11	18	19	13	16
N.S.	1	1.00	1.00	1.42	0.92	1.50	1.58	1.08	1.33
time (sec)	N/A	0.005	0.011	0.035	0.270	0.258	0.063	0.313	2.944

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	23	15	18	17	15	226	14
N.S.	1	1.00	1.64	1.07	1.29	1.21	1.07	16.14	1.00
time (sec)	N/A	0.009	0.004	0.031	0.286	0.248	0.074	0.423	2.793

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	28	31	27	32	216	30
N.S.	1	1.00	0.93	1.04	1.15	1.00	1.19	8.00	1.11
time (sec)	N/A	0.013	0.036	0.050	0.191	0.249	0.082	0.588	2.797

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	38	27	29	26	27	585	24
N.S.	1	1.00	1.36	0.96	1.04	0.93	0.96	20.89	0.86
time (sec)	N/A	0.016	0.005	0.046	0.278	0.252	0.104	0.842	2.881

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	37	38	54	39	44	462	38
N.S.	1	1.00	0.86	0.88	1.26	0.91	1.02	10.74	0.88
time (sec)	N/A	0.026	0.041	0.088	0.322	0.231	0.125	1.373	2.739

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	53	39	41	38	39	989	35
N.S.	1	1.00	1.20	0.89	0.93	0.86	0.89	22.48	0.80
time (sec)	N/A	0.034	0.017	0.063	0.509	0.225	0.140	2.163	2.907

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	47	49	74	51	56	740	49
N.S.	1	1.00	0.82	0.86	1.30	0.89	0.98	12.98	0.86
time (sec)	N/A	0.040	0.076	0.104	0.590	0.237	0.172	3.112	2.561

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	68	49	51	48	51	1441	44
N.S.	1	1.00	1.17	0.84	0.88	0.83	0.88	24.84	0.76
time (sec)	N/A	0.037	0.011	0.086	0.277	0.231	0.196	5.001	2.986

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	174	169	186	203	0	0	93
N.S.	1	1.00	0.75	0.73	0.80	0.88	0.00	0.00	0.40
time (sec)	N/A	0.213	0.684	0.195	0.279	0.235	0.000	0.000	3.383

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	101	154	176	202	0	0	74
N.S.	1	1.00	0.48	0.73	0.83	0.95	0.00	0.00	0.35
time (sec)	N/A	0.168	0.195	0.056	0.284	0.240	0.000	0.000	3.150

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	160	149	170	177	0	0	73
N.S.	1	1.00	0.76	0.71	0.81	0.84	0.00	0.00	0.35
time (sec)	N/A	0.165	0.126	0.053	0.293	0.245	0.000	0.000	3.135

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	71	136	153	164	0	176	49
N.S.	1	1.00	0.37	0.71	0.80	0.85	0.00	0.92	0.26
time (sec)	N/A	0.136	0.059	0.168	0.299	0.232	0.000	0.369	2.773

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	131	138	155	152	0	184	59
N.S.	1	1.00	0.68	0.72	0.81	0.79	0.00	0.96	0.31
time (sec)	N/A	0.143	0.081	0.099	0.313	0.229	0.000	0.385	3.121

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	82	157	167	235	0	0	76
N.S.	1	1.00	0.39	0.74	0.79	1.11	0.00	0.00	0.36
time (sec)	N/A	0.165	0.099	0.057	0.316	0.235	0.000	0.000	2.971

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	86	157	168	236	0	0	75
N.S.	1	1.00	0.40	0.73	0.79	1.10	0.00	0.00	0.35
time (sec)	N/A	0.177	0.145	0.056	0.296	0.247	0.000	0.000	3.355

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	96	171	195	256	0	0	92
N.S.	1	1.00	0.41	0.73	0.83	1.09	0.00	0.00	0.39
time (sec)	N/A	0.197	0.221	0.055	0.309	0.248	0.000	0.000	3.328

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	205	215	185	312	0	209	247
N.S.	1	1.00	0.84	0.88	0.76	1.28	0.00	0.86	1.02
time (sec)	N/A	0.429	0.246	0.174	0.305	0.241	0.000	0.445	3.339

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	185	191	168	314	0	206	259
N.S.	1	1.00	0.83	0.85	0.75	1.40	0.00	0.92	1.16
time (sec)	N/A	0.478	0.156	0.163	0.316	0.233	0.000	0.397	3.778

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	106	108	98	124	0	127	146
N.S.	1	1.00	0.81	0.82	0.75	0.95	0.00	0.97	1.11
time (sec)	N/A	0.114	0.228	0.106	0.304	0.233	0.000	0.385	3.387

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	100	108	99	299	0	125	128
N.S.	1	1.00	0.76	0.82	0.76	2.28	0.00	0.95	0.98
time (sec)	N/A	0.120	0.098	0.095	0.308	0.272	0.000	0.362	3.174

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	189	203	170	282	0	0	230
N.S.	1	1.00	0.84	0.91	0.76	1.26	0.00	0.00	1.03
time (sec)	N/A	0.412	0.170	0.135	0.315	0.233	0.000	0.000	3.393

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	254	215	182	430	0	227	278
N.S.	1	1.00	1.04	0.88	0.74	1.76	0.00	0.93	1.13
time (sec)	N/A	0.564	0.229	0.098	0.305	0.247	0.000	0.477	3.166

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	51	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.044	0.154	0.000	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	56	58	47	74	0	646	0
N.S.	1	1.00	0.57	0.59	0.48	0.76	0.00	6.59	0.00
time (sec)	N/A	0.055	0.280	0.161	0.296	0.239	0.000	1.181	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	48	34	52	0	226	0
N.S.	1	1.00	0.77	0.79	0.56	0.85	0.00	3.70	0.00
time (sec)	N/A	0.045	0.117	0.065	0.288	0.241	0.000	0.611	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	19	38	0	23	0
N.S.	1	1.00	1.00	1.16	0.59	1.19	0.00	0.72	0.00
time (sec)	N/A	0.018	0.029	0.066	0.309	0.241	0.000	0.361	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	45	33	50	0	0	34
N.S.	1	1.00	1.26	1.45	1.06	1.61	0.00	0.00	1.10
time (sec)	N/A	0.017	0.066	0.072	0.364	0.233	0.000	0.000	3.136

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	56	63	46	69	0	169	0
N.S.	1	1.00	0.85	0.95	0.70	1.05	0.00	2.56	0.00
time (sec)	N/A	0.030	0.257	0.075	0.318	0.251	0.000	0.428	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	66	74	66	82	0	225	0
N.S.	1	1.00	0.68	0.76	0.68	0.85	0.00	2.32	0.00
time (sec)	N/A	0.045	0.194	0.083	0.322	0.237	0.000	0.440	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	204	263	178	332	0	291	0
N.S.	1	1.00	0.56	0.72	0.49	0.91	0.00	0.80	0.00
time (sec)	N/A	0.179	0.678	0.173	0.339	0.236	0.000	0.504	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	115	236	140	279	0	253	0
N.S.	1	1.00	0.40	0.83	0.49	0.98	0.00	0.88	0.00
time (sec)	N/A	0.144	0.480	0.072	0.331	0.239	0.000	0.391	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	162	205	133	272	0	195	0
N.S.	1	1.00	0.64	0.80	0.52	1.07	0.00	0.76	0.00
time (sec)	N/A	0.133	0.176	0.073	0.325	0.238	0.000	0.354	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	87	211	126	307	0	251	0
N.S.	1	1.00	0.34	0.83	0.49	1.20	0.00	0.98	0.00
time (sec)	N/A	0.147	0.263	0.075	0.359	0.240	0.000	0.465	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	98	233	163	320	0	279	0
N.S.	1	1.00	0.33	0.78	0.55	1.07	0.00	0.94	0.00
time (sec)	N/A	0.149	0.212	0.075	0.490	0.237	0.000	0.618	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	139	272	172	348	0	305	0
N.S.	1	1.00	0.38	0.75	0.47	0.96	0.00	0.84	0.00
time (sec)	N/A	0.179	0.401	0.079	0.421	0.256	0.000	0.860	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	86	84	79	96	0	960	0
N.S.	1	1.00	0.47	0.46	0.43	0.53	0.00	5.27	0.00
time (sec)	N/A	0.080	0.598	0.233	0.380	0.245	0.000	6.981	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	66	64	53	62	0	992	0
N.S.	1	1.00	0.60	0.58	0.48	0.56	0.00	9.02	0.00
time (sec)	N/A	0.054	0.547	0.062	0.325	0.234	0.000	2.937	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	41	42	26	37	0	229	0
N.S.	1	1.00	0.82	0.84	0.52	0.74	0.00	4.58	0.00
time (sec)	N/A	0.027	0.075	0.063	0.302	0.231	0.000	0.458	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	43	40	27	39	0	45	0
N.S.	1	1.00	0.84	0.78	0.53	0.76	0.00	0.88	0.00
time (sec)	N/A	0.023	0.048	0.069	0.293	0.238	0.000	0.472	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	45	63	50	62	0	114	0
N.S.	1	1.00	0.38	0.53	0.42	0.52	0.00	0.96	0.00
time (sec)	N/A	0.048	0.038	0.071	0.306	0.240	0.000	0.653	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	45	83	70	82	0	169	0
N.S.	1	1.00	0.25	0.45	0.38	0.45	0.00	0.92	0.00
time (sec)	N/A	0.085	0.036	0.094	0.459	0.259	0.000	1.280	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	51	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.077	0.000	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.073	0.000	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	55	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.081	0.000	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	63	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	0.102	0.000	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	0.085	0.000	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	0.072	0.000	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	60	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	61	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	63	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	32	32	32	5979	0	23	0	0	0
N.S.	1	1.00	1.00	186.84	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.022	0.020	6.385	0.000	0.243	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.083	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	257	257	122	619	225	947	0	245	0
N.S.	1	1.00	0.47	2.41	0.88	3.68	0.00	0.95	0.00
time (sec)	N/A	0.230	0.490	13.869	0.406	0.401	0.000	0.314	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	227	227	104	529	194	934	0	219	0
N.S.	1	1.00	0.46	2.33	0.85	4.11	0.00	0.96	0.00
time (sec)	N/A	0.177	0.324	0.920	0.353	0.383	0.000	0.309	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	37	0	16	48
N.S.	1	1.00	1.00	0.94	1.28	2.06	0.00	0.89	2.67
time (sec)	N/A	0.044	0.324	0.207	0.250	0.239	0.000	0.341	3.172

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	30	43	33	63	0	43	102
N.S.	1	1.00	0.73	1.05	0.80	1.54	0.00	1.05	2.49
time (sec)	N/A	0.049	0.322	0.800	0.276	0.243	0.000	0.336	8.404

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	50	52	48	82	0	58	356
N.S.	1	1.00	0.79	0.83	0.76	1.30	0.00	0.92	5.65
time (sec)	N/A	0.055	0.366	0.751	0.303	0.267	0.000	0.355	8.128

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	105	105	139	1740	0	0	0	0	0
N.S.	1	1.00	1.32	16.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	11.785	9.250	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	57	200	0	0	0	0	0
N.S.	1	1.00	0.76	2.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	3.786	0.740	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	73	108	0	54	0	0	0
N.S.	1	1.00	1.55	2.30	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.081	0.251	0.630	0.000	0.086	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	115	218	0	113	0	0	0
N.S.	1	1.00	1.49	2.83	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.113	0.667	0.750	0.000	0.090	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	124	345	0	157	0	0	0
N.S.	1	1.00	1.18	3.29	0.00	1.50	0.00	0.00	0.00
time (sec)	N/A	0.160	1.344	0.699	0.000	0.095	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	277	277	123	890	235	954	0	252	0
N.S.	1	1.00	0.44	3.21	0.85	3.44	0.00	0.91	0.00
time (sec)	N/A	0.230	0.769	3.462	0.366	0.386	0.000	0.378	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	247	247	113	1083	204	942	0	226	0
N.S.	1	1.00	0.46	4.38	0.83	3.81	0.00	0.91	0.00
time (sec)	N/A	0.196	0.464	2.329	0.385	0.383	0.000	0.330	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	24	0	16	43
N.S.	1	1.00	1.00	0.94	1.28	1.33	0.00	0.89	2.39
time (sec)	N/A	0.045	0.342	0.159	0.245	0.234	0.000	0.324	2.882

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	30	38	34	51	0	43	100
N.S.	1	1.00	0.73	0.93	0.83	1.24	0.00	1.05	2.44
time (sec)	N/A	0.054	0.287	0.638	0.252	0.249	0.000	0.359	3.741

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	42	47	58	71	0	64	292
N.S.	1	1.00	0.67	0.75	0.92	1.13	0.00	1.02	4.63
time (sec)	N/A	0.064	0.383	0.578	0.214	0.251	0.000	0.365	7.078

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	90	409	0	0	0	0	0
N.S.	1	1.00	0.82	3.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.168	4.041	0.815	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	58	395	0	0	0	0	0
N.S.	1	1.00	0.76	5.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	0.318	0.769	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	61	381	0	136	0	0	0
N.S.	1	1.00	0.80	5.01	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.111	0.339	0.625	0.000	0.098	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	71	371	0	176	0	0	0
N.S.	1	1.00	0.70	3.64	0.00	1.73	0.00	0.00	0.00
time (sec)	N/A	0.170	0.643	0.668	0.000	0.096	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	277	277	142	526	240	1048	0	278	0
N.S.	1	1.00	0.51	1.90	0.87	3.78	0.00	1.00	0.00
time (sec)	N/A	0.238	0.830	48.450	0.315	0.416	0.000	0.354	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	247	247	126	500	209	1035	0	252	0
N.S.	1	1.00	0.51	2.02	0.85	4.19	0.00	1.02	0.00
time (sec)	N/A	0.201	0.461	3.589	0.323	0.387	0.000	0.352	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	40	0	24	56
N.S.	1	1.00	1.00	0.85	1.15	2.00	0.00	1.20	2.80
time (sec)	N/A	0.050	0.400	0.296	0.211	0.248	0.000	0.362	3.127

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	32	42	36	58	0	42	64
N.S.	1	1.00	0.78	1.02	0.88	1.41	0.00	1.02	1.56
time (sec)	N/A	0.055	0.378	4.887	0.215	0.254	0.000	0.421	3.612

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	42	55	56	82	0	70	134
N.S.	1	1.00	0.67	0.87	0.89	1.30	0.00	1.11	2.13
time (sec)	N/A	0.071	0.504	73.866	0.219	0.248	0.000	0.394	6.155

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	137	137	153	1840	0	0	0	0	0
N.S.	1	1.00	1.12	13.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	2.609	7.734	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	133	233	0	0	0	0	0
N.S.	1	1.00	1.23	2.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.148	1.773	5.351	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	215	0	103	0	0	0
N.S.	1	1.00	0.89	2.69	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.129	0.509	1.633	0.000	0.087	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	215	0	103	0	0	0
N.S.	1	1.00	0.89	2.69	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.120	0.612	2.502	0.000	0.086	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	259	0	162	0	0	0
N.S.	1	1.00	1.00	2.35	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.162	0.697	20.387	0.000	0.093	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	130	456	0	209	0	0	0
N.S.	1	1.00	0.93	3.26	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.209	1.844	151.869	0.000	0.091	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	257	257	122	603	220	892	0	246	0
N.S.	1	1.00	0.47	2.35	0.86	3.47	0.00	0.96	0.00
time (sec)	N/A	0.200	0.616	9.156	0.316	0.352	0.000	0.351	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	227	227	109	524	188	877	0	218	0
N.S.	1	1.00	0.48	2.31	0.83	3.86	0.00	0.96	0.00
time (sec)	N/A	0.176	0.391	1.731	0.332	0.396	0.000	0.349	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	46	0	23	102
N.S.	1	1.00	1.00	0.85	1.15	2.30	0.00	1.15	5.10
time (sec)	N/A	0.039	0.290	0.158	0.238	0.246	0.000	0.322	3.431

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	43	35	70	0	45	530
N.S.	1	1.00	0.93	1.00	0.81	1.63	0.00	1.05	12.33
time (sec)	N/A	0.046	0.279	0.772	0.234	0.253	0.000	0.361	7.663

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	50	52	48	93	0	58	831
N.S.	1	1.00	0.77	0.80	0.74	1.43	0.00	0.89	12.78
time (sec)	N/A	0.055	0.310	0.865	0.270	0.271	0.000	0.349	12.236

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	86	390	0	0	0	0	0
N.S.	1	1.00	0.80	3.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.154	0.938	1.116	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	98	377	0	0	0	0	0
N.S.	1	1.00	1.24	4.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.106	0.766	0.988	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	60	363	0	0	0	0	0
N.S.	1	1.00	1.28	7.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.233	0.829	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	69	349	0	169	0	0	0
N.S.	1	1.00	0.96	4.85	0.00	2.35	0.00	0.00	0.00
time (sec)	N/A	0.115	0.319	0.885	0.000	0.095	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	104	370	0	237	0	0	0
N.S.	1	1.00	1.02	3.63	0.00	2.32	0.00	0.00	0.00
time (sec)	N/A	0.158	0.702	1.033	0.000	0.097	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	257	257	123	621	225	986	0	257	0
N.S.	1	1.00	0.48	2.42	0.88	3.84	0.00	1.00	0.00
time (sec)	N/A	0.213	0.589	10.056	0.368	0.397	0.000	0.428	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	227	227	105	529	193	971	0	228	0
N.S.	1	1.00	0.46	2.33	0.85	4.28	0.00	1.00	0.00
time (sec)	N/A	0.194	0.350	13.241	0.330	0.393	0.000	0.388	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	58	0	26	381
N.S.	1	1.00	1.00	0.85	1.15	2.90	0.00	1.30	19.05
time (sec)	N/A	0.047	0.329	0.150	0.229	0.252	0.000	0.405	7.067

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	48	35	84	0	45	684
N.S.	1	1.00	0.98	1.12	0.81	1.95	0.00	1.05	15.91
time (sec)	N/A	0.053	0.282	0.700	0.250	0.254	0.000	0.434	9.492

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	54	57	48	109	0	58	987
N.S.	1	1.00	0.83	0.88	0.74	1.68	0.00	0.89	15.18
time (sec)	N/A	0.076	0.314	0.769	0.270	0.275	0.000	0.500	14.657

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	112	112	102	1048	0	0	0	0	0
N.S.	1	1.00	0.91	9.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.153	0.498	3.034	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	126	151	0	0	0	0	0
N.S.	1	1.00	1.59	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.101	0.669	2.535	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	110	195	0	119	0	0	0
N.S.	1	1.00	1.34	2.38	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.122	0.686	0.792	0.000	0.094	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	136	345	0	163	0	0	0
N.S.	1	1.00	1.21	3.08	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.189	1.545	0.865	0.000	0.089	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	257	257	123	605	219	956	0	248	0
N.S.	1	1.00	0.48	2.35	0.85	3.72	0.00	0.96	0.00
time (sec)	N/A	0.214	0.868	15.087	0.320	0.356	0.000	0.379	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	227	227	113	527	189	943	0	220	0
N.S.	1	1.00	0.50	2.32	0.83	4.15	0.00	0.97	0.00
time (sec)	N/A	0.188	0.596	14.338	0.328	0.358	0.000	0.397	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	63	0	26	530
N.S.	1	1.00	1.00	0.85	1.15	3.15	0.00	1.30	26.50
time (sec)	N/A	0.047	0.428	0.154	0.261	0.251	0.000	0.382	8.183

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	50	48	35	91	0	45	831
N.S.	1	1.00	1.16	1.12	0.81	2.12	0.00	1.05	19.33
time (sec)	N/A	0.052	0.387	0.863	0.236	0.273	0.000	0.516	13.898

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	60	57	48	114	0	58	1132
N.S.	1	1.00	0.92	0.88	0.74	1.75	0.00	0.89	17.42
time (sec)	N/A	0.065	0.462	0.952	0.226	0.313	0.000	0.512	15.050

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	122	444	0	0	0	0	0
N.S.	1	1.00	0.85	3.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	1.857	1.460	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	100	431	0	0	0	0	0
N.S.	1	1.00	0.88	3.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.167	1.274	1.177	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	97	417	0	0	0	0	0
N.S.	1	1.00	1.15	4.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	0.767	0.945	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	366	0	0	0	0	0
N.S.	1	1.00	0.88	4.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.101	0.492	0.948	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	105	373	0	246	0	0	0
N.S.	1	1.00	0.95	3.39	0.00	2.24	0.00	0.00	0.00
time (sec)	N/A	0.164	1.398	0.963	0.000	0.103	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	116	392	0	309	0	0	0
N.S.	1	1.00	0.83	2.80	0.00	2.21	0.00	0.00	0.00
time (sec)	N/A	0.217	0.910	1.004	0.000	0.103	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	51	444	0	65	0	0	80
N.S.	1	1.00	0.75	6.53	0.00	0.96	0.00	0.00	1.18
time (sec)	N/A	0.103	0.709	1.862	0.000	0.241	0.000	0.000	5.357

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	80	156	0	102	0	0	0
N.S.	1	1.00	0.91	1.77	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.126	5.605	2.746	0.000	0.087	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	71	0	47	0	0	60
N.S.	1	1.00	1.00	2.37	0.00	1.57	0.00	0.00	2.00
time (sec)	N/A	0.049	0.458	1.624	0.000	0.238	0.000	0.000	3.300

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	60	79	0	64	0	0	0
N.S.	1	1.00	1.20	1.58	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.059	0.373	1.408	0.000	0.080	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	72	164	0	413	0	0	0
N.S.	1	1.00	0.67	1.53	0.00	3.86	0.00	0.00	0.00
time (sec)	N/A	0.100	0.567	1.392	0.000	0.416	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	79	153	0	147	0	0	0
N.S.	1	1.00	0.92	1.78	0.00	1.71	0.00	0.00	0.00
time (sec)	N/A	0.119	0.592	1.591	0.000	0.091	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	99	467	0	127	0	0	0
N.S.	1	1.00	0.79	3.71	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.193	5.629	3.939	0.000	0.095	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	45	436	0	57	0	0	69
N.S.	1	1.00	0.66	6.41	0.00	0.84	0.00	0.00	1.01
time (sec)	N/A	0.127	0.450	1.786	0.000	0.250	0.000	0.000	4.109

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	83	310	0	100	0	0	0
N.S.	1	1.00	0.99	3.69	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.124	0.508	2.641	0.000	0.085	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	268	0	45	0	0	39
N.S.	1	1.00	1.00	8.93	0.00	1.50	0.00	0.00	1.30
time (sec)	N/A	0.055	0.384	2.073	0.000	0.253	0.000	0.000	3.558

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	92	329	0	103	0	0	0
N.S.	1	1.00	1.02	3.66	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.127	0.549	2.130	0.000	0.093	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	104	221	0	524	0	0	0
N.S.	1	1.00	0.72	1.52	0.00	3.61	0.00	0.00	0.00
time (sec)	N/A	0.166	0.621	1.992	0.000	0.480	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	100	464	0	136	0	0	0
N.S.	1	1.00	0.81	3.77	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.189	1.170	5.078	0.000	0.093	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	52	48	0	71	0	0	88
N.S.	1	1.00	0.76	0.71	0.00	1.04	0.00	0.00	1.29
time (sec)	N/A	0.115	0.612	0.850	0.000	0.262	0.000	0.000	5.393

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	87	432	0	117	0	0	0
N.S.	1	1.00	0.99	4.91	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.137	0.580	2.606	0.000	0.092	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	33	0	53	0	0	69
N.S.	1	1.00	1.00	1.03	0.00	1.66	0.00	0.00	2.16
time (sec)	N/A	0.075	0.502	0.826	0.000	0.264	0.000	0.000	3.948

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	69	306	0	71	0	0	0
N.S.	1	1.00	1.38	6.12	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.073	0.419	1.820	0.000	0.086	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	80	158	0	419	0	0	0
N.S.	1	1.00	0.75	1.49	0.00	3.95	0.00	0.00	0.00
time (sec)	N/A	0.129	0.404	0.773	0.000	0.406	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	89	275	0	157	0	0	0
N.S.	1	1.00	1.02	3.16	0.00	1.80	0.00	0.00	0.00
time (sec)	N/A	0.167	0.631	1.529	0.000	0.095	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	112	292	0	605	0	0	0
N.S.	1	1.00	0.77	2.00	0.00	4.14	0.00	0.00	0.00
time (sec)	N/A	0.209	0.831	1.147	0.000	0.482	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	67	63	0	84	0	0	296
N.S.	1	1.00	0.46	0.43	0.00	0.58	0.00	0.00	2.03
time (sec)	N/A	0.380	1.800	0.951	0.000	0.276	0.000	0.000	9.517

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	57	53	0	71	0	0	94
N.S.	1	1.00	0.52	0.49	0.00	0.65	0.00	0.00	0.86
time (sec)	N/A	0.180	0.958	1.006	0.000	0.264	0.000	0.000	6.002

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	45	40	0	55	0	0	81
N.S.	1	1.00	1.41	1.25	0.00	1.72	0.00	0.00	2.53
time (sec)	N/A	0.062	0.595	0.845	0.000	0.254	0.000	0.000	4.821

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	88	207	0	529	0	0	0
N.S.	1	1.00	0.62	1.47	0.00	3.75	0.00	0.00	0.00
time (sec)	N/A	0.165	0.624	1.108	0.000	0.478	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	103	288	0	608	0	0	0
N.S.	1	1.00	0.68	1.91	0.00	4.03	0.00	0.00	0.00
time (sec)	N/A	0.170	0.597	1.098	0.000	0.468	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	118	178	0	140	0	0	0
N.S.	1	1.00	0.71	1.07	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.248	1.631	6.367	0.000	0.100	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	97	256	0	126	0	0	0
N.S.	1	1.00	0.75	1.97	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.189	0.846	3.795	0.000	0.089	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	80	131	0	104	0	0	0
N.S.	1	1.00	0.86	1.41	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.129	0.594	2.099	0.000	0.089	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	79	152	0	149	0	0	0
N.S.	1	1.00	0.92	1.77	0.00	1.73	0.00	0.00	0.00
time (sec)	N/A	0.122	0.506	1.794	0.000	0.093	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	96	171	0	200	0	0	0
N.S.	1	1.00	0.74	1.32	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.188	0.679	1.906	0.000	0.100	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	106	193	0	264	0	0	0
N.S.	1	1.00	0.63	1.16	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.279	0.953	2.543	0.000	0.114	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	10.770	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.115	10.616	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	10.581	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	67	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	10.709	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	63	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.128	10.753	0.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	63	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.108	10.645	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	63	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	10.629	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	10.829	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	11.213	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	66	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.099	10.573	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	66	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	10.643	0.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	121	0	0	0	0	0	0
N.S.	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	31.134	0.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	85	0	0	0	0	0	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.118	10.766	0.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	72	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	10.694	0.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	67	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.121	10.864	0.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	70	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.110	10.705	0.000	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	53	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.065	0.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	53	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	0.026	0.000	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	18	17	0	17	17
N.S.	1	1.00	1.00	1.06	1.06	1.00	0.00	1.00	1.00
time (sec)	N/A	0.033	0.007	0.480	0.290	0.263	0.000	0.308	2.887

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	46	46	37	2751	47	57	0	0	91
N.S.	1	1.00	0.80	59.80	1.02	1.24	0.00	0.00	1.98
time (sec)	N/A	0.054	0.045	1.190	0.254	0.255	0.000	0.000	4.088

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	72	72	62	6931	71	112	0	0	219
N.S.	1	1.00	0.86	96.26	0.99	1.56	0.00	0.00	3.04
time (sec)	N/A	0.071	0.264	2.018	0.267	0.262	0.000	0.000	8.690

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	71	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	0.094	0.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	71	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.101	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	66	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.110	0.062	0.000	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.107	0.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.137	0.890	0.000	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	0.593	0.000	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	0.545	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	0.939	0.000	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	260	0	0	0	0	0	0
N.S.	1	1.00	3.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.107	1.823	0.000	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	53	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.426	0.000	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	53	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.289	0.000	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	30	28	42	0	0	53
N.S.	1	1.00	0.88	1.20	1.12	1.68	0.00	0.00	2.12
time (sec)	N/A	0.049	0.281	1.824	0.293	0.249	0.000	0.000	2.997

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	53	53	46	5281	55	86	0	0	138
N.S.	1	1.00	0.87	99.64	1.04	1.62	0.00	0.00	2.60
time (sec)	N/A	0.062	0.290	20.026	0.291	0.248	0.000	0.000	4.743

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	80	80	69	10580	81	144	0	0	0
N.S.	1	1.00	0.86	132.25	1.01	1.80	0.00	0.00	0.00
time (sec)	N/A	0.070	0.390	151.818	0.288	0.255	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	456	0	0	0	0	0	0
N.S.	1	1.00	5.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	2.297	0.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	252	0	0	0	0	0	0
N.S.	1	1.00	3.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	1.575	0.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	64	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.079	0.298	0.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	743	0	0	0	0	0	0
N.S.	1	1.00	9.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.086	13.223	0.000	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	1017	0	0	0	0	0	0
N.S.	1	1.00	13.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.088	14.172	0.000	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	297	0	0	0	0	0	0
N.S.	1	1.00	3.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	32.845	0.000	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	91	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	11.648	0.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.122	11.343	0.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	90	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.132	9.652	0.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	81	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.104	0.616	0.000	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	64	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	0.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	232	232	109	707	207	212	0	0	97
N.S.	1	1.00	0.47	3.05	0.89	0.91	0.00	0.00	0.42
time (sec)	N/A	0.253	0.581	17.605	0.331	0.275	0.000	0.000	2.814

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	214	214	91	568	190	187	0	0	83
N.S.	1	1.00	0.43	2.65	0.89	0.87	0.00	0.00	0.39
time (sec)	N/A	0.229	0.165	17.697	0.371	0.248	0.000	0.000	2.875

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	210	210	80	656	189	200	0	0	80
N.S.	1	1.00	0.38	3.12	0.90	0.95	0.00	0.00	0.38
time (sec)	N/A	0.206	0.150	18.903	0.327	0.263	0.000	0.000	3.600

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	192	192	132	448	167	156	0	0	61
N.S.	1	1.00	0.69	2.33	0.87	0.81	0.00	0.00	0.32
time (sec)	N/A	0.164	0.142	14.924	0.465	0.258	0.000	0.000	0.251

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	71	136	165	232	0	0	50
N.S.	1	1.00	0.37	0.71	0.86	1.21	0.00	0.00	0.26
time (sec)	N/A	0.136	0.070	0.151	0.594	0.264	0.000	0.000	3.028

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	164	149	178	253	0	0	74
N.S.	1	1.00	0.78	0.71	0.85	1.21	0.00	0.00	0.35
time (sec)	N/A	0.197	0.142	0.125	0.347	0.263	0.000	0.000	2.960

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	100	156	187	328	0	0	76
N.S.	1	1.00	0.47	0.73	0.87	1.53	0.00	0.00	0.36
time (sec)	N/A	0.201	0.212	0.133	0.318	0.275	0.000	0.000	3.102

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	172	171	199	339	0	0	90
N.S.	1	1.00	0.74	0.74	0.86	1.47	0.00	0.00	0.39
time (sec)	N/A	0.244	0.302	0.129	0.335	0.265	0.000	0.000	3.782

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	234	234	109	632	207	223	0	0	97
N.S.	1	1.00	0.47	2.70	0.88	0.95	0.00	0.00	0.41
time (sec)	N/A	0.265	0.563	3.407	0.475	0.263	0.000	0.000	2.784

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	214	214	91	569	190	196	0	0	83
N.S.	1	1.00	0.43	2.66	0.89	0.92	0.00	0.00	0.39
time (sec)	N/A	0.214	0.178	2.089	0.533	0.278	0.000	0.000	2.692

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	212	212	82	572	189	209	0	0	82
N.S.	1	1.00	0.39	2.70	0.89	0.99	0.00	0.00	0.39
time (sec)	N/A	0.206	0.149	3.089	0.557	0.261	0.000	0.000	3.108

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	192	192	134	448	167	164	0	0	61
N.S.	1	1.00	0.70	2.33	0.87	0.85	0.00	0.00	0.32
time (sec)	N/A	0.175	0.073	2.046	0.338	0.255	0.000	0.000	3.055

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	72	138	167	180	0	0	54
N.S.	1	1.00	0.38	0.72	0.87	0.94	0.00	0.00	0.28
time (sec)	N/A	0.165	0.059	0.187	0.385	0.259	0.000	0.000	2.864

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	161	149	179	262	0	0	75
N.S.	1	1.00	0.77	0.71	0.85	1.25	0.00	0.00	0.36
time (sec)	N/A	0.159	0.089	0.122	0.346	0.252	0.000	0.000	3.261

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	100	152	184	338	0	0	73
N.S.	1	1.00	0.47	0.72	0.87	1.60	0.00	0.00	0.35
time (sec)	N/A	0.183	0.180	0.130	0.520	0.253	0.000	0.000	3.051

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	172	171	199	350	0	0	91
N.S.	1	1.00	0.74	0.74	0.86	1.51	0.00	0.00	0.39
time (sec)	N/A	0.225	0.198	0.135	0.470	0.261	0.000	0.000	3.646

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	94	714	207	223	0	0	97
N.S.	1	1.00	0.41	3.09	0.90	0.97	0.00	0.00	0.42
time (sec)	N/A	0.231	0.242	3.461	0.686	0.254	0.000	0.000	3.044

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	212	212	84	484	190	198	0	0	81
N.S.	1	1.00	0.40	2.28	0.90	0.93	0.00	0.00	0.38
time (sec)	N/A	0.207	0.192	2.273	0.348	0.268	0.000	0.000	2.985

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	79	663	189	211	0	0	79
N.S.	1	1.00	0.38	3.17	0.90	1.01	0.00	0.00	0.38
time (sec)	N/A	0.189	0.044	3.127	0.328	0.234	0.000	0.000	0.223

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	131	138	165	220	0	0	57
N.S.	1	1.00	0.68	0.72	0.86	1.15	0.00	0.00	0.30
time (sec)	N/A	0.138	0.053	0.189	0.352	0.243	0.000	0.000	2.955

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	74	135	164	236	0	0	58
N.S.	1	1.00	0.39	0.70	0.85	1.23	0.00	0.00	0.30
time (sec)	N/A	0.148	0.004	0.187	0.452	0.246	0.000	0.000	3.097

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	161	151	181	264	0	0	77
N.S.	1	1.00	0.76	0.71	0.85	1.25	0.00	0.00	0.36
time (sec)	N/A	0.193	0.098	0.148	0.550	0.257	0.000	0.000	3.409

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	100	156	187	339	0	0	76
N.S.	1	1.00	0.47	0.73	0.87	1.58	0.00	0.00	0.36
time (sec)	N/A	0.185	0.122	0.150	0.719	0.255	0.000	0.000	3.030

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	232	232	97	734	207	231	0	0	93
N.S.	1	1.00	0.42	3.16	0.89	1.00	0.00	0.00	0.40
time (sec)	N/A	0.239	0.216	3.276	0.335	0.252	0.000	0.000	3.081

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	211	211	83	571	190	214	0	0	80
N.S.	1	1.00	0.39	2.71	0.90	1.01	0.00	0.00	0.38
time (sec)	N/A	0.214	0.097	2.717	0.354	0.262	0.000	0.000	0.322

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	82	157	187	379	0	0	76
N.S.	1	1.00	0.39	0.74	0.88	1.79	0.00	0.00	0.36
time (sec)	N/A	0.171	0.001	0.141	0.355	0.248	0.000	0.000	3.787

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	134	138	164	228	0	0	57
N.S.	1	1.00	0.70	0.72	0.85	1.19	0.00	0.00	0.30
time (sec)	N/A	0.161	0.016	0.194	0.601	0.245	0.000	0.000	2.846

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	74	138	167	236	0	0	58
N.S.	1	1.00	0.39	0.72	0.87	1.23	0.00	0.00	0.30
time (sec)	N/A	0.163	0.004	0.184	0.449	0.242	0.000	0.000	2.745

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	161	151	181	280	0	0	77
N.S.	1	1.00	0.76	0.71	0.85	1.32	0.00	0.00	0.36
time (sec)	N/A	0.191	0.115	0.158	0.345	0.247	0.000	0.000	3.339

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	100	156	187	347	0	0	76
N.S.	1	1.00	0.47	0.73	0.87	1.62	0.00	0.00	0.36
time (sec)	N/A	0.197	0.157	0.142	0.338	0.252	0.000	0.000	3.554

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	172	171	199	381	0	0	93
N.S.	1	1.00	0.74	0.73	0.85	1.63	0.00	0.00	0.40
time (sec)	N/A	0.233	0.330	0.148	0.317	0.257	0.000	0.000	3.321

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	64	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.087	0.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.085	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	67	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.113	0.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	52	52	51	59	0	76	334
N.S.	1	1.00	0.78	0.78	0.76	0.88	0.00	1.13	4.99
time (sec)	N/A	0.084	0.394	0.331	0.333	0.263	0.000	0.371	8.455

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	34	37	36	49	0	53	218
N.S.	1	1.00	0.76	0.82	0.80	1.09	0.00	1.18	4.84
time (sec)	N/A	0.073	0.268	0.253	0.639	0.270	0.000	0.357	7.261

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	37	0	21	53
N.S.	1	1.00	1.00	0.86	0.82	1.68	0.00	0.95	2.41
time (sec)	N/A	0.061	0.026	0.129	0.202	0.252	0.000	0.337	3.297

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	71	136	153	164	0	176	49
N.S.	1	1.00	0.37	0.71	0.80	0.85	0.00	0.92	0.26
time (sec)	N/A	0.191	0.057	0.166	0.284	0.270	0.000	0.317	3.406

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	227	227	102	528	193	924	0	218	0
N.S.	1	1.00	0.45	2.33	0.85	4.07	0.00	0.96	0.00
time (sec)	N/A	0.255	0.330	1.166	0.314	0.404	0.000	0.312	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	102	401	0	184	0	0	0
N.S.	1	1.00	0.95	3.75	0.00	1.72	0.00	0.00	0.00
time (sec)	N/A	0.227	0.489	1.461	0.000	0.103	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	61	367	0	131	0	0	0
N.S.	1	1.00	0.81	4.89	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.099	0.282	1.416	0.000	0.097	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	57	377	0	0	0	0	0
N.S.	1	1.00	1.21	8.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.077	0.179	0.946	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	94	390	0	0	0	0	0
N.S.	1	1.00	1.16	4.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	0.527	1.077	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	86	404	0	0	0	0	0
N.S.	1	1.00	0.77	3.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.172	0.832	1.375	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	52	52	51	68	0	78	392
N.S.	1	1.00	0.78	0.78	0.76	1.01	0.00	1.16	5.85
time (sec)	N/A	0.065	0.334	0.394	0.341	0.297	0.000	0.393	11.537

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	37	36	56	0	55	276
N.S.	1	1.00	0.93	0.82	0.80	1.24	0.00	1.22	6.13
time (sec)	N/A	0.060	0.234	0.323	0.426	0.281	0.000	0.351	8.382

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	45	0	24	100
N.S.	1	1.00	1.00	0.86	0.82	2.05	0.00	1.09	4.55
time (sec)	N/A	0.052	0.041	0.224	0.225	0.279	0.000	0.334	4.379

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	160	149	170	177	0	0	73
N.S.	1	1.00	0.76	0.71	0.81	0.84	0.00	0.00	0.35
time (sec)	N/A	0.172	0.179	0.175	0.325	0.246	0.000	0.000	3.142

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	225	225	110	525	188	927	0	210	0
N.S.	1	1.00	0.49	2.33	0.84	4.12	0.00	0.93	0.00
time (sec)	N/A	0.191	0.408	2.173	0.312	0.384	0.000	0.321	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	90	265	0	127	0	0	0
N.S.	1	1.00	0.66	1.95	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.223	0.769	1.669	0.000	0.103	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	80	244	0	114	0	0	0
N.S.	1	1.00	0.74	2.26	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.188	0.497	1.473	0.000	0.095	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	69	221	0	97	0	0	0
N.S.	1	1.00	0.86	2.76	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.113	0.331	1.386	0.000	0.088	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	219	0	0	0	0	0
N.S.	1	1.00	0.74	2.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.119	0.213	3.019	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	108	108	96	1939	0	0	0	0	0
N.S.	1	1.00	0.89	17.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.179	0.986	3.336	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	136	136	131	1958	0	0	0	0	0
N.S.	1	1.00	0.96	14.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	2.077	6.391	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	52	52	51	82	0	78	474
N.S.	1	1.00	0.78	0.78	0.76	1.22	0.00	1.16	7.07
time (sec)	N/A	0.063	0.645	0.214	0.390	0.301	0.000	0.415	18.326

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	37	36	69	0	55	352
N.S.	1	1.00	0.93	0.82	0.80	1.53	0.00	1.22	7.82
time (sec)	N/A	0.059	0.363	236.850	0.201	0.292	0.000	0.401	8.533

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	55	0	26	230
N.S.	1	1.00	1.00	0.86	0.82	2.50	0.00	1.18	10.45
time (sec)	N/A	0.049	0.037	1.696	0.204	0.257	0.000	0.357	6.706

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	101	154	176	202	0	0	74
N.S.	1	1.00	0.48	0.73	0.83	0.95	0.00	0.00	0.35
time (sec)	N/A	0.178	0.164	0.165	0.305	0.266	0.000	0.000	3.421

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	225	225	107	551	194	969	0	231	0
N.S.	1	1.00	0.48	2.45	0.86	4.31	0.00	1.03	0.00
time (sec)	N/A	0.197	0.400	2.482	0.298	0.423	0.000	0.425	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	253	253	125	641	225	985	0	257	0
N.S.	1	1.00	0.49	2.53	0.89	3.89	0.00	1.02	0.00
time (sec)	N/A	0.208	0.320	59.813	0.595	0.427	0.000	0.430	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	79	219	0	115	0	0	0
N.S.	1	1.00	0.72	2.01	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.154	0.679	1.497	0.000	0.093	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	68	200	0	97	0	0	0
N.S.	1	1.00	0.86	2.53	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.120	0.394	1.436	0.000	0.092	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	77	103	0	57	0	0	0
N.S.	1	1.00	1.64	2.19	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.071	0.185	0.967	0.000	0.087	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	126	192	0	0	0	0	0
N.S.	1	1.00	1.66	2.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.108	0.535	3.098	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	109	109	94	1906	0	0	0	0	0
N.S.	1	1.00	0.86	17.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.154	0.993	6.227	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	41	54	64	0	80	268
N.S.	1	1.00	0.69	0.63	0.83	0.98	0.00	1.23	4.12
time (sec)	N/A	0.071	0.534	1.559	0.257	0.291	0.000	0.522	7.656

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	31	36	54	0	44	64
N.S.	1	1.00	0.74	0.72	0.84	1.26	0.00	1.02	1.49
time (sec)	N/A	0.059	0.277	1.224	0.250	0.254	0.000	0.482	3.440

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	40	0	18	51
N.S.	1	1.00	1.00	0.95	0.90	2.00	0.00	0.90	2.55
time (sec)	N/A	0.049	0.045	0.252	0.239	0.267	0.000	0.425	2.922

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	82	157	167	235	0	0	76
N.S.	1	1.00	0.39	0.74	0.79	1.11	0.00	0.00	0.36
time (sec)	N/A	0.155	0.090	0.161	0.578	0.246	0.000	0.000	2.825

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	249	249	115	937	204	1034	0	252	0
N.S.	1	1.00	0.46	3.76	0.82	4.15	0.00	1.01	0.00
time (sec)	N/A	0.208	0.495	19.772	0.513	0.425	0.000	0.415	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	104	380	0	222	0	0	0
N.S.	1	1.00	0.75	2.75	0.00	1.61	0.00	0.00	0.00
time (sec)	N/A	0.194	0.876	1.615	0.000	0.104	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	93	367	0	172	0	0	0
N.S.	1	1.00	0.89	3.53	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	0.161	0.539	1.483	0.000	0.095	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	352	0	169	0	0	0
N.S.	1	1.00	0.88	4.51	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	0.114	0.428	1.111	0.000	0.120	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	366	0	0	0	0	0
N.S.	1	1.00	0.85	4.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.115	0.408	1.106	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	77	380	0	0	0	0	0
N.S.	1	1.00	0.69	3.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.187	0.639	1.489	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	89	393	0	0	0	0	0
N.S.	1	1.00	0.63	2.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	0.910	1.388	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	113	299	0	119	0	0	0
N.S.	1	1.00	1.38	3.65	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.099	0.741	1.095	0.000	0.095	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	103	372	0	241	0	0	0
N.S.	1	1.00	0.94	3.38	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	0.172	1.989	1.281	0.000	0.102	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	77	0	0	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	0.283	0.000	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	56	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.065	0.152	0.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	56	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	0.139	0.000	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	56	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	0.134	0.000	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	54	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.066	0.126	0.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	89	0	0	0	0	0	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	0.941	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	78	0	0	0	0	0	0
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.072	0.755	0.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	77	0	0	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.066	0.659	0.000	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	78	0	0	0	0	0	0
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	0.261	0.000	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	77	0	0	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.072	0.284	0.000	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.084	0.000	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	0.110	0.000	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	0.111	0.000	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	58	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.169	0.000	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.124	0.000	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.112	0.000	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.298	0.000	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	59	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.065	0.291	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	131	195	0	788	0	0	0
N.S.	1	1.00	0.74	1.10	0.00	4.43	0.00	0.00	0.00
time (sec)	N/A	0.239	0.823	17.762	0.000	0.402	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	73	454	0	105	0	0	0
N.S.	1	1.00	0.78	4.88	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.125	10.548	2.479	0.000	0.097	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	101	129	0	654	0	0	0
N.S.	1	1.00	0.77	0.98	0.00	4.95	0.00	0.00	0.00
time (sec)	N/A	0.114	0.551	17.512	0.000	0.375	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	66	342	0	66	0	0	0
N.S.	1	1.00	1.20	6.22	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.067	0.518	2.737	0.000	0.088	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	35	0	50	53	0	55
N.S.	1	1.00	1.00	1.03	0.00	1.47	1.56	0.00	1.62
time (sec)	N/A	0.054	0.457	1.334	0.000	0.258	4.948	0.000	3.747

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	70	454	0	112	0	0	0
N.S.	1	1.00	0.74	4.78	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.145	0.917	2.902	0.000	0.091	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	53	47	0	63	0	0	69
N.S.	1	1.00	0.74	0.65	0.00	0.88	0.00	0.00	0.96
time (sec)	N/A	0.124	0.709	1.557	0.000	0.250	0.000	0.000	3.797

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	86	468	0	125	0	0	0
N.S.	1	1.00	0.65	3.55	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.205	1.144	2.596	0.000	0.099	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	83	295	0	140	0	0	0
N.S.	1	1.00	0.63	2.25	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.207	5.621	2.604	0.000	0.097	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	130	189	0	769	0	0	0
N.S.	1	1.00	0.77	1.12	0.00	4.55	0.00	0.00	0.00
time (sec)	N/A	0.206	0.900	20.763	0.000	0.374	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	65	272	0	91	0	0	0
N.S.	1	1.00	0.74	3.09	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.130	0.565	2.668	0.000	0.089	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	66	292	0	741	0	0	0
N.S.	1	1.00	0.40	1.75	0.00	4.44	0.00	0.00	0.00
time (sec)	N/A	0.141	0.703	19.504	0.000	0.548	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	67	272	0	104	0	0	0
N.S.	1	1.00	0.70	2.83	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.134	0.641	2.550	0.000	0.089	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	43	38	0	58	53	0	65
N.S.	1	1.00	1.26	1.12	0.00	1.71	1.56	0.00	1.91
time (sec)	N/A	0.059	0.949	1.456	0.000	0.250	45.793	0.000	3.706

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	81	292	0	116	0	0	0
N.S.	1	1.00	0.62	2.23	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.211	0.932	3.122	0.000	0.092	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	55	50	0	70	0	0	78
N.S.	1	1.00	0.53	0.49	0.00	0.68	0.00	0.00	0.76
time (sec)	N/A	0.179	0.994	1.403	0.000	0.279	0.000	0.000	3.955

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	153	300	0	852	0	0	0
N.S.	1	1.00	0.74	1.44	0.00	4.10	0.00	0.00	0.00
time (sec)	N/A	0.252	2.331	169.046	0.000	0.447	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	80	487	0	155	0	0	0
N.S.	1	1.00	0.61	3.72	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.195	0.737	1.891	0.000	0.093	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	130	233	0	788	0	0	0
N.S.	1	1.00	0.77	1.38	0.00	4.66	0.00	0.00	0.00
time (sec)	N/A	0.174	0.738	12.085	0.000	0.386	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	65	471	0	114	0	0	0
N.S.	1	1.00	0.74	5.35	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.140	0.635	2.212	0.000	0.095	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	140	168	0	766	0	0	0
N.S.	1	1.00	0.83	1.00	0.00	4.56	0.00	0.00	0.00
time (sec)	N/A	0.199	1.012	10.714	0.000	0.593	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	79	458	0	122	0	0	0
N.S.	1	1.00	0.82	4.77	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.151	0.836	1.852	0.000	0.093	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	45	40	0	68	0	0	72
N.S.	1	1.00	1.32	1.18	0.00	2.00	0.00	0.00	2.12
time (sec)	N/A	0.066	0.878	1.392	0.000	0.246	0.000	0.000	3.923

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	91	472	0	137	0	0	0
N.S.	1	1.00	0.69	3.60	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.225	1.266	2.188	0.000	0.104	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	133	203	0	782	0	0	0
N.S.	1	1.00	0.75	1.14	0.00	4.39	0.00	0.00	0.00
time (sec)	N/A	0.206	1.031	12.214	0.000	0.421	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	83	239	0	100	0	0	0
N.S.	1	1.00	0.90	2.60	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.138	0.657	1.873	0.000	0.082	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	99	133	0	653	0	0	0
N.S.	1	1.00	0.76	1.02	0.00	4.98	0.00	0.00	0.00
time (sec)	N/A	0.128	0.674	10.273	0.000	0.388	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	68	130	0	58	0	0	0
N.S.	1	1.00	1.24	2.36	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.071	0.541	2.008	0.000	0.081	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	32	0	47	51	0	52
N.S.	1	1.00	1.00	1.00	0.00	1.47	1.59	0.00	1.62
time (sec)	N/A	0.053	0.627	1.273	0.000	0.241	4.291	0.000	3.505

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	70	239	0	101	0	0	0
N.S.	1	1.00	0.74	2.52	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.133	0.737	1.929	0.000	0.092	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	50	45	0	58	88	0	64
N.S.	1	1.00	0.69	0.62	0.00	0.81	1.22	0.00	0.89
time (sec)	N/A	0.118	0.798	1.522	0.000	0.240	57.254	0.000	3.721

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	118	239	0	794	0	0	0
N.S.	1	1.00	0.69	1.40	0.00	4.64	0.00	0.00	0.00
time (sec)	N/A	0.201	1.076	12.633	0.000	0.411	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	97	97	80	365	0	130	0	0	0
N.S.	1	1.00	0.82	3.76	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.144	0.816	1.978	0.000	0.086	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	0	52	53	0	46
N.S.	1	1.00	1.00	0.91	0.00	1.62	1.66	0.00	1.44
time (sec)	N/A	0.056	0.436	1.446	0.000	0.242	1.581	0.000	3.690

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	78	365	0	129	0	0	0
N.S.	1	1.00	0.86	4.01	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.132	0.644	2.239	0.000	0.092	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	67	52	47	0	66	90	0	60
N.S.	1	0.93	0.72	0.65	0.00	0.92	1.25	0.00	0.83
time (sec)	N/A	0.124	0.604	1.474	0.000	0.243	26.170	0.000	3.861

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	89	452	0	141	0	0	0
N.S.	1	1.00	0.68	3.48	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.200	0.889	2.451	0.000	0.104	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	151	235	0	850	0	0	0
N.S.	1	1.00	0.88	1.37	0.00	4.94	0.00	0.00	0.00
time (sec)	N/A	0.201	1.225	17.886	0.000	0.468	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	81	151	0	154	0	0	0
N.S.	1	1.00	0.80	1.50	0.00	1.52	0.00	0.00	0.00
time (sec)	N/A	0.139	0.922	2.270	0.000	0.092	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	36	0	61	54	0	55
N.S.	1	1.00	1.00	1.06	0.00	1.79	1.59	0.00	1.62
time (sec)	N/A	0.065	0.609	1.267	0.000	0.270	44.265	0.000	4.114

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	80	239	0	136	0	0	0
N.S.	1	1.00	0.84	2.52	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.127	0.820	2.069	0.000	0.090	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	44	52	0	75	92	0	81
N.S.	1	1.00	0.64	0.75	0.00	1.09	1.33	0.00	1.17
time (sec)	N/A	0.114	0.789	1.356	0.000	0.266	56.796	0.000	4.306

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	95	259	0	153	0	0	0
N.S.	1	1.00	0.72	1.96	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.198	1.086	2.092	0.000	0.100	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	65	66	0	89	0	0	93
N.S.	1	1.00	0.61	0.62	0.00	0.84	0.00	0.00	0.88
time (sec)	N/A	0.185	1.125	1.390	0.000	0.260	0.000	0.000	5.107

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.076	0.000	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.075	0.000	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.086	0.000	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	0.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	0.097	0.000	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	0.050	0.000	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	71	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.068	0.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	0.101	0.000	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	0.064	0.000	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.089	0.000	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.171	0.000	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	0.107	0.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	0.080	0.000	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	64	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.086	0.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	67	67	62	6067	77	80	0	0	199
N.S.	1	1.00	0.93	90.55	1.15	1.19	0.00	0.00	2.97
time (sec)	N/A	0.087	0.329	8.101	0.327	0.261	0.000	0.000	8.878

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	43	43	37	2423	51	50	0	0	87
N.S.	1	1.00	0.86	56.35	1.19	1.16	0.00	0.00	2.02
time (sec)	N/A	0.070	0.151	2.058	0.361	0.258	0.000	0.000	4.058

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	20	19	42	0	19
N.S.	1	1.00	1.00	1.06	1.18	1.12	2.47	0.00	1.12
time (sec)	N/A	0.031	0.056	0.462	0.538	0.281	0.165	0.000	0.137

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.051	0.000	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.123	0.000	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	0.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	0.149	0.000	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	62	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.059	0.184	0.000	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.164	0.000	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	62	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.438	0.000	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	80	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.110	0.000	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	101	88	77	85	0	0	0
N.S.	1	1.00	1.36	1.19	1.04	1.15	0.00	0.00	0.00
time (sec)	N/A	0.081	1.615	0.339	0.290	0.265	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	78	58	51	61	0	0	139
N.S.	1	1.00	1.59	1.18	1.04	1.24	0.00	0.00	2.84
time (sec)	N/A	0.065	0.863	56.741	0.283	0.259	0.000	0.000	4.538

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	25	25	24	40	0	0	49
N.S.	1	1.00	1.04	1.04	1.00	1.67	0.00	0.00	2.04
time (sec)	N/A	0.041	0.015	2.755	0.386	0.251	0.000	0.000	3.278

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	51	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	0.078	0.000	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	51	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.399	0.000	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	51	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.341	0.000	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.046	0.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	0.067	0.000	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	64	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.051	0.000	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	452	0	0	0	0	0	0
N.S.	1	1.00	6.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	2.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	1313	0	0	0	0	0	0
N.S.	1	1.00	16.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	4.847	0.000	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	52	0	0	0	0	0	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	0.143	0.000	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	39	39	52	0	0	0	0	0	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	21	20	54	20	43
N.S.	1	1.00	1.00	1.06	1.17	1.11	3.00	1.11	2.39
time (sec)	N/A	0.024	0.022	0.281	0.623	0.263	0.179	0.354	3.254

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	43	43	36	3514	50	60	0	0	92
N.S.	1	1.00	0.84	81.72	1.16	1.40	0.00	0.00	2.14
time (sec)	N/A	0.053	0.066	1.159	0.308	0.268	0.000	0.000	4.149

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	69	69	63	8846	78	115	0	0	222
N.S.	1	1.00	0.91	128.20	1.13	1.67	0.00	0.00	3.22
time (sec)	N/A	0.068	0.245	2.139	0.285	0.260	0.000	0.000	8.935

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	240	0	0	0	0	0	0
N.S.	1	1.00	3.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.046	8.730	0.000	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	79	0	0	0	0	0	0
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	1.474	0.000	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	186	0	0	0	0	0	0
N.S.	1	1.00	2.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	1.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	106	0	0	0	0	0	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	0.561	0.000	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	1.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.711	0.000	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.196	0.735	0.000	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.201	0.790	0.000	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	287	0	0	0	0	0	0
N.S.	1	1.00	3.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.176	2.004	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [9] had the largest ratio of [.7500000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	2	1.00	8	0.250
4	A	3	2	1.00	8	0.250
5	A	3	2	1.00	8	0.250
6	A	4	2	1.00	8	0.250
7	A	4	2	1.00	8	0.250
8	A	5	2	1.00	8	0.250
9	A	13	9	1.00	12	0.750
10	A	12	9	1.00	12	0.750
11	A	12	9	1.00	12	0.750
12	A	11	8	1.00	12	0.667
13	A	11	8	1.00	12	0.667
14	A	12	9	1.00	12	0.750
15	A	12	9	1.00	12	0.750
16	A	13	9	1.00	12	0.750
17	A	13	9	1.00	12	0.750
18	A	12	8	1.00	12	0.667
19	A	9	9	1.00	12	0.750
20	A	9	9	1.00	12	0.750
21	A	12	8	1.00	12	0.667
22	A	13	9	1.00	12	0.750
23	A	2	2	1.00	10	0.200
24	A	4	3	1.00	14	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	3	1.00	14	0.214
26	A	2	2	1.00	14	0.143
27	A	2	2	1.00	14	0.143
28	A	3	3	1.00	14	0.214
29	A	4	3	1.00	14	0.214
30	A	16	10	1.00	14	0.714
31	A	14	10	1.00	14	0.714
32	A	13	10	1.00	14	0.714
33	A	13	10	1.00	14	0.714
34	A	14	10	1.00	14	0.714
35	A	16	10	1.00	14	0.714
36	A	7	3	1.00	14	0.214
37	A	5	3	1.00	14	0.214
38	A	3	3	1.00	14	0.214
39	A	3	3	1.00	14	0.214
40	A	5	3	1.00	14	0.214
41	A	7	3	1.00	14	0.214
42	A	3	3	1.00	12	0.250
43	A	3	3	1.00	12	0.250
44	A	3	3	1.00	12	0.250
45	A	3	3	1.00	12	0.250
46	A	3	3	1.00	14	0.214
47	A	3	3	1.00	14	0.214
48	A	3	3	1.00	14	0.214
49	A	3	3	1.00	14	0.214
50	A	3	3	1.00	14	0.214
51	A	3	3	1.00	14	0.214
52	A	2	2	1.00	14	0.143
53	A	3	3	1.00	14	0.214
54	A	13	9	1.00	21	0.429
55	A	12	9	1.00	21	0.429
56	A	2	2	1.00	21	0.095
57	A	3	2	1.00	21	0.095
58	A	3	2	1.00	21	0.095
59	A	5	4	1.00	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	4	4	1.00	19	0.210
61	A	3	3	1.00	19	0.158
62	A	4	4	1.00	21	0.190
63	A	5	4	1.00	21	0.190
64	A	14	10	1.00	21	0.476
65	A	13	10	1.00	21	0.476
66	A	2	2	1.00	21	0.095
67	A	3	2	1.00	21	0.095
68	A	3	2	1.00	21	0.095
69	A	5	5	1.00	21	0.238
70	A	4	4	1.00	19	0.210
71	A	4	4	1.00	19	0.210
72	A	5	5	1.00	21	0.238
73	A	14	10	1.00	21	0.476
74	A	13	10	1.00	21	0.476
75	A	2	2	1.00	21	0.095
76	A	3	2	1.00	21	0.095
77	A	3	2	1.00	21	0.095
78	A	6	5	1.00	21	0.238
79	A	5	5	1.00	19	0.263
80	A	4	4	1.00	19	0.210
81	A	4	4	1.00	21	0.190
82	A	5	5	1.00	21	0.238
83	A	6	5	1.00	21	0.238
84	A	13	9	1.00	21	0.429
85	A	12	9	1.00	21	0.429
86	A	2	2	1.00	21	0.095
87	A	3	2	1.00	21	0.095
88	A	3	2	1.00	21	0.095
89	A	5	4	1.00	21	0.190
90	A	4	4	1.00	21	0.190
91	A	3	3	1.00	19	0.158
92	A	4	4	1.00	19	0.210
93	A	5	5	1.00	21	0.238
94	A	13	10	1.00	21	0.476

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	12	9	1.00	21	0.429
96	A	2	2	1.00	21	0.095
97	A	3	2	1.00	21	0.095
98	A	3	2	1.00	21	0.095
99	A	5	5	1.00	21	0.238
100	A	4	4	1.00	19	0.210
101	A	4	4	1.00	19	0.210
102	A	5	5	1.00	21	0.238
103	A	13	10	1.00	21	0.476
104	A	12	9	1.00	21	0.429
105	A	2	2	1.00	21	0.095
106	A	3	2	1.00	21	0.095
107	A	3	2	1.00	21	0.095
108	A	6	5	1.00	21	0.238
109	A	5	5	1.00	21	0.238
110	A	4	4	1.00	21	0.190
111	A	4	4	1.00	19	0.210
112	A	5	5	1.00	19	0.263
113	A	6	6	1.00	21	0.286
114	A	2	2	1.00	25	0.080
115	A	3	3	1.00	25	0.120
116	A	1	1	1.00	25	0.040
117	A	2	2	1.00	25	0.080
118	A	7	7	1.00	25	0.280
119	A	3	3	1.00	25	0.120
120	A	4	4	1.00	25	0.160
121	A	2	2	1.00	25	0.080
122	A	3	3	1.00	25	0.120
123	A	1	1	1.00	25	0.040
124	A	3	3	1.00	25	0.120
125	A	8	8	1.00	25	0.320
126	A	4	3	1.00	25	0.120
127	A	2	2	1.00	25	0.080
128	A	3	3	1.00	25	0.120
129	A	1	1	1.00	25	0.040

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	2	2	1.00	25	0.080
131	A	7	7	1.00	25	0.280
132	A	3	3	1.00	25	0.120
133	A	8	8	1.00	25	0.320
134	A	4	3	1.00	25	0.120
135	A	3	3	1.00	25	0.120
136	A	1	1	1.00	25	0.040
137	A	8	8	1.00	25	0.320
138	A	8	8	1.00	25	0.320
139	A	5	4	1.00	25	0.160
140	A	4	4	1.00	25	0.160
141	A	3	3	1.00	25	0.120
142	A	3	3	1.00	25	0.120
143	A	4	4	1.00	25	0.160
144	A	5	4	1.00	25	0.160
145	A	2	2	1.00	25	0.080
146	A	2	2	1.00	25	0.080
147	A	2	2	1.00	25	0.080
148	A	2	2	1.00	25	0.080
149	A	2	2	1.00	25	0.080
150	A	2	2	1.00	25	0.080
151	A	2	2	1.00	25	0.080
152	A	2	2	1.00	25	0.080
153	A	2	2	1.00	25	0.080
154	A	2	2	1.00	25	0.080
155	A	2	2	1.00	25	0.080
156	A	2	2	1.00	25	0.080
157	A	2	2	1.00	25	0.080
158	A	2	2	1.00	25	0.080
159	A	2	2	1.00	25	0.080
160	A	2	2	1.00	25	0.080
161	A	2	2	1.00	19	0.105
162	A	2	2	1.00	17	0.118
163	A	2	2	1.00	17	0.118
164	A	3	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	3	2	1.00	19	0.105
166	A	2	2	1.00	19	0.105
167	A	2	2	1.00	19	0.105
168	A	2	2	1.00	19	0.105
169	A	2	2	1.00	19	0.105
170	A	2	2	1.00	23	0.087
171	A	2	2	1.00	23	0.087
172	A	2	2	1.00	23	0.087
173	A	2	2	1.00	23	0.087
174	A	2	2	1.00	21	0.095
175	A	2	2	1.00	19	0.105
176	A	2	2	1.00	19	0.105
177	A	2	2	1.00	19	0.105
178	A	3	2	1.00	19	0.105
179	A	3	2	1.00	19	0.105
180	A	2	2	1.00	19	0.105
181	A	2	2	1.00	17	0.118
182	A	2	2	1.00	17	0.118
183	A	2	2	1.00	19	0.105
184	A	2	2	1.00	19	0.105
185	A	2	2	1.00	23	0.087
186	A	2	2	1.00	23	0.087
187	A	2	2	1.00	23	0.087
188	A	2	2	1.00	23	0.087
189	A	2	2	1.00	21	0.095
190	A	3	3	1.00	21	0.143
191	A	14	10	1.00	21	0.476
192	A	13	10	1.00	21	0.476
193	A	13	10	1.00	21	0.476
194	A	12	9	1.00	19	0.474
195	A	11	8	1.00	12	0.667
196	A	13	10	1.00	19	0.526
197	A	13	10	1.00	21	0.476
198	A	14	10	1.00	21	0.476
199	A	14	10	1.00	21	0.476

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	13	10	1.00	21	0.476
201	A	13	10	1.00	21	0.476
202	A	12	9	1.00	21	0.429
203	A	12	9	1.00	19	0.474
204	A	12	9	1.00	12	0.750
205	A	13	10	1.00	19	0.526
206	A	14	10	1.00	21	0.476
207	A	14	10	1.00	21	0.476
208	A	13	10	1.00	21	0.476
209	A	13	10	1.00	19	0.526
210	A	11	8	1.00	12	0.667
211	A	12	9	1.00	19	0.474
212	A	13	10	1.00	21	0.476
213	A	13	10	1.00	21	0.476
214	A	14	10	1.00	21	0.476
215	A	13	10	1.00	19	0.526
216	A	12	9	1.00	12	0.750
217	A	12	9	1.00	19	0.474
218	A	12	9	1.00	21	0.429
219	A	13	10	1.00	21	0.476
220	A	13	10	1.00	21	0.476
221	A	14	10	1.00	21	0.476
222	A	3	3	1.00	17	0.176
223	A	3	3	1.00	19	0.158
224	A	3	3	1.00	19	0.158
225	A	3	3	1.00	21	0.143
226	A	3	2	1.00	21	0.095
227	A	3	2	1.00	21	0.095
228	A	2	2	1.00	21	0.095
229	A	11	8	1.00	12	0.667
230	A	12	9	1.00	21	0.429
231	A	5	4	1.00	21	0.190
232	A	4	4	1.00	19	0.210
233	A	3	3	1.00	19	0.158
234	A	4	4	1.00	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	5	4	1.00	21	0.190
236	A	3	2	1.00	21	0.095
237	A	3	2	1.00	21	0.095
238	A	2	2	1.00	21	0.095
239	A	12	9	1.00	12	0.750
240	A	12	9	1.00	21	0.429
241	A	6	5	1.00	21	0.238
242	A	5	5	1.00	21	0.238
243	A	4	4	1.00	19	0.210
244	A	4	4	1.00	19	0.210
245	A	5	5	1.00	21	0.238
246	A	6	5	1.00	21	0.238
247	A	3	2	1.00	21	0.095
248	A	3	2	1.00	21	0.095
249	A	2	2	1.00	21	0.095
250	A	12	9	1.00	12	0.750
251	A	12	9	1.00	21	0.429
252	A	13	10	1.00	21	0.476
253	A	5	4	1.00	21	0.190
254	A	4	4	1.00	21	0.190
255	A	3	3	1.00	19	0.158
256	A	4	4	1.00	19	0.210
257	A	5	4	1.00	21	0.190
258	A	3	2	1.00	21	0.095
259	A	3	2	1.00	21	0.095
260	A	2	2	1.00	21	0.095
261	A	12	9	1.00	12	0.750
262	A	13	10	1.00	21	0.476
263	A	6	5	1.00	21	0.238
264	A	5	5	1.00	21	0.238
265	A	4	4	1.00	19	0.210
266	A	4	4	1.00	19	0.210
267	A	5	5	1.00	21	0.238
268	A	6	5	1.00	21	0.238
269	A	4	4	1.00	19	0.210

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	5	4	1.00	21	0.190
271	A	2	2	1.00	19	0.105
272	A	2	2	1.00	19	0.105
273	A	2	2	1.00	19	0.105
274	A	2	2	1.00	19	0.105
275	A	2	2	1.00	19	0.105
276	A	2	2	1.00	19	0.105
277	A	2	2	1.00	19	0.105
278	A	2	2	1.00	19	0.105
279	A	2	2	1.00	19	0.105
280	A	2	2	1.00	19	0.105
281	A	1	1	1.00	21	0.048
282	A	1	1	1.00	21	0.048
283	A	1	1	1.00	21	0.048
284	A	1	1	1.00	21	0.048
285	A	1	1	1.00	21	0.048
286	A	1	1	1.00	21	0.048
287	A	1	1	1.00	21	0.048
288	A	1	1	1.00	21	0.048
289	A	1	1	1.00	21	0.048
290	A	1	1	1.00	21	0.048
291	A	7	7	1.00	25	0.280
292	A	4	4	1.00	25	0.160
293	A	6	6	1.00	25	0.240
294	A	3	3	1.00	25	0.120
295	A	1	1	1.00	25	0.040
296	A	4	4	1.00	25	0.160
297	A	2	2	1.00	25	0.080
298	A	5	4	1.00	25	0.160
299	A	5	5	1.00	25	0.200
300	A	7	7	1.00	25	0.280
301	A	4	4	1.00	25	0.160
302	A	7	7	1.00	25	0.280
303	A	4	4	1.00	25	0.160
304	A	1	1	1.00	25	0.040

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	5	5	1.00	25	0.200
306	A	3	3	1.00	25	0.120
307	A	8	8	1.00	25	0.320
308	A	5	5	1.00	25	0.200
309	A	7	7	1.00	25	0.280
310	A	4	4	1.00	25	0.160
311	A	7	7	1.00	25	0.280
312	A	4	4	1.00	25	0.160
313	A	1	1	1.00	25	0.040
314	A	5	5	1.00	25	0.200
315	A	7	7	1.00	25	0.280
316	A	4	4	1.00	25	0.160
317	A	6	6	1.00	25	0.240
318	A	3	3	1.00	25	0.120
319	A	1	1	1.00	25	0.040
320	A	4	4	1.00	25	0.160
321	A	2	2	1.00	25	0.080
322	A	7	7	1.00	25	0.280
323	A	4	4	1.00	25	0.160
324	A	1	1	1.00	25	0.040
325	A	4	4	1.00	25	0.160
326	A	2	2	0.93	25	0.080
327	A	5	5	1.00	25	0.200
328	A	7	7	1.00	25	0.280
329	A	4	4	1.00	25	0.160
330	A	1	1	1.00	25	0.040
331	A	4	4	1.00	25	0.160
332	A	2	2	1.00	25	0.080
333	A	5	5	1.00	25	0.200
334	A	3	3	1.00	25	0.120
335	A	1	1	1.00	25	0.040
336	A	1	1	1.00	25	0.040
337	A	1	1	1.00	25	0.040
338	A	1	1	1.00	25	0.040
339	A	1	1	1.00	25	0.040

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	1	1	1.00	25	0.040
341	A	1	1	1.00	25	0.040
342	A	1	1	1.00	25	0.040
343	A	1	1	1.00	25	0.040
344	A	1	1	1.00	25	0.040
345	A	1	1	1.00	25	0.040
346	A	1	1	1.00	25	0.040
347	A	1	1	1.00	25	0.040
348	A	1	1	1.00	25	0.040
349	A	1	1	1.00	25	0.040
350	A	1	1	1.00	25	0.040
351	A	3	2	1.00	19	0.105
352	A	3	2	1.00	19	0.105
353	A	2	2	1.00	17	0.118
354	A	2	2	1.00	17	0.118
355	A	2	2	1.00	19	0.105
356	A	2	2	1.00	19	0.105
357	A	1	1	1.00	19	0.053
358	A	1	1	1.00	19	0.053
359	A	1	1	1.00	19	0.053
360	A	1	1	1.00	19	0.053
361	A	1	1	1.00	19	0.053
362	A	1	1	1.00	21	0.048
363	A	3	2	1.00	19	0.105
364	A	3	2	1.00	19	0.105
365	A	2	2	1.00	19	0.105
366	A	2	2	1.00	10	0.200
367	A	2	2	1.00	19	0.105
368	A	2	2	1.00	19	0.105
369	A	1	1	1.00	19	0.053
370	A	1	1	1.00	19	0.053
371	A	1	1	1.00	17	0.059
372	A	1	1	1.00	17	0.059
373	A	1	1	1.00	19	0.053
374	A	2	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	2	2	1.00	17	0.118
376	A	2	2	1.00	17	0.118
377	A	3	2	1.00	19	0.105
378	A	3	2	1.00	19	0.105
379	A	1	1	1.00	19	0.053
380	A	1	1	1.00	19	0.053
381	A	1	1	1.00	19	0.053
382	A	1	1	1.00	19	0.053
383	A	3	3	1.00	23	0.130
384	A	3	3	1.00	23	0.130
385	A	3	3	1.00	23	0.130
386	A	3	3	1.00	23	0.130
387	A	3	3	1.00	21	0.143

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \tan(c+dx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 12

$$\int \tan(c+dx)\,dx=-\frac{\log(\cos(c+dx))}{d}$$

```
[Out] -ln(cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3556}

$$\int \tan(c+dx)\,dx=-\frac{\log(\cos(c+dx))}{d}$$

```
[In] Int[Tan[c + d*x],x]
```

```
[Out] -(Log[Cos[c + d*x]]/d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral}=-\frac{\log(\cos(c+dx))}{d}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \tan(c+dx)\,dx=-\frac{\log(\cos(c+dx))}{d}$$

```
[In] Integrate[Tan[c + d*x],x]
[Out] -(Log[Cos[c + d*x]]/d)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

method	result	size
derivativedivides	$\frac{\ln(1+\tan^2(dx+c))}{2d}$	17
default	$\frac{\ln(1+\tan^2(dx+c))}{2d}$	17
norman	$\frac{\ln(1+\tan^2(dx+c))}{2d}$	17
parallelrisch	$\frac{\ln(1+\tan^2(dx+c))}{2d}$	17
risch	$ix+\frac{2ic}{d}-\frac{\ln(e^{2i(dx+c)}+1)}{d}$	30

```
[In] int(tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/2/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \tan(c+dx)\,dx=-\frac{\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

```
[In] integrate(tan(d*x+c),x, algorithm="fricas")
[Out] -1/2*log(1/(tan(d*x + c)^2 + 1))/d
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \tan(c+dx)\,dx = \begin{cases} \frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} & \text{for } d \neq 0 \\ x\tan\left(c\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(tan(d*x+c),x)
[Out] Piecewise((log(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*tan(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \tan(c+dx)\,dx = \frac{\log\left(\sec\left(dx+c\right)\right)}{d}$$

```
[In] integrate(tan(d*x+c),x, algorithm="maxima")
[Out] log(sec(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \tan(c+dx)\,dx = -\frac{\log\left(\left|\cos\left(dx+c\right)\right|\right)}{d}$$

```
[In] integrate(tan(d*x+c),x, algorithm="giac")
[Out] -log(abs(cos(d*x + c)))/d
```

Mupad [B] (verification not implemented)

Time = 2.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan(c+dx)\,dx = \frac{\ln\left(\tan(c+d\,x)^2+1\right)}{2\,d}$$

```
[In] int(tan(c + d*x),x)
[Out] log(tan(c + d*x)^2 + 1)/(2*d)
```

3.2 $\int \tan^2(c+dx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 14

$$\int \tan^2(c+dx)\,dx = -x + \frac{\tan(c+dx)}{d}$$

```
[Out] -x+tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3554, 8}

$$\int \tan^2(c+dx)\,dx = \frac{\tan(c+dx)}{d} - x$$

```
[In] Int[Tan[c + d*x]^2,x]
```

```
[Out] -x + Tan[c + d*x]/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\tan(c+dx)}{d} - \int 1\,dx \\ &= -x + \frac{\tan(c+dx)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \tan^2(c+dx)\,dx = -\frac{\arctan(\tan(c+dx))}{d} + \frac{\tan(c+dx)}{d}$$

```
[In] Integrate[Tan[c + d*x]^2,x]
[Out] -(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
norman	$-x+\frac{\tan(dx+c)}{d}$	15
parallelrisch	$-\frac{dx-\tan(dx+c)}{d}$	18
derivativedivides	$\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{d}$	21
default	$\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{d}$	21
risch	$-x+\frac{2i}{d(e^{2i(dx+c)}+1)}$	24

```
[In] int(tan(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] -x+tan(d*x+c)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \tan^2(c+dx)\,dx = -\frac{dx-\tan(dx+c)}{d}$$

```
[In] integrate(tan(d*x+c)^2,x, algorithm="fricas")
[Out] -(d*x - tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \tan^2(c+dx)\,dx = \begin{cases} -x+\frac{\tan(c+dx)}{d} & \text{for } d\neq 0 \\ x\tan^2(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(tan(d*x+c)**2,x)
[Out] Piecewise((-x + tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \tan^2(c+dx)\,dx = -\frac{dx+c-\tan(dx+c)}{d}$$

```
[In] integrate(tan(d*x+c)^2,x, algorithm="maxima")
[Out] -(d*x + c - tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(14) = 28.

Time = 0.42 (sec) , antiderivative size = 226, normalized size of antiderivative = 16.14

$$\int \tan^2(c+dx)\,dx$$

$$= \frac{\pi - 4\,dx\tan(dx)\tan(c) - \pi\mathrm{sgn}\left(2\tan(dx)^2\tan(c) + 2\tan(dx)\tan(c)^2 - 2\tan(dx) - 2\tan(c)\right)\tan(d}{}$$

```
[In] integrate(tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(pi - 4*d*x*tan(d*x)*tan(c) - pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*t
an(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - pi*tan(d*x)*tan(c) + 2*a
rctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 2*arctan
((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 4*d*x + pi*sg
n(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 2*ar
ctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 2*arctan((tan(d*x) + tan(
c))/(tan(d*x)*tan(c) - 1)) - 4*tan(d*x) - 4*tan(c))/(d*tan(d*x)*tan(c) - d)
```

Mupad [B] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \tan^2(c+dx)\,dx=\frac{\tan(c+d\,x)}{d}-x$$

```
[In] int(tan(c + d*x)^2,x)
```

```
[Out] tan(c + d*x)/d - x
```

3.3 $\int \tan^3(c+dx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \tan^3(c+dx)\,dx = \frac{\log(\cos(c+dx))}{d} + \frac{\tan^2(c+dx)}{2d}$$

```
[Out] ln(cos(d*x+c))/d+1/2*tan(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3554, 3556}

$$\int \tan^3(c+dx)\,dx = \frac{\tan^2(c+dx)}{2d} + \frac{\log(\cos(c+dx))}{d}$$

```
[In] Int[Tan[c + d*x]^3,x]
```

```
[Out] Log[Cos[c + d*x]]/d + Tan[c + d*x]^2/(2*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\tan^2(c+dx)}{2d} - \int \tan(c+dx)\,dx$$
$$= \frac{\log(\cos(c+dx))}{d} + \frac{\tan^2(c+dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \tan^3(c+dx)\,dx = \frac{2\log(\cos(c+dx)) + \tan^2(c+dx)}{2d}$$

```
[In] Integrate[Tan[c + d*x]^3,x]
[Out] (2*Log[Cos[c + d*x]] + Tan[c + d*x]^2)/(2*d)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$-\frac{-(\tan^2(dx+c))+\ln(1+\tan^2(dx+c))}{2d}$	28
derivativedivides	$\frac{\frac{(\tan^2(dx+c))}{2}-\frac{\ln(1+\tan^2(dx+c))}{2}}{d}$	29
default	$\frac{\frac{(\tan^2(dx+c))}{2}-\frac{\ln(1+\tan^2(dx+c))}{2}}{d}$	29
norman	$\frac{\tan^2(dx+c)}{2d}-\frac{\ln(1+\tan^2(dx+c))}{2d}$	31
risch	$-ix-\frac{2ic}{d}+\frac{2\,\mathrm{e}^{2i(dx+c)}}{d(\mathrm{e}^{2i(dx+c)}+1)^2}+\frac{\ln(\mathrm{e}^{2i(dx+c)}+1)}{d}$	56

```
[In] int(tan(d*x+c)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*(-tan(d*x+c)^2+ln(1+tan(d*x+c)^2))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \tan^3(c+dx)\,dx = \frac{\tan\left(dx+c\right)^2 + \log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

```
[In] integrate(tan(d*x+c)^3,x, algorithm="fricas")
[Out] 1/2*(tan(d*x + c)^2 + log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \tan^3(c+dx)\,dx = \begin{cases} -\frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{\tan^2\left(c+dx\right)}{2d} & \text{for } d \neq 0 \\ x\tan^3\left(c\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(tan(d*x+c)**3,x)
[Out] Piecewise((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**2/(2*d), Ne(d, 0
)), (x*tan(c)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \tan^3(c+dx)\,dx = -\frac{\frac{1}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)^2 - 1\right)}{2\,d}$$

```
[In] integrate(tan(d*x+c)^3,x, algorithm="maxima")
[Out] -1/2*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(25) = 50$.

Time = 0.59 (sec) , antiderivative size = 216, normalized size of antiderivative = 8.00

$$\int \tan^3(c+dx)\,dx$$
$$= \frac{\log\left(\frac{4\left(\tan(dx)^2\tan(c)^2-2\tan(dx)\tan(c)+1\right)}{\tan(dx)^2\tan(c)^2+\tan(dx)^2+\tan(c)^2+1}\right)\tan(dx)^2\tan(c)^2+\tan(dx)^2\tan(c)^2-2\log\left(\frac{4\left(\tan(dx)^2\tan(c)^2-2\tan(\right.}{\tan(dx)^2\tan(c)^2+\tan(dx)}\right.}{2\left(d\tan(dx)^2\tan(c)^2-2\,d\,\mathrm{ta}\right.}$$

```
[In] integrate(tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)
^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + tan(d*x)^2*tan(c)^2
- 2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^
2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(c) + tan(d*x)^2 + tan(c)^2 + l
og(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 + tan(c)^2 + 1)) + 1)/(d*tan(d*x)^2*tan(c)^2 - 2*d*tan(d*x)*tan(c
) + d)
```

Mupad [B] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \tan^3(c+dx)\,dx = \frac{\tan(c+d\,x)^2}{2\,d} - \frac{\ln\left(\tan(c+d\,x)^2+1\right)}{2\,d}$$

```
[In] int(tan(c + d*x)^3,x)
```

```
[Out] tan(c + d*x)^2/(2*d) - log(tan(c + d*x)^2 + 1)/(2*d)
```

3.4 $\int \tan^4(c+dx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 28

$$\int \tan^4(c+dx)\,dx = x - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d}$$

```
[Out] x-tan(d*x+c)/d+1/3*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 8}

$$\int \tan^4(c+dx)\,dx = \frac{\tan^3(c+dx)}{3d} - \frac{\tan(c+dx)}{d} + x$$

```
[In] Int[Tan[c + d*x]^4,x]
```

```
[Out] x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\tan^3(c+dx)}{3d} - \int \tan^2(c+dx)\,dx \\ &= -\frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d} + \int 1\,dx \\ &= x - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \tan^4(c+dx)\,dx = \frac{\arctan(\tan(c+dx))}{d} - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d}$$

```
[In] Integrate[Tan[c + d*x]^4,x]
[Out] ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
norman	$x-\frac{\tan(dx+c)}{d}+\frac{\tan^3(dx+c)}{3d}$	27
parallelrisch	$\frac{\tan^3(dx+c)+3dx-3\tan(dx+c)}{3d}$	27
derivativedivides	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$	31
default	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$	31
risch	$x-\frac{4i\left(3\,e^{4i(dx+c)}+3\,e^{2i(dx+c)}+2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}$	46

```
[In] int(tan(d*x+c)^4,x,method=_RETURNVERBOSE)
[Out] x-tan(d*x+c)/d+1/3*tan(d*x+c)^3/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \tan^4(c+dx)\,dx = \frac{\tan\left(dx+c\right)^3 + 3\,dx - 3\,\tan\left(dx+c\right)}{3\,d}$$

```
[In] integrate(tan(d*x+c)^4,x, algorithm="fricas")
[Out] 1/3*(tan(d*x + c)^3 + 3*d*x - 3*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \tan^4(c+dx)\,dx = \begin{cases} x + \frac{\tan^3\left(c+dx\right)}{3d} - \frac{\tan\left(c+dx\right)}{d} & \text{for}\ d \neq 0 \\ x\tan^4\left(c\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(tan(d*x+c)**4,x)
[Out] Piecewise((x + tan(c + d*x)**3/(3*d) - tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)
**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \tan^4(c+dx)\,dx = \frac{\tan\left(dx+c\right)^3 + 3\,dx + 3\,c - 3\,\tan\left(dx+c\right)}{3\,d}$$

```
[In] integrate(tan(d*x+c)^4,x, algorithm="maxima")
[Out] 1/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(26) = 52$.

Time = 0.84 (sec) , antiderivative size = 585, normalized size of antiderivative = 20.89

$$\int \tan^4(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/12*(3*pi + 12*d*x*tan(d*x)^3*tan(c)^3 - 3*pi*sgn(2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 - 3*pi*tan(d
*x)^3*tan(c)^3 + 6*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*
x)^3*tan(c)^3 + 6*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x
)^3*tan(c)^3 - 36*d*x*tan(d*x)^2*tan(c)^2 + 9*pi*sgn(2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 9*pi*tan
(d*x)^2*tan(c)^2 - 18*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan
(d*x)^2*tan(c)^2 - 18*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan
(d*x)^2*tan(c)^2 + 12*tan(d*x)^3*tan(c)^2 + 12*tan(d*x)^2*tan(c)^3 + 36*d*x
*tan(d*x)*tan(c) - 9*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*t
an(d*x) - 2*tan(c))*tan(d*x)*tan(c) - 4*tan(d*x)^3 - 9*pi*tan(d*x)*tan(c) +
 18*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 18*
arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) - 36*tan(
d*x)^2*tan(c) - 36*tan(d*x)*tan(c)^2 - 4*tan(c)^3 - 12*d*x + 3*pi*sgn(2*tan
(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 6*arctan((t
an(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 6*arctan((tan(d*x) + tan(c))/(ta
n(d*x)*tan(c) - 1)) + 12*tan(d*x) + 12*tan(c))/(d*tan(d*x)^3*tan(c)^3 - 3*d
*tan(d*x)^2*tan(c)^2 + 3*d*tan(d*x)*tan(c) - d)
```

Mupad [B] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \tan^4(c+dx)\,dx = x - \frac{\tan(c+d\,x) - \frac{\tan(c+d\,x)^3}{3}}{d}$$

```
[In] int(tan(c + d*x)^4,x)
```

```
[Out] x - (tan(c + d*x) - tan(c + d*x)^3/3)/d
```

3.5 $\int \tan^5(c+dx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \tan^5(c+dx)\,dx = -\frac{\log(\cos(c+dx))}{d} - \frac{\tan^2(c+dx)}{2d} + \frac{\tan^4(c+dx)}{4d}$$

```
[Out] -ln(cos(d*x+c))/d-1/2*tan(d*x+c)^2/d+1/4*tan(d*x+c)^4/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3554, 3556}

$$\int \tan^5(c+dx)\,dx = \frac{\tan^4(c+dx)}{4d} - \frac{\tan^2(c+dx)}{2d} - \frac{\log(\cos(c+dx))}{d}$$

```
[In] Int[Tan[c + d*x]^5,x]
```

```
[Out] -(Log[Cos[c + d*x]]/d) - Tan[c + d*x]^2/(2*d) + Tan[c + d*x]^4/(4*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan^4(c+dx)}{4d} - \int \tan^3(c+dx)\,dx \\
&= -\frac{\tan^2(c+dx)}{2d} + \frac{\tan^4(c+dx)}{4d} + \int \tan(c+dx)\,dx \\
&= -\frac{\log(\cos(c+dx))}{d} - \frac{\tan^2(c+dx)}{2d} + \frac{\tan^4(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \tan^5(c+dx)\,dx = -\frac{4\log(\cos(c+dx)) + 2\tan^2(c+dx) - \tan^4(c+dx)}{4d}$$

```
[In] Integrate[Tan[c + d*x]^5,x]
[Out] -1/4*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4)/d
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$\frac{\tan^4(dx+c)-2\left(\tan^2(dx+c)\right)+2\ln\left(1+\tan^2(dx+c)\right)}{4d}$	38
derivativedivides	$\frac{\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right)}{2}+\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}}{d}$	39
default	$\frac{\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right)}{2}+\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}}{d}$	39
norman	$-\frac{\tan^2(dx+c)}{2d}+\frac{\tan^4(dx+c)}{4d}+\frac{\ln\left(1+\tan^2(dx+c)\right)}{2d}$	44
risch	$ix+\frac{2ic}{d}-\frac{4\left(e^{6i(dx+c)}+e^{4i(dx+c)}+e^{2i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^4}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{d}$	76

```
[In] int(tan(d*x+c)^5,x,method=_RETURNVERBOSE)
[Out] 1/4*(tan(d*x+c)^4-2*tan(d*x+c)^2+2*ln(1+tan(d*x+c)^2))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \tan^5(c+dx)\,dx = \frac{\tan(dx+c)^4 - 2\,\tan(dx+c)^2 - 2\,\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{4\,d}$$

```
[In] integrate(tan(d*x+c)^5,x, algorithm="fricas")
[Out] 1/4*(tan(d*x + c)^4 - 2*tan(d*x + c)^2 - 2*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \tan^5(c+dx)\,dx = \begin{cases} \frac{\log(\tan^2(c+dx)+1)}{2d} + \frac{\tan^4(c+dx)}{4d} - \frac{\tan^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\tan^5(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(tan(d*x+c)**5,x)
[Out] Piecewise((log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**4/(4*d) - tan(c +
 d*x)**2/(2*d), Ne(d, 0)), (x*tan(c)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \tan^5(c+dx)\,dx = \frac{\frac{4\,\sin(dx+c)^2-3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - 2\,\log\left(\sin(dx+c)^2-1\right)}{4\,d}$$

```
[In] integrate(tan(d*x+c)^5,x, algorithm="maxima")
[Out] 1/4*((4*sin(d*x + c)^2 - 3)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 2*log
(sin(d*x + c)^2 - 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(39) = 78.

Time = 1.37 (sec) , antiderivative size = 462, normalized size of antiderivative = 10.74

$$\int \tan^5(c+dx)\,dx =$$

$$-\frac{2\log\left(\frac{4\left(\tan(dx)^2\tan(c)^2-2\tan(dx)\tan(c)+1\right)}{\tan(dx)^2\tan(c)^2+\tan(dx)^2+\tan(c)^2+1}\right)\tan(dx)^4\tan(c)^4+3\tan(dx)^4\tan(c)^4-8\log\left(\frac{4\left(\tan(dx)^2\tan(c)^2\right.}{\tan(dx)^2\tan(c)^2+}\right.}{}$$

```
[In] integrate(tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/4*(2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 3*tan(d*x)^4*tan(
c)^4 - 8*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*ta
n(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 2*tan(d*x)^4*tan
(c)^2 - 8*tan(d*x)^3*tan(c)^3 + 2*tan(d*x)^2*tan(c)^4 + 12*log(4*(tan(d*x)^
2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan
(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - tan(d*x)^4 - 8*tan(d*x)^3*tan(c) + 4*tan(
d*x)^2*tan(c)^2 - 8*tan(d*x)*tan(c)^3 - tan(c)^4 - 8*log(4*(tan(d*x)^2*tan(
c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2
+ 1))*tan(d*x)*tan(c) + 2*tan(d*x)^2 - 8*tan(d*x)*tan(c) + 2*tan(c)^2 + 2*l
og(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 + tan(c)^2 + 1)) + 3)/(d*tan(d*x)^4*tan(c)^4 - 4*d*tan(d*x)^3*tan
(c)^3 + 6*d*tan(d*x)^2*tan(c)^2 - 4*d*tan(d*x)*tan(c) + d)
```

Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \tan^5(c+dx)\,dx = \frac{\frac{\ln\left(\tan(c+d\,x)^2+1\right)}{2}-\frac{\tan(c+d\,x)^2}{2}+\frac{\tan(c+d\,x)^4}{4}}{d}$$

```
[In] int(tan(c + d*x)^5,x)
```

```
[Out] (log(tan(c + d*x)^2 + 1)/2 - tan(c + d*x)^2/2 + tan(c + d*x)^4/4)/d
```

3.6 $\int \tan^6(c+dx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 44

$$\int \tan^6(c+dx)\,dx=-x+\frac{\tan(c+dx)}{d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan^5(c+dx)}{5d}$$

```
[Out] -x+tan(d*x+c)/d-1/3*tan(d*x+c)^3/d+1/5*tan(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 8}

$$\int \tan^6(c+dx)\,dx=\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}-x$$

```
[In] Int[Tan[c + d*x]^6,x]
```

```
[Out] -x + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) + Tan[c + d*x]^5/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tan^5(c+dx)}{5d} - \int \tan^4(c+dx)\,dx \\
&= -\frac{\tan^3(c+dx)}{3d} + \frac{\tan^5(c+dx)}{5d} + \int \tan^2(c+dx)\,dx \\
&= \frac{\tan(c+dx)}{d} - \frac{\tan^3(c+dx)}{3d} + \frac{\tan^5(c+dx)}{5d} - \int 1\,dx \\
&= -x + \frac{\tan(c+dx)}{d} - \frac{\tan^3(c+dx)}{3d} + \frac{\tan^5(c+dx)}{5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$
\int \tan^6(c+dx)\,dx = -\frac{\arctan(\tan(c+dx))}{d} + \frac{\tan(c+dx)}{d} - \frac{\tan^3(c+dx)}{3d} + \frac{\tan^5(c+dx)}{5d}
$$

```
[In] Integrate[Tan[c + d*x]^6,x]
[Out] -(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) + Tan[c + 
 d*x]^5/(5*d)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$-\frac{-3\left(\tan^5(dx+c)\right)+5\left(\tan^3(dx+c)\right)+15dx-15\tan(dx+c)}{15d}$	39
derivativedivides	$\frac{\frac{\left(\tan^5(dx+c)\right)}{5}-\frac{\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)-\arctan(\tan(dx+c))}{d}$	41
default	$\frac{\frac{\left(\tan^5(dx+c)\right)}{5}-\frac{\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)-\arctan(\tan(dx+c))}{d}$	41
norman	$-x+\frac{\tan(dx+c)}{d}-\frac{\tan^3(dx+c)}{3d}+\frac{\tan^5(dx+c)}{5d}$	41
risch	$-x+\frac{2i\left(45\,e^{8i(dx+c)}+90\,e^{6i(dx+c)}+140\,e^{4i(dx+c)}+70\,e^{2i(dx+c)}+23\right)}{15d\left(e^{2i(dx+c)}+1\right)^5}$	70

```
[In] int(tan(d*x+c)^6,x,method=_RETURNVERBOSE)
[Out] -1/15*(-3*tan(d*x+c)^5+5*tan(d*x+c)^3+15*d*x-15*tan(d*x+c))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \tan^6(c+dx)\,dx = \frac{3\,\tan\left(dx+c\right)^5 - 5\,\tan\left(dx+c\right)^3 - 15\,dx + 15\,\tan\left(dx+c\right)}{15\,d}$$

```
[In] integrate(tan(d*x+c)^6,x, algorithm="fricas")
[Out] 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x + 15*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \tan^6(c+dx)\,dx = \begin{cases} -x + \frac{\tan^5(c+dx)}{5d} - \frac{\tan^3(c+dx)}{3d} + \frac{\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\tan^6(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(tan(d*x+c)**6,x)
[Out] Piecewise((-x + tan(c + d*x)**5/(5*d) - tan(c + d*x)**3/(3*d) + tan(c + d*x
)/d, Ne(d, 0)), (x*tan(c)**6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \tan^6(c+dx)\,dx = \frac{3\,\tan\left(dx+c\right)^5 - 5\,\tan\left(dx+c\right)^3 - 15\,dx - 15\,c + 15\,\tan\left(dx+c\right)}{15\,d}$$

```
[In] integrate(tan(d*x+c)^6,x, algorithm="maxima")
[Out] 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c)
)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. $2(40) = 80$.

Time = 2.16 (sec) , antiderivative size = 989, normalized size of antiderivative = 22.48

$$\int \tan^6(c + dx)\, dx = \text{Too large to display}$$

```
[In] integrate(tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/60*(15*pi - 60*d*x*tan(d*x)^5*tan(c)^5 - 15*pi*sgn(2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^5 - 15*pi*ta
n(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*ta
n(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*ta
n(d*x)^5*tan(c)^5 + 300*d*x*tan(d*x)^4*tan(c)^4 + 75*pi*sgn(2*tan(d*x)^2*ta
n(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 7
5*pi*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan
(c)))*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c)
 - 1))*tan(d*x)^4*tan(c)^4 - 60*tan(d*x)^5*tan(c)^4 - 60*tan(d*x)^4*tan(c)^
5 - 600*d*x*tan(d*x)^3*tan(c)^3 - 150*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*
x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 + 20*tan(d*x)^5*ta
n(c)^2 - 150*pi*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x)*tan(c) - 1)/(tan
(d*x) + tan(c)))*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x) + tan(c))/(tan(
d*x)*tan(c) - 1))*tan(d*x)^3*tan(c)^3 + 300*tan(d*x)^4*tan(c)^3 + 300*tan(d
*x)^3*tan(c)^4 + 20*tan(d*x)^2*tan(c)^5 + 600*d*x*tan(d*x)^2*tan(c)^2 + 150
*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*
tan(d*x)^2*tan(c)^2 - 12*tan(d*x)^5 - 100*tan(d*x)^4*tan(c) + 150*pi*tan(d*
x)^2*tan(c)^2 - 300*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d
*x)^2*tan(c)^2 - 300*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(
d*x)^2*tan(c)^2 - 600*tan(d*x)^3*tan(c)^2 - 600*tan(d*x)^2*tan(c)^3 - 100*t
an(d*x)*tan(c)^4 - 12*tan(c)^5 - 300*d*x*tan(d*x)*tan(c) - 75*pi*sgn(2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c
) + 20*tan(d*x)^3 - 75*pi*tan(d*x)*tan(c) + 150*arctan((tan(d*x)*tan(c) - 1
)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 150*arctan((tan(d*x) + tan(c))/(ta
n(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 300*tan(d*x)^2*tan(c) + 300*tan(d*x)*
tan(c)^2 + 20*tan(c)^3 + 60*d*x + 15*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x
)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 30*arctan((tan(d*x)*tan(c) - 1)/(tan(
d*x) + tan(c))) - 30*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) - 60
*tan(d*x) - 60*tan(c))/(d*tan(d*x)^5*tan(c)^5 - 5*d*tan(d*x)^4*tan(c)^4 + 1
0*d*tan(d*x)^3*tan(c)^3 - 10*d*tan(d*x)^2*tan(c)^2 + 5*d*tan(d*x)*tan(c) -
d)
```

Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \tan^6(c+dx)\,dx = \frac{\frac{\tan(c+d\,x)^5}{5}-\frac{\tan(c+d\,x)^3}{3}+\tan(c+d\,x)}{d}-x$$

```
[In] int(tan(c + d*x)^6,x)
[Out] (tan(c + d*x) - tan(c + d*x)^3/3 + tan(c + d*x)^5/5)/d - x
```

3.7 $\int \tan^7(c+dx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 57

$$\int \tan^7(c+dx)\,dx=\frac{\log(\cos(c+dx))}{d}+\frac{\tan^2(c+dx)}{2d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^6(c+dx)}{6d}$$

```
[Out] ln(cos(d*x+c))/d+1/2*tan(d*x+c)^2/d-1/4*tan(d*x+c)^4/d+1/6*tan(d*x+c)^6/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 3556}

$$\int \tan^7(c+dx)\,dx=\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}$$

```
[In] Int[Tan[c + d*x]^7,x]
```

```
[Out] Log[Cos[c + d*x]]/d + Tan[c + d*x]^2/(2*d) - Tan[c + d*x]^4/(4*d) + Tan[c +
 d*x]^6/(6*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan^6(c+dx)}{6d} - \int \tan^5(c+dx)\,dx \\
&= -\frac{\tan^4(c+dx)}{4d} + \frac{\tan^6(c+dx)}{6d} + \int \tan^3(c+dx)\,dx \\
&= \frac{\tan^2(c+dx)}{2d} - \frac{\tan^4(c+dx)}{4d} + \frac{\tan^6(c+dx)}{6d} - \int \tan(c+dx)\,dx \\
&= \frac{\log(\cos(c+dx))}{d} + \frac{\tan^2(c+dx)}{2d} - \frac{\tan^4(c+dx)}{4d} + \frac{\tan^6(c+dx)}{6d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \tan^7(c+dx)\,dx = \frac{12\log(\cos(c+dx)) + 6\tan^2(c+dx) - 3\tan^4(c+dx) + 2\tan^6(c+dx)}{12d}$$

```
[In] Integrate[Tan[c + d*x]^7,x]
[Out] (12*Log[Cos[c + d*x]] + 6*Tan[c + d*x]^2 - 3*Tan[c + d*x]^4 + 2*Tan[c + d*x
]^6)/(12*d)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^6(dx+c)\right)}{6} - \frac{\left(\tan^4(dx+c)\right)}{4} + \frac{\left(\tan^2(dx+c)\right)}{2} - \frac{\ln\left(1+\tan^2(dx+c)\right)}{2}}{d}$	49
default	$\frac{\frac{\left(\tan^6(dx+c)\right)}{6} - \frac{\left(\tan^4(dx+c)\right)}{4} + \frac{\left(\tan^2(dx+c)\right)}{2} - \frac{\ln\left(1+\tan^2(dx+c)\right)}{2}}{d}$	49
parallelrisch	$-\frac{-2\left(\tan^6(dx+c)\right)+3\left(\tan^4(dx+c)\right)-6\left(\tan^2(dx+c)\right)+6\ln\left(1+\tan^2(dx+c)\right)}{12d}$	50
norman	$\frac{\tan^2(dx+c)}{2d} - \frac{\tan^4(dx+c)}{4d} + \frac{\tan^6(dx+c)}{6d} - \frac{\ln\left(1+\tan^2(dx+c)\right)}{2d}$	57
risch	$-ix - \frac{2ic}{d} + \frac{6\,\mathrm{e}^{10i(dx+c)}+12\,\mathrm{e}^{8i(dx+c)}+\frac{68\,\mathrm{e}^{6i(dx+c)}}{3}+12\,\mathrm{e}^{4i(dx+c)}+6\,\mathrm{e}^{2i(dx+c)}}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^6} + \frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	103

```
[In] int(tan(d*x+c)^7,x,method=_RETURNVERBOSE)
[Out] 1/d*(1/6*tan(d*x+c)^6-1/4*tan(d*x+c)^4+1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+c)
^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \tan^7(c+dx)\,dx = \frac{2\,\tan\left(dx+c\right)^6 - 3\,\tan\left(dx+c\right)^4 + 6\,\tan\left(dx+c\right)^2 + 6\,\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{12\,d}$$

```
[In] integrate(tan(d*x+c)^7,x, algorithm="fricas")
[Out] 1/12*(2*tan(d*x + c)^6 - 3*tan(d*x + c)^4 + 6*tan(d*x + c)^2 + 6*log(1/(tan
(d*x + c)^2 + 1)))/d
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \tan^7(c+dx)\,dx = \begin{cases} -\frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{\tan^6\left(c+dx\right)}{6d} - \frac{\tan^4\left(c+dx\right)}{4d} + \frac{\tan^2\left(c+dx\right)}{2d} & \text{for } d \neq 0 \\ x\tan^7\left(c\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(tan(d*x+c)**7,x)
[Out] Piecewise((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**6/(6*d) - tan(c
+ d*x)**4/(4*d) + tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*tan(c)**7, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \tan^7(c+dx)\,dx = -\frac{\frac{18\,\sin(dx+c)^4 - 27\,\sin(dx+c)^2 + 11}{\sin(dx+c)^6 - 3\,\sin(dx+c)^4 + 3\,\sin(dx+c)^2 - 1} - 6\,\log\left(\sin\left(dx+c\right)^2 - 1\right)}{12\,d}$$

```
[In] integrate(tan(d*x+c)^7,x, algorithm="maxima")
[Out] -1/12*((18*sin(d*x + c)^4 - 27*sin(d*x + c)^2 + 11)/(sin(d*x + c)^6 - 3*sin
(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 6*log(sin(d*x + c)^2 - 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(51) = 102$.

Time = 3.11 (sec) , antiderivative size = 740, normalized size of antiderivative = 12.98

$$\int \tan^7(c + dx)\, dx = \text{Too large to display}$$

```
[In] integrate(tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/12*(6*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^6*tan(c)^6 + 11*tan(d*x)^6*tan
(c)^6 - 36*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*
tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 + 6*tan(d*x)^6*t
an(c)^4 - 54*tan(d*x)^5*tan(c)^5 + 6*tan(d*x)^4*tan(c)^6 + 90*log(4*(tan(d*
x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 +
tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 3*tan(d*x)^6*tan(c)^2 - 36*tan(d*x)^5*
tan(c)^3 + 99*tan(d*x)^4*tan(c)^4 - 36*tan(d*x)^3*tan(c)^5 - 3*tan(d*x)^2*t
an(c)^6 - 120*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)
^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 2*tan(d*x)^
6 + 18*tan(d*x)^5*tan(c) + 90*tan(d*x)^4*tan(c)^2 - 72*tan(d*x)^3*tan(c)^3
+ 90*tan(d*x)^2*tan(c)^4 + 18*tan(d*x)*tan(c)^5 + 2*tan(c)^6 + 90*log(4*(ta
n(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^
2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 3*tan(d*x)^4 - 36*tan(d*x)^3*tan(c
) + 99*tan(d*x)^2*tan(c)^2 - 36*tan(d*x)*tan(c)^3 - 3*tan(c)^4 - 36*log(4*(
tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x
)^2 + tan(c)^2 + 1))*tan(d*x)*tan(c) + 6*tan(d*x)^2 - 54*tan(d*x)*tan(c) +
6*tan(c)^2 + 6*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x
)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1)) + 11)/(d*tan(d*x)^6*tan(c)^6 - 6
*d*tan(d*x)^5*tan(c)^5 + 15*d*tan(d*x)^4*tan(c)^4 - 20*d*tan(d*x)^3*tan(c)^
3 + 15*d*tan(d*x)^2*tan(c)^2 - 6*d*tan(d*x)*tan(c) + d)
```

Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \tan^7(c + dx)\, dx = -\frac{\frac{\ln\left(\tan(c+d\,x)^2+1\right)}{2} - \frac{\tan(c+d\,x)^2}{2} + \frac{\tan(c+d\,x)^4}{4} - \frac{\tan(c+d\,x)^6}{6}}{d}$$

```
[In] int(tan(c + d*x)^7,x)
```

```
[Out] -(log(tan(c + d*x)^2 + 1)/2 - tan(c + d*x)^2/2 + tan(c + d*x)^4/4 - tan(c +
 d*x)^6/6)/d
```

3.8 $\int \tan^8(c+dx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 58

$$\int \tan^8(c+dx)\,dx = x-\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^7(c+dx)}{7d}$$

```
[Out] x-tan(d*x+c)/d+1/3*tan(d*x+c)^3/d-1/5*tan(d*x+c)^5/d+1/7*tan(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 8}

$$\int \tan^8(c+dx)\,dx = \frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}+x$$

```
[In] Int[Tan[c + d*x]^8,x]
```

```
[Out] x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c + d*x]^5/(5*d) + Tan[c +
d*x]^7/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tan^7(c+dx)}{7d} - \int \tan^6(c+dx)\,dx \\
&= -\frac{\tan^5(c+dx)}{5d} + \frac{\tan^7(c+dx)}{7d} + \int \tan^4(c+dx)\,dx \\
&= \frac{\tan^3(c+dx)}{3d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^7(c+dx)}{7d} - \int \tan^2(c+dx)\,dx \\
&= -\frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^7(c+dx)}{7d} + \int 1\,dx \\
&= x - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^7(c+dx)}{7d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

$$
\int \tan^8(c+dx)\,dx = \frac{\arctan(\tan(c+dx))}{d} - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^7(c+dx)}{7d}
$$

```
[In] Integrate[Tan[c + d*x]^8,x]
[Out] ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c + d*
x]^5/(5*d) + Tan[c + d*x]^7/(7*d)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$\frac{15(\tan^7(dx+c))-21(\tan^5(dx+c))+35(\tan^3(dx+c))+105dx-105\tan(dx+c)}{105d}$	49
derivativedivides	$\frac{\frac{(\tan^7(dx+c))}{7}-\frac{(\tan^5(dx+c))}{5}+\frac{(\tan^3(dx+c))}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$	51
default	$\frac{\frac{(\tan^7(dx+c))}{7}-\frac{(\tan^5(dx+c))}{5}+\frac{(\tan^3(dx+c))}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$	51
norman	$x-\frac{\tan(dx+c)}{d}+\frac{\tan^3(dx+c)}{3d}-\frac{\tan^5(dx+c)}{5d}+\frac{\tan^7(dx+c)}{7d}$	53
risch	$x-\frac{8i(105\,e^{12i(dx+c)}+315\,e^{10i(dx+c)}+770\,e^{8i(dx+c)}+770\,e^{6i(dx+c)}+609\,e^{4i(dx+c)}+203\,e^{2i(dx+c)}+44)}{105d(e^{2i(dx+c)}+1)^7}$	90

```
[In] int(tan(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(15*tan(d*x+c)^7-21*tan(d*x+c)^5+35*tan(d*x+c)^3+105*d*x-105*tan(d*x+
c))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \tan^8(c+dx)\,dx = \frac{15\,\tan(dx+c)^7 - 21\,\tan(dx+c)^5 + 35\,\tan(dx+c)^3 + 105\,dx - 105\,\tan(dx+c)}{105\,d}$$

```
[In] integrate(tan(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/105*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x
- 105*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \tan^8(c+dx)\,dx = \begin{cases} x + \frac{\tan^7(c+dx)}{7d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^3(c+dx)}{3d} - \frac{\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\tan^8(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(tan(d*x+c)**8,x)
```

```
[Out] Piecewise((x + tan(c + d*x)**7/(7*d) - tan(c + d*x)**5/(5*d) + tan(c + d*x)
**3/(3*d) - tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)**8, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \tan^8(c+dx)\,dx = \frac{15\,\tan(dx+c)^7 - 21\,\tan(dx+c)^5 + 35\,\tan(dx+c)^3 + 105\,dx + 105\,c - 105\,\tan(dx+c)}{105\,d}$$

```
[In] integrate(tan(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/105*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x
+ 105*c - 105*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1441 vs. $2(52) = 104$.

Time = 5.00 (sec) , antiderivative size = 1441, normalized size of antiderivative = 24.84

$$\int \tan^8(c + dx)\, dx = \text{Too large to display}$$

```
[In] integrate(tan(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 1/420*(105*pi + 420*d*x*tan(d*x)^7*tan(c)^7 - 105*pi*sgn(2*tan(d*x)^2*tan(c
) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^7*tan(c)^7 - 105*
pi*tan(d*x)^7*tan(c)^7 + 210*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c
)))*tan(d*x)^7*tan(c)^7 + 210*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) -
 1))*tan(d*x)^7*tan(c)^7 - 2940*d*x*tan(d*x)^6*tan(c)^6 + 735*pi*sgn(2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan
(c)^6 + 735*pi*tan(d*x)^6*tan(c)^6 - 1470*arctan((tan(d*x)*tan(c) - 1)/(tan
(d*x) + tan(c)))*tan(d*x)^6*tan(c)^6 - 1470*arctan((tan(d*x) + tan(c))/(tan
(d*x)*tan(c) - 1))*tan(d*x)^6*tan(c)^6 + 420*tan(d*x)^7*tan(c)^6 + 420*tan(
d*x)^6*tan(c)^7 + 8820*d*x*tan(d*x)^5*tan(c)^5 - 2205*pi*sgn(2*tan(d*x)^2*t
an(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^5 -
140*tan(d*x)^7*tan(c)^4 - 2205*pi*tan(d*x)^5*tan(c)^5 + 4410*arctan((tan(d*
x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)^5*tan(c)^5 + 4410*arctan((tan(
d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^5*tan(c)^5 - 2940*tan(d*x)^6
*tan(c)^5 - 2940*tan(d*x)^5*tan(c)^6 - 140*tan(d*x)^4*tan(c)^7 - 14700*d*x*
tan(d*x)^4*tan(c)^4 + 3675*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2
 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 84*tan(d*x)^7*tan(c)^2 + 98
0*tan(d*x)^6*tan(c)^3 + 3675*pi*tan(d*x)^4*tan(c)^4 - 7350*arctan((tan(d*x)
*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)^4*tan(c)^4 - 7350*arctan((tan(d*
x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 + 8820*tan(d*x)^5*t
an(c)^4 + 8820*tan(d*x)^4*tan(c)^5 + 980*tan(d*x)^3*tan(c)^6 + 84*tan(d*x)^
2*tan(c)^7 + 14700*d*x*tan(d*x)^3*tan(c)^3 - 3675*pi*sgn(2*tan(d*x)^2*tan(c
) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 - 60*t
an(d*x)^7 - 588*tan(d*x)^6*tan(c) - 2940*tan(d*x)^5*tan(c)^2 - 3675*pi*tan(
d*x)^3*tan(c)^3 + 7350*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*ta
n(d*x)^3*tan(c)^3 + 7350*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*
tan(d*x)^3*tan(c)^3 - 14700*tan(d*x)^4*tan(c)^3 - 14700*tan(d*x)^3*tan(c)^4
 - 2940*tan(d*x)^2*tan(c)^5 - 588*tan(d*x)*tan(c)^6 - 60*tan(c)^7 - 8820*d*
x*tan(d*x)^2*tan(c)^2 + 2205*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 84*tan(d*x)^5 + 980*tan(d
*x)^4*tan(c) + 2205*pi*tan(d*x)^2*tan(c)^2 - 4410*arctan((tan(d*x)*tan(c) -
 1)/(tan(d*x) + tan(c)))*tan(d*x)^2*tan(c)^2 - 4410*arctan((tan(d*x) + tan(
c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 + 8820*tan(d*x)^3*tan(c)^2 +
 8820*tan(d*x)^2*tan(c)^3 + 980*tan(d*x)*tan(c)^4 + 84*tan(c)^5 + 2940*d*x*
tan(d*x)*tan(c) - 735*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*
```

```
tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - 140*tan(d*x)^3 - 735*pi*tan(d*x)*tan
(c) + 1470*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c
) + 1470*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c)
- 2940*tan(d*x)^2*tan(c) - 2940*tan(d*x)*tan(c)^2 - 140*tan(c)^3 - 420*d*x
+ 105*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan
(c)) - 210*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 210*arctan((
tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) + 420*tan(d*x) + 420*tan(c))/(d*t
an(d*x)^7*tan(c)^7 - 7*d*tan(d*x)^6*tan(c)^6 + 21*d*tan(d*x)^5*tan(c)^5 - 3
5*d*tan(d*x)^4*tan(c)^4 + 35*d*tan(d*x)^3*tan(c)^3 - 21*d*tan(d*x)^2*tan(c)
^2 + 7*d*tan(d*x)*tan(c) - d)
```

Mupad [B] (verification not implemented)

Time = 2.99 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \tan^8(c+dx)\,dx = x - \frac{-\frac{\tan(c+d\,x)^7}{7} + \frac{\tan(c+d\,x)^5}{5} - \frac{\tan(c+d\,x)^3}{3} + \tan(c+d\,x)}{d}$$

```
[In] int(tan(c + d*x)^8,x)
```

```
[Out] x - (tan(c + d*x) - tan(c + d*x)^3/3 + tan(c + d*x)^5/5 - tan(c + d*x)^7/7)
/d
```

3.9 $\int (b\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 232

$$\begin{aligned}\int (b\tan(c+dx))^{7/2}\,dx =& -\frac{b^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} \\ &+ \frac{b^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} \\ &- \frac{b^{7/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} \\ &+ \frac{b^{7/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} \\ &- \frac{2b^3\sqrt{b\tan(c+dx)}}{d}+\frac{2b(b\tan(c+dx))^{5/2}}{5d}\end{aligned}$$

```
[Out] -1/2*b^(7/2)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)+1/2*b
^(7/2)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)-1/4*b^(7/2)
*ln(b^(1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2)+1/4*
b^(7/2)*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/
2)-2*b^3*(b*tan(d*x+c))^(1/2)/d+2/5*b*(b*tan(d*x+c))^(5/2)/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int (b\tan(c+dx))^{7/2}\,dx = &-\frac{b^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}\\ &+\frac{b^{7/2}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}d}\\ &-\frac{b^{7/2}\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}d}\\ &+\frac{b^{7/2}\log\left(\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}d}\\ &-\frac{2b^3\sqrt{b\tan(c+dx)}}{d}+\frac{2b(b\tan(c+dx))^{5/2}}{5d}\end{aligned}$$

```
[In] Int[(b*Tan[c + d*x])^(7/2),x]
```

```
[Out] -((b^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d))
 + (b^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d)
 - (b^(7/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x
]]])/(2*Sqrt[2]*d) + (b^(7/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*
Sqrt[b*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (2*b^3*Sqrt[b*Tan[c + d*x]])/d + (2*
b*(b*Tan[c + d*x])^(5/2))/(5*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{2b(b\tan(c+dx))^{5/2}}{5d} - b^2\int (b\tan(c+dx))^{3/2}\,dx$$

$$=-\frac{2b^3\sqrt{b\tan(c+dx)}}{d}+\frac{2b(b\tan(c+dx))^{5/2}}{5d}+b^4\int\frac{1}{\sqrt{b\tan(c+dx)}}\,dx$$

$$=-\frac{2b^3\sqrt{b\tan(c+dx)}}{d}+\frac{2b(b\tan(c+dx))^{5/2}}{5d}+\frac{b^5\text{Subst}\left(\int\frac{1}{\sqrt{x}(b^2+x^2)}\,dx,x,b\tan(c+dx)\right)}{d}$$

$$=-\frac{2b^3\sqrt{b\tan(c+dx)}}{d}+\frac{2b(b\tan(c+dx))^{5/2}}{5d}+\frac{(2b^5)\,\text{Subst}\left(\int\frac{1}{b^2+x^4}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{d}$$

$$=-\frac{2b^3\sqrt{b\tan(c+dx)}}{d}+\frac{2b(b\tan(c+dx))^{5/2}}{5d}+\frac{b^4\text{Subst}\left(\int\frac{b-x^2}{b^2+x^4}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{d}+\frac{b^4\text{Subst}\left(\int\frac{b+x^2}{b^2+x^4}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{d}$$

$$=-\frac{2b^3\sqrt{b\tan(c+dx)}}{d}+\frac{2b(b\tan(c+dx))^{5/2}}{5d}-\frac{b^{7/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}-\frac{b^{7/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}+\frac{b^4\text{Subst}\left(\int\frac{1}{b-\sqrt{2}\sqrt{b}x+x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2d}+\frac{b^4\text{Subst}\left(\int\frac{1}{b+\sqrt{2}\sqrt{b}x+x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2d}$$

$$=-\frac{b^{7/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}+\frac{b^{7/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}-\frac{2b^3\sqrt{b\tan(c+dx)}}{d}+\frac{2b(b\tan(c+dx))^{5/2}}{5d}+\frac{b^{7/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}-\frac{b^{7/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}$$

$$= -\frac{b^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}+\frac{b^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}$$
$$-\frac{b^{7/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}$$
$$+\frac{b^{7/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}$$
$$-\frac{2b^3\sqrt{b\tan(c+dx)}}{d}+\frac{2b(b\tan(c+dx))^{5/2}}{5d}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.75

$$\int (b\tan(c+dx))^{7/2}\,dx = \frac{(b\tan(c+dx))^{7/2}\left(-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+\right.}{2\sqrt{2}}\right.}{d\tan^{\frac{7}{2}}(c+dx)}$$

```
[In] Integrate[(b*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((b*Tan[c + d*x])^(7/2)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])
+ ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]]/(2*Sqrt[2]) - 2*Sqrt[Tan[c + d*x]] + (2*Tan[c + d*x]^(5/2)
)/5))/(d*Tan[c + d*x]^(7/2))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{5}{2}}}{5}-b^2\sqrt{b\tan(dx+c)}+\frac{b^2(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(d}}{(b^2)^{\frac{1}{4}}}\right.\right.}{8}\right.}{d}$
default	$\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{5}{2}}}{5}-b^2\sqrt{b\tan(dx+c)}+\frac{b^2(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(d}}{(b^2)^{\frac{1}{4}}}\right.\right.}{8}\right.}{d}$

```
[In] int((b*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*(1/5*(b*tan(d*x+c))^(5/2)-b^2*(b*tan(d*x+c))^(1/2)+1/8*b^2*(b^2)^(1/4
)*2^(1/2)*(ln((b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^
(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))
+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^
2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.88

$$\int (b\tan(c+dx))^{7/2}\,dx = \frac{5\left(-\frac{b^{14}}{d^4}\right)^{\frac{1}{4}} d\log\left(\sqrt{b\tan(dx+c)}b^3+\left(-\frac{b^{14}}{d^4}\right)^{\frac{1}{4}} d\right)+5i\left(-\frac{b^{14}}{d^4}\right)^{\frac{1}{4}} d\log\left(\sqrt{b\tan(dx+c)}b^3+i\right.}{}$$

```
[In] integrate((b*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/10*(5*(-b^14/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b^3 + (-b^14/d^4)^(1/4
)*d) + 5*I*(-b^14/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b^3 + I*(-b^14/d^4)
^(1/4)*d) - 5*I*(-b^14/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b^3 - I*(-b^14
/d^4)^(1/4)*d) - 5*(-b^14/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b^3 - (-b^1
4/d^4)^(1/4)*d) + 4*(b^3*tan(d*x + c)^2 - 5*b^3)*sqrt(b*tan(d*x + c)))/d
```

Sympy [F]

$$\int (b\tan(c+dx))^{7/2}\,dx = \int (b\tan(c+dx))^{\frac{7}{2}}\,dx$$

```
[In] integrate((b*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.80

$$\int (b\tan(c+dx))^{7/2}\,dx = \frac{10\sqrt{2}b^{\frac{9}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+10\sqrt{2}b^{\frac{9}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+5\sqrt{}$$

```
[In] integrate((b*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/20*(10*sqrt(2)*b^(9/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan
(d*x + c)))/sqrt(b)) + 10*sqrt(2)*b^(9/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(b) - 2*sqrt(b*tan(d*x + c)))/sqrt(b)) + 5*sqrt(2)*b^(9/2)*log(b*tan(d*x +
c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - 5*sqrt(2)*b^(9/2)*log(b*ta
n(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) + 8*(b*tan(d*x + c))
^(5/2)*b^2 - 40*sqrt(b*tan(d*x + c))*b^4)/(b*d)
```

Giac [F(-1)]

Timed out.

$$\int (b\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((b*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.40

$$\int (b\tan(c+dx))^{7/2}\,dx = \frac{2\,b\,(b\tan(c+d\,x))^{5/2}}{5\,d} - \frac{2\,b^3\,\sqrt{b\tan(c+d\,x)}}{d} - \frac{(-1)^{1/4}\,b^{7/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1i}{d} - \frac{(-1)^{1/4}\,b^{7/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}\,1i}{\sqrt{b}}\right)}{d}$$

```
[In] int((b*tan(c + d*x))^(7/2),x)
```

```
[Out] (2*b*(b*tan(c + d*x))^(5/2))/(5*d) - (2*b^3*(b*tan(c + d*x))^(1/2))/d - ((-
1)^(1/4)*b^(7/2)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/d -
((-1)^(1/4)*b^(7/2)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2)*1i)/b^(1/2)))/d
```

3.10 $\int (b\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 212

$$\int (b\tan(c+dx))^{5/2}\,dx = \frac{b^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{b^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{b^{5/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{b^{5/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{2b(b\tan(c+dx))^{3/2}}{3d}$$

```
[Out] 1/2*b^(5/2)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)-1/2*b^
(5/2)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)-1/4*b^(5/2)*
ln(b^(1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2)+1/4*b
^(5/2)*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2
)+2/3*b*(b*tan(d*x+c))^(3/2)/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used

= {3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int (b\tan(c+dx))^{5/2}\,dx = \frac{b^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}d} - \frac{b^{5/2}\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}d} + \frac{b^{5/2}\log\left(\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}d} + \frac{2b(b\tan(c+dx))^{3/2}}{3d}$$

```
[In] Int[(b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (b^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) -
(b^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) -
(b^(5/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]
)/(2*Sqrt[2]*d) + (b^(5/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqr
t[b*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (2*b*(b*Tan[c + d*x])^(3/2))/(3*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(b\tan(c+dx))^{3/2}}{3d} - b^2\int\sqrt{b\tan(c+dx)}\,dx\\
&= \frac{2b(b\tan(c+dx))^{3/2}}{3d} - \frac{b^3\text{Subst}\left(\int\frac{\sqrt{x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}\\
&= \frac{2b(b\tan(c+dx))^{3/2}}{3d} - \frac{(2b^3)\,\text{Subst}\left(\int\frac{x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d}
\end{aligned}$$

$$=\frac{2b(b\tan(c+dx))^{3/2}}{3d}+\frac{b^3\text{Subst}\left(\int\frac{b-x^2}{b^2+x^4}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{d}-\frac{b^3\text{Subst}\left(\int\frac{b+x^2}{b^2+x^4}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{d}$$

$$=\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{b^{5/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}-\frac{b^{5/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}-\frac{b^3\text{Subst}\left(\int\frac{1}{b-\sqrt{2}\sqrt{b}x+x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2d}-\frac{b^3\text{Subst}\left(\int\frac{1}{b+\sqrt{2}\sqrt{b}x+x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2d}$$

$$=-\frac{b^{5/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}+\frac{b^{5/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}+\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{b^{5/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}+\frac{b^{5/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}$$

$$=\frac{b^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}-\frac{b^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}-\frac{b^{5/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}+\frac{b^{5/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}+\frac{2b(b\tan(c+dx))^{3/2}}{3d}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.48

$$\int (b\tan(c+dx))^{5/2}\,dx = \frac{b(b\tan(c+dx))^{3/2}\left(-3\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan(c+dx)}+3\text{arctanh}\left(\sqrt[4]{-\tan^2(c\right.\right.}{3d\tan^{\frac{7}{4}}(c+dx)}$$

[In] Integrate[(b*Tan[c + d*x])^(5/2),x]

[Out] (b*(b*Tan[c + d*x])^(3/2)*(-3*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x
])^(1/4) + 3*ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x])^(1/4) + 2*Tan
[c + d*x]^(7/4)))/(3*d*Tan[c + d*x]^(7/4))

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{b^2\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{b}}{(}\right.\right.}{8(b^2)^{\frac{1}{4}}}\right.}{d}$
default	$\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{b^2\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{b}}{(}\right.\right.}{8(b^2)^{\frac{1}{4}}}\right.}{d}$

[In] int((b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/d*b*(1/3*(b*tan(d*x+c))^(3/2)-1/8*b^2/(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+
c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2
)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(
1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(
1/2)+1)))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95

$$\int (b\tan(c+dx))^{5/2}\,dx = \frac{4\sqrt{b\tan(dx+c)}b^2\tan(dx+c) - 3\left(-\frac{b^{10}}{d^4}\right)^{\frac{1}{4}} d\log\left(\sqrt{b\tan(dx+c)}b^7 + \left(-\frac{b^{10}}{d^4}\right)^{\frac{3}{4}} d^3\right) + 3i\left(\right.}{}$$

```
[In] integrate((b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(b*tan(d*x + c))*b^2*tan(d*x + c) - 3*(-b^10/d^4)^(1/4)*d*log(sq
rt(b*tan(d*x + c))*b^7 + (-b^10/d^4)^(3/4)*d^3) + 3*I*(-b^10/d^4)^(1/4)*d*l
og(sqrt(b*tan(d*x + c))*b^7 + I*(-b^10/d^4)^(3/4)*d^3) - 3*I*(-b^10/d^4)^(1
/4)*d*log(sqrt(b*tan(d*x + c))*b^7 - I*(-b^10/d^4)^(3/4)*d^3) + 3*(-b^10/d^
4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b^7 - (-b^10/d^4)^(3/4)*d^3))/d
```

Sympy [F]

$$\int (b\tan(c+dx))^{5/2}\,dx = \int (b\tan(c+dx))^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.83

$$\int (b\tan(c+dx))^{5/2}\,dx = -\frac{3\,b^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}} - \frac{\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}\right.}{\sqrt{b}}\right.}{12\,bd}$$

```
[In] integrate((b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(
d*x + c)))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b
```

```
) - 2*sqrt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c) +
 sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(d*x
+ c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b)) - 8*(b*tan(d*x +
c))^(3/2)*b^2)/(b*d)
```

Giac [F(-1)]

Timed out.

$$\int (b\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

$$\int (b\tan(c+dx))^{5/2}\,dx = \frac{2\,b\,(b\tan(c+d\,x))^{3/2}}{3\,d} - \frac{(-1)^{1/4}\,b^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{d} + \frac{(-1)^{1/4}\,b^{5/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{d}$$

```
[In] int((b*tan(c + d*x))^(5/2),x)
```

```
[Out] (2*b*(b*tan(c + d*x))^(3/2))/(3*d) - ((-1)^(1/4)*b^(5/2)*atan(((-1)^(1/4)*(
b*tan(c + d*x))^(1/2))/b^(1/2)))/d + ((-1)^(1/4)*b^(5/2)*atanh(((-1)^(1/4)*
(b*tan(c + d*x))^(1/2))/b^(1/2)))/d
```

3.11 $\int (b\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 210

$$\int (b\tan(c+dx))^{3/2}\,dx = \frac{b^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{b^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{b^{3/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{b^{3/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{2b\sqrt{b\tan(c+dx)}}{d}$$

```
[Out] 1/2*b^(3/2)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)-1/2*b^
(3/2)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)+1/4*b^(3/2)*
ln(b^(1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2)-1/4*b
^(3/2)*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2
)+2*b*(b*tan(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used

$= \{3554, 3557, 335, 217, 1179, 642, 1176, 631, 210\}$

$$\int (b\tan(c+dx))^{3/2}\,dx = \frac{b^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{b^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}d} + \frac{b^{3/2}\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}d} - \frac{b^{3/2}\log\left(\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}d} + \frac{2b\sqrt{b\tan(c+dx)}}{d}$$

```
[In] Int[(b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) -
(b^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) +
(b^(3/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]
)/(2*Sqrt[2]*d) - (b^(3/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqr
t[b*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (2*b*Sqrt[b*Tan[c + d*x]])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2b\sqrt{b\tan(c+dx)}}{d} - b^2\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx\\ &= \frac{2b\sqrt{b\tan(c+dx)}}{d} - \frac{b^3\text{Subst}\left(\int \frac{1}{\sqrt{x}(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}\\ &= \frac{2b\sqrt{b\tan(c+dx)}}{d} - \frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d}\end{aligned}$$

$$=\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{b^2\text{Subst}\left(\int\frac{b-x^2}{b^2+x^4}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{d}-\frac{b^2\text{Subst}\left(\int\frac{b+x^2}{b^2+x^4}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{d}$$

$$=\frac{2b\sqrt{b\tan(c+dx)}}{d}+\frac{b^{3/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}+\frac{b^{3/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}-\frac{b^2\text{Subst}\left(\int\frac{1}{b-\sqrt{2}\sqrt{b}x+x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2d}-\frac{b^2\text{Subst}\left(\int\frac{1}{b+\sqrt{2}\sqrt{b}x+x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2d}$$

$$=\frac{b^{3/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}-\frac{b^{3/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}+\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{b^{3/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}+\frac{b^{3/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}$$

$$=\frac{b^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}-\frac{b^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}+\frac{b^{3/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}-\frac{b^{3/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}+\frac{2b\sqrt{b\tan(c+dx)}}{d}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.76

$$\int (b\tan(c+dx))^{3/2}\,dx = \frac{\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(\right.}{2}\right.}{d\tan^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[(b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[
Tan[c + d*x]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]
/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2
]) + 2*Sqrt[Tan[c + d*x]])*(b*Tan[c + d*x])^(3/2))/(d*Tan[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{2b\left(\sqrt{b\tan(dx+c)} - \frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8}\right.}{d}$
default	$\frac{2b\left(\sqrt{b\tan(dx+c)} - \frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8}\right.}{d}$

```
[In] int((b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*((b*tan(d*x+c))^(1/2)-1/8*(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)+(b^2)
^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*
(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*
tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.84

$$\int (b\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{\left(-\frac{b^6}{d^4}\right)^{\frac{1}{4}} d\log\left(\sqrt{b\tan\left(dx+c\right)}b+\left(-\frac{b^6}{d^4}\right)^{\frac{1}{4}} d\right)+i\left(-\frac{b^6}{d^4}\right)^{\frac{1}{4}} d\log\left(\sqrt{b\tan\left(dx+c\right)}b+i\left(-\frac{b^6}{d^4}\right)^{\frac{1}{4}} d\right)-i\left(-\right.}{}$$

```
[In] integrate((b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*((-b^6/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b + (-b^6/d^4)^(1/4)*d) +
 I*(-b^6/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b + I*(-b^6/d^4)^(1/4)*d) -
I*(-b^6/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b - I*(-b^6/d^4)^(1/4)*d) - (
-b^6/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b - (-b^6/d^4)^(1/4)*d) - 4*sqrt
(b*tan(d*x + c))*b)/d
```

Sympy [F]

$$\int (b\tan(c+dx))^{3/2}\,dx = \int \left(b\tan\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.81

$$\int (b\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{2\sqrt{2}b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+2\sqrt{2}b^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+\sqrt{2}b^{\frac{5}{2}}\log\left(b\tan\left(dx\right.\right.}{}$$

```
[In] integrate((b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*b^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(
d*x + c)))/sqrt(b)) + 2*sqrt(2)*b^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b
) - 2*sqrt(b*tan(d*x + c)))/sqrt(b)) + sqrt(2)*b^(5/2)*log(b*tan(d*x + c) +
 sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - sqrt(2)*b^(5/2)*log(b*tan(d*x
+ c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - 8*sqrt(b*tan(d*x + c))*b
^2)/(b*d)
```

Giac [F]

$$\int (b\tan(c+dx))^{3/2}\,dx = \int (b\tan(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

$$\int (b\tan(c+dx))^{3/2}\,dx = \frac{2\,b\,\sqrt{b\tan(c+d\,x)}}{d} + \frac{(-1)^{1/4}\,b^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{d} + \frac{(-1)^{1/4}\,b^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{d}$$

```
[In] int((b*tan(c + d*x))^(3/2),x)
```

```
[Out] (2*b*(b*tan(c + d*x))^(1/2))/d + ((-1)^(1/4)*b^(3/2)*atan(((-1)^(1/4)*(b*ta
n(c + d*x))^(1/2))/b^(1/2))*1i)/d + ((-1)^(1/4)*b^(3/2)*atanh(((-1)^(1/4)*(
b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/d
```

3.12 $\int \sqrt{b \tan(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 192

$$\int \sqrt{b \tan(c+dx)}\,dx = -\frac{\sqrt{b}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{\sqrt{b}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d*2^(1/2)+1/2*a
rctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d*2^(1/2)+1/4*ln(b^(1
/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))*b^(1/2)/d*2^(1/2)-1/4*
ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))*b^(1/2)/d*2^(1/
2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used

= {3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \sqrt{b\tan(c+dx)}\,dx = -\frac{\sqrt{b}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}d} + \frac{\sqrt{b}\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}d} - \frac{\sqrt{b}\log\left(\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}d}$$

```
[In] Int[Sqrt[b*Tan[c + d*x]],x]
```

```
[Out] -((Sqrt[b]*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d))
 + (Sqrt[b]*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d)
 + (Sqrt[b]*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x
]]])/(2*Sqrt[2]*d) - (Sqrt[b]*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*
Sqrt[b*Tan[c + d*x]]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{\sqrt{x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d} \\
&= \frac{(2b)\text{Subst}\left(\int \frac{x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} \\
&= -\frac{b\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} + \frac{b\text{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} \\
&= \frac{\sqrt{b}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad + \frac{\sqrt{b}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad + \frac{b\text{Subst}\left(\int \frac{1}{b-\sqrt{2}\sqrt{b}x+x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2d} \\
&\quad + \frac{b\text{Subst}\left(\int \frac{1}{b+\sqrt{2}\sqrt{b}x+x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2d}
\end{aligned}$$

$$= \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{\sqrt{b}\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{\sqrt{b}\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}$$

$$= -\frac{\sqrt{b}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{\sqrt{b}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.37

$$\int\sqrt{b\tan(c+dx)}\,dx = \frac{\left(\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)-\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(c+dx)}\right)\right)\sqrt[4]{-\tan(c+dx)}\sqrt{b\tan(c+dx)}}{d\tan^{\frac{3}{4}}(c+dx)}$$

```
[In] Integrate[Sqrt[b*Tan[c + d*x]],x]
[Out] ((ArcTan[(-Tan[c + d*x]^2)^(1/4)] - ArcTanh[(-Tan[c + d*x]^2)^(1/4)])*(-Tan
[c + d*x])^(1/4)*Sqrt[b*Tan[c + d*x]])/(d*Tan[c + d*x]^(3/4))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{b\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^{2}}}{b\tan(dx+c)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^{2}\right)^{\frac{1}{4}}}+1\right)\right.}{4d\left(b^{2}\right)^{\frac{1}{4}}}$
default	$\frac{b\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^{2}}}{b\tan(dx+c)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^{2}\right)^{\frac{1}{4}}}+1\right)\right.}{4d\left(b^{2}\right)^{\frac{1}{4}}}$

```
[In] int((b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*b/(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1
/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(
1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*a
rctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\int\sqrt{b\tan(c+dx)}\,dx=\frac{1}{2}\left(-\frac{b^{2}}{d^{4}}\right)^{\frac{1}{4}}\log\left(d^{3}\left(-\frac{b^{2}}{d^{4}}\right)^{\frac{3}{4}}+\sqrt{b\tan\left(dx+c\right)}b\right)-\frac{1}{2}i\left(-\frac{b^{2}}{d^{4}}\right)^{\frac{1}{4}}\log\left(i\,d^{3}\left(-\frac{b^{2}}{d^{4}}\right)^{\frac{3}{4}}+\sqrt{b\tan\left(dx+c\right)}b\right)+\frac{1}{2}i\left(-\frac{b^{2}}{d^{4}}\right)^{\frac{1}{4}}\log\left(-i\,d^{3}\left(-\frac{b^{2}}{d^{4}}\right)^{\frac{3}{4}}+\sqrt{b\tan\left(dx+c\right)}b\right)-\frac{1}{2}\left(-\frac{b^{2}}{d^{4}}\right)^{\frac{1}{4}}\log\left(-d^{3}\left(-\frac{b^{2}}{d^{4}}\right)^{\frac{3}{4}}+\sqrt{b\tan\left(dx+c\right)}b\right)$$

```
[In] integrate((b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-b^2/d^4)^(1/4)*log(d^3*(-b^2/d^4)^(3/4) + sqrt(b*tan(d*x + c))*b) - 1
/2*I*(-b^2/d^4)^(1/4)*log(I*d^3*(-b^2/d^4)^(3/4) + sqrt(b*tan(d*x + c))*b)
+ 1/2*I*(-b^2/d^4)^(1/4)*log(-I*d^3*(-b^2/d^4)^(3/4) + sqrt(b*tan(d*x + c))
*b) - 1/2*(-b^2/d^4)^(1/4)*log(-d^3*(-b^2/d^4)^(3/4) + sqrt(b*tan(d*x + c))
*b)
```

Sympy [F]

$$\int \sqrt{b \tan(c + dx)}\, dx = \int \sqrt{b \tan{(c + dx)}}\, dx$$

[In] integrate((b*tan(d*x+c))**(1/2),x)

[Out] Integral(sqrt(b*tan(c + d*x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.80

$$\int \sqrt{b \tan(c + dx)}\, dx = \frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}} - \frac{\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{\sqrt{b}}\right.}{4\,d}$$

[In] integrate((b*tan(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x + c
)))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) - 2*s
qrt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c) + sqrt(2
)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(d*x + c) -
sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b))/d

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.92

$$\int \sqrt{b \tan(c + dx)}\, dx = \frac{\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d} + \frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d} - \frac{\sqrt{2}|b|^{\frac{3}{2}}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(d\right.}{d}}{4\,b}$$

[In] integrate((b*tan(d*x+c))^(1/2),x, algorithm="giac")

```
[Out] 1/4*(2*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*sq
rt(b*tan(d*x + c)))/sqrt(abs(b)))/d + 2*sqrt(2)*abs(b)^(3/2)*arctan(-1/2*sq
rt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d - sqr
t(2)*abs(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(ab
s(b)) + abs(b))/d + sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) - sqrt(2)*sqrt(
b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/d)/b
```

Mupad [B] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.26

$$\int \sqrt{b\tan(c+dx)}\,dx = \frac{(-1)^{1/4}\sqrt{b}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)\right)}{d}$$

```
[In] int((b*tan(c + d*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*b^(1/2)*(atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)) - at
anh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))))/d
```

3.13 $\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 192

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}d}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)/b^(1/2)+1/2*a
rctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)/b^(1/2)-1/4*ln(b^(1
/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2)/b^(1/2)+1/4*
ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2)/b^(1/
2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used

= {3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}\sqrt{b}d} - \frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\sqrt{b}d} + \frac{\log\left(\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\sqrt{b}d}$$

```
[In] Int[1/Sqrt[b*Tan[c + d*x]],x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*Sqrt[b]*d)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*Sqrt[b]*d) - L
og[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2
]*Sqrt[b]*d) + Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c +
d*x]]]/(2*Sqrt[2]*Sqrt[b]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{x}(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d} \\
&= \frac{(2b)\text{Subst}\left(\int \frac{1}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{b-\sqrt{2}\sqrt{b}x+x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{b+\sqrt{2}\sqrt{b}x+x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2d} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}d} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}d}
\end{aligned}
$$

$$= -\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}d}$$
$$+\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}d}$$
$$+\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d}$$
$$-\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d}$$
$$-\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}d}$$
$$+\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}d}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.68

$$\int\frac{1}{\sqrt{b\tan(c+dx)}}\,dx$$
$$=\frac{\left(-2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\text{ta}\right.\right.}{2\sqrt{2}d\sqrt{b\tan(c+dx)}}$$

```
[In] Integrate[1/Sqrt[b*Tan[c + d*x]],x]
```

```
[Out] ((-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan
[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + Log[1 +
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*
d*Sqrt[b*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}\right.\right.}{4db}$
default	$\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}\right.\right.}{4db}$

```
[In] int(1/(b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/b*(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1
/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(
1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*a
rctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx &= \frac{1}{2}\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\log\left(bd\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}+\sqrt{b\tan(dx+c)}\right)\\ &+\frac{1}{2}i\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\log\left(i\,bd\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}+\sqrt{b\tan(dx+c)}\right)\\ &-\frac{1}{2}i\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\log\left(-i\,bd\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}+\sqrt{b\tan(dx+c)}\right)\\ &-\frac{1}{2}\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\log\left(-bd\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}+\sqrt{b\tan(dx+c)}\right)\end{aligned}$$

```
[In] integrate(1/(b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-1/(b^2*d^4))^(1/4)*log(b*d*(-1/(b^2*d^4))^(1/4) + sqrt(b*tan(d*x + c)
)) + 1/2*I*(-1/(b^2*d^4))^(1/4)*log(I*b*d*(-1/(b^2*d^4))^(1/4) + sqrt(b*tan
(d*x + c))) - 1/2*I*(-1/(b^2*d^4))^(1/4)*log(-I*b*d*(-1/(b^2*d^4))^(1/4) +
sqrt(b*tan(d*x + c))) - 1/2*(-1/(b^2*d^4))^(1/4)*log(-b*d*(-1/(b^2*d^4))^(1
/4) + sqrt(b*tan(d*x + c)))
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan{(c+dx)}}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$$
$$= \frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+\sqrt{2}\sqrt{b}\log\left(b\tan\right.}{4\,bd}$$

```
[In] integrate(1/(b*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d
*x + c)))/sqrt(b)) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b)
 - 2*sqrt(b*tan(d*x + c)))/sqrt(b)) + sqrt(2)*sqrt(b)*log(b*tan(d*x + c) +
sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - sqrt(2)*sqrt(b)*log(b*tan(d*x +
 c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b))/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{2\,bd} + \frac{\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{2\,bd} + \frac{\sqrt{2}\sqrt{|b|}\log\left(b\tan{(dx+c)}+\sqrt{2}\sqrt{b\tan{(dx+c)}}\sqrt{|b|}+|b|\right)}{4\,bd} - \frac{\sqrt{2}\sqrt{|b|}\log\left(b\tan{(dx+c)}-\sqrt{2}\sqrt{b\tan{(dx+c)}}\sqrt{|b|}+|b|\right)}{4\,bd}$$

```
[In] integrate(1/(b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*sqrt(
b*tan(d*x + c)))/sqrt(abs(b)))/(b*d) + 1/2*sqrt(2)*sqrt(abs(b))*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b*d
) + 1/4*sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x +
c))*sqrt(abs(b)) + abs(b))/(b*d) - 1/4*sqrt(2)*sqrt(abs(b))*log(b*tan(d*x +
 c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b*d)
```

Mupad [B] (verification not implemented)

Time = 3.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.31

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = -\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{\sqrt{b}\,d} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{\sqrt{b}\,d}$$

```
[In] int(1/(b*tan(c + d*x))^(1/2),x)
```

```
[Out] - ((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/(b^(1/2
)*d) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/(
b^(1/2)*d)
```

3.14 $\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 212

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{3/2}d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{3/2}d} - \frac{2}{bd\sqrt{b\tan(c+dx)}}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d*2^(1/2)-1/2*ar
ctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d*2^(1/2)-1/4*ln(b^(1/
2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(3/2)/d*2^(1/2)+1/4*l
n(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(3/2)/d*2^(1/2
)-2/b/d/(b*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used

= {3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}b^{3/2}d} - \frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}b^{3/2}d} + \frac{\log\left(\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}b^{3/2}d} - \frac{2}{bd\sqrt{b\tan(c+dx)}}$$

```
[In] Int[(b*Tan[c + d*x])^(-3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(3/2)*d) - Ar
cTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(3/2)*d) - Log[
Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2]*b
^(3/2)*d) + Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c + d*x
]]]/(2*Sqrt[2]*b^(3/2)*d) - 2/(b*d*Sqrt[b*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{2}{bd\sqrt{b\tan(c+dx)}} - \frac{\int \sqrt{b\tan(c+dx)}\,dx}{b^2}$$

$$= -\frac{2}{bd\sqrt{b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{b\tan(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{b\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{bd} - \frac{\text{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{3/2}d} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{3/2}d} - \frac{\text{Subst}\left(\int \frac{1}{b-\sqrt{2}\sqrt{b}x+x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2bd} - \frac{\text{Subst}\left(\int \frac{1}{b+\sqrt{2}\sqrt{b}x+x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2bd}$$

$$= -\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{3/2}d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{3/2}d} - \frac{2}{bd\sqrt{b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d}$$

$$= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{3/2}d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{3/2}d} - \frac{2}{bd\sqrt{b\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.39

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = \frac{-2-\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan^2(c+dx)}+\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(c+dx)}\right.}{bd\sqrt{b\tan(c+dx)}}$$

[In] Integrate[(b*Tan[c + d*x])^(-3/2),x]

[Out] (-2 - ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4) + ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4))/(b*d*Sqrt[b*Tan[c + d*x]])

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{2b\left(-\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^2\right)^{\frac{1}{4}}}+1\right)\right)}{8b^2\left(b^2\right)^{\frac{1}{4}}}\right.}{d}$
default	$\frac{2b\left(-\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^2\right)^{\frac{1}{4}}}+1\right)\right)}{8b^2\left(b^2\right)^{\frac{1}{4}}}\right.}{d}$

[In] int(1/(b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/d*b*(-1/8/b^2/(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))-1/b^2/(b*tan(d*x+c))^(1/2))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.11

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx =$$
$$-\frac{b^2d\left(-\frac{1}{b^6d^4}\right)^{\frac{1}{4}}\log\left(b^5d^3\left(-\frac{1}{b^6d^4}\right)^{\frac{3}{4}}+\sqrt{b\tan\left(dx+c\right)}\right)\tan\left(dx+c\right)-i\,b^2d\left(-\frac{1}{b^6d^4}\right)^{\frac{1}{4}}\log\left(i\,b^5d^3\left(-\frac{1}{b^6d^4}\right)^{\frac{3}{4}}+\sqrt{}\right.}{}$$

```
[In] integrate(1/(b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d*(-1/(b^6*d^4))^(1/4)*log(b^5*d^3*(-1/(b^6*d^4))^(3/4) + sqrt(b*
tan(d*x + c)))*tan(d*x + c) - I*b^2*d*(-1/(b^6*d^4))^(1/4)*log(I*b^5*d^3*(-
1/(b^6*d^4))^(3/4) + sqrt(b*tan(d*x + c)))*tan(d*x + c) + I*b^2*d*(-1/(b^6*
d^4))^(1/4)*log(-I*b^5*d^3*(-1/(b^6*d^4))^(3/4) + sqrt(b*tan(d*x + c)))*tan
(d*x + c) - b^2*d*(-1/(b^6*d^4))^(1/4)*log(-b^5*d^3*(-1/(b^6*d^4))^(3/4) +
sqrt(b*tan(d*x + c)))*tan(d*x + c) + 4*sqrt(b*tan(d*x + c)))/(b^2*d*tan(d*x
 + c))
```

Sympy [F]

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.79

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx =$$
$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}-\frac{\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{\sqrt{b}}+}{4\,bd}$$

```
[In] integrate(1/(b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x + c)
))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) - 2*sq
rt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c) + sqrt(2)
*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(d*x + c) - s
qrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + 8/sqrt(b*tan(d*x + c)))/
(b*d)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.97 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d} - \frac{2}{b\,d\,\sqrt{b\tan\left(c+d\,x\right)}}$$

```
[In] int(1/(b*tan(c + d*x))^(3/2),x)
```

```
[Out] ((-1)^(1/4)*atanh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(3/2)*d)
 - ((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(3/2)*
d) - 2/(b*d*(b*tan(c + d*x))^(1/2))
```

3.15 $\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 214

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{5/2}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{5/2}d} - \frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d*2^(1/2)-1/2*ar
ctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d*2^(1/2)+1/4*ln(b^(1/
2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(5/2)/d*2^(1/2)-1/4*l
n(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(5/2)/d*2^(1/2
)-2/3/b/d/(b*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used

= {3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}b^{5/2}d} + \frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}b^{5/2}d} - \frac{\log\left(\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}b^{5/2}d} - \frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

```
[In] Int[(b*Tan[c + d*x])^(-5/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(5/2)*d) - Ar
cTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(5/2)*d) + Log[
Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2]*b
^(5/2)*d) - Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c + d*x
]]]/(2*Sqrt[2]*b^(5/2)*d) - 2/(3*b*d*(b*Tan[c + d*x])^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2}{3bd(b\tan(c+dx))^{3/2}} - \frac{\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx}{b^2} \\
&= -\frac{2}{3bd(b\tan(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{bd} \\
&= -\frac{2}{3bd(b\tan(c+dx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{bd}
\end{aligned}$$

$$= -\frac{2}{3bd(b\tan(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{b^2d} - \frac{\text{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{b^2d}$$

$$= -\frac{2}{3bd(b\tan(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{5/2}d} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{5/2}d} - \frac{\text{Subst}\left(\int \frac{1}{b-\sqrt{2}\sqrt{b}x+x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2b^2d} - \frac{\text{Subst}\left(\int \frac{1}{b+\sqrt{2}\sqrt{b}x+x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2b^2d}$$

$$= \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{5/2}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{5/2}d} - \frac{2}{3bd(b\tan(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d}$$

$$= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{5/2}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{5/2}d} - \frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.40

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = \frac{-2+3\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)(-\tan^2(c+dx))^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\tan^2(c\right.}{3bd(b\tan(c+dx))^{3/2}}$$

```
[In] Integrate[(b*Tan[c + d*x])^(-5/2),x]
```

```
[Out] (-2 + 3*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(3/4) + 3*ArcTanh
[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(3/4))/(3*b*d*(b*Tan[c + d*x])^
(3/2))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{2b\left(-\frac{1}{3b^2(b\tan(dx+c))^{\frac{3}{2}}}-\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\,\text{arct}\right.}{8b^4}\right.}{d}$
default	$\frac{2b\left(-\frac{1}{3b^2(b\tan(dx+c))^{\frac{3}{2}}}-\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\,\text{arct}\right.}{8b^4}\right.}{d}$

```
[In] int(1/(b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*(-1/3/b^2/(b*tan(d*x+c))^(3/2)-1/8/b^4*(b^2)^(1/4)*2^(1/2)*(ln((b*tan
(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)
-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b
^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+
c))^(1/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.10

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = -\frac{3\,b^3d\left(-\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}\log\left(b^3d\left(-\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}+\sqrt{b\tan\left(dx+c\right)}\right)\tan\left(dx+c\right)^2+3i\,b^3d\left(-\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}\log\left(i\,b^3d\left(-\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}+\cdots\right.}{\cdots}$$

```
[In] integrate(1/(b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*b^3*d*(-1/(b^10*d^4))^(1/4)*log(b^3*d*(-1/(b^10*d^4))^(1/4) + sqrt(
b*tan(d*x + c)))*tan(d*x + c)^2 + 3*I*b^3*d*(-1/(b^10*d^4))^(1/4)*log(I*b^3
*d*(-1/(b^10*d^4))^(1/4) + sqrt(b*tan(d*x + c)))*tan(d*x + c)^2 - 3*I*b^3*d
*(-1/(b^10*d^4))^(1/4)*log(-I*b^3*d*(-1/(b^10*d^4))^(1/4) + sqrt(b*tan(d*x
+ c)))*tan(d*x + c)^2 - 3*b^3*d*(-1/(b^10*d^4))^(1/4)*log(-b^3*d*(-1/(b^10*
d^4))^(1/4) + sqrt(b*tan(d*x + c)))*tan(d*x + c)^2 + 4*sqrt(b*tan(d*x + c))
)/(b^3*d*tan(d*x + c)^2)
```

Sympy [F]

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.79

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = -\frac{\frac{6\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{b^{\frac{3}{2}}}+\frac{6\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{b^{\frac{3}{2}}}+\frac{3\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{b^{\frac{3}{2}}}-\cdots}{12\,bd}$$

```
[In] integrate(1/(b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x + c
)))/sqrt(b))/b^(3/2) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) - 2*s
qrt(b*tan(d*x + c)))/sqrt(b))/b^(3/2) + 3*sqrt(2)*log(b*tan(d*x + c) + sqrt
(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/b^(3/2) - 3*sqrt(2)*log(b*tan(d*x + c
) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/b^(3/2) + 8/(b*tan(d*x + c))^
(3/2))/(b*d)
```

Giac [F]

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\tan(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c))^(-5/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.35

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = -\frac{2}{3\,b\,d\,(b\tan(c+d\,x))^{3/2}} + \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{b^{5/2}\,d} + \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{b^{5/2}\,d}$$

```
[In] int(1/(b*tan(c + d*x))^(5/2),x)
```

```
[Out] ((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/(b^(5/2)*
d) - 2/(3*b*d*(b*tan(c + d*x))^(3/2)) + ((-1)^(1/4)*atanh(((-1)^(1/4)*(b*ta
n(c + d*x))^(1/2))/b^(1/2))*1i)/(b^(5/2)*d)
```

3.16 $\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 234

$$\begin{aligned}\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx &= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d} \\ &+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d}+\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{7/2}d} \\ &-\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{7/2}d} \\ &-\frac{2}{5bd(b\tan(c+dx))^{5/2}}+\frac{2}{b^3d\sqrt{b\tan(c+dx)}}\end{aligned}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d*2^(1/2)+1/2*a
rctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d*2^(1/2)+1/4*ln(b^(1
/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(7/2)/d*2^(1/2)-1/4*
ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(7/2)/d*2^(1/
2)+2/b^3/d/(b*tan(d*x+c))^(1/2)-2/5/b/d/(b*tan(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used

= {3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}b^{7/2}d} + \frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}b^{7/2}d} - \frac{\log\left(\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}b^{7/2}d} + \frac{2}{b^3d\sqrt{b\tan(c+dx)}} - \frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

```
[In] Int[(b*Tan[c + d*x])^(-7/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(7/2)*d)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(7/2)*d) + L
og[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2
]*b^(7/2)*d) - Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c +
d*x]]]/(2*Sqrt[2]*b^(7/2)*d) - 2/(5*b*d*(b*Tan[c + d*x])^(5/2)) + 2/(b^3*d*
Sqrt[b*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2}{5bd(b\tan(c+dx))^{5/2}} - \frac{\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx}{b^2} \\
&= -\frac{2}{5bd(b\tan(c+dx))^{5/2}} + \frac{2}{b^3d\sqrt{b\tan(c+dx)}} + \frac{\int \sqrt{b\tan(c+dx)}\,dx}{b^4} \\
&= -\frac{2}{5bd(b\tan(c+dx))^{5/2}} + \frac{2}{b^3d\sqrt{b\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{b^3d}
\end{aligned}
$$

$$= -\frac{2}{5bd(b\tan(c+dx))^{5/2}}+\frac{2}{b^3d\sqrt{b\tan(c+dx)}}+\frac{2\text{Subst}\left(\int\frac{x^2}{b^2+x^4}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{b^3d}$$

$$= -\frac{2}{5bd(b\tan(c+dx))^{5/2}}+\frac{2}{b^3d\sqrt{b\tan(c+dx)}} - \frac{\text{Subst}\left(\int\frac{b-x^2}{b^2+x^4}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{b^3d}+\frac{\text{Subst}\left(\int\frac{b+x^2}{b^2+x^4}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{b^3d}$$

$$= -\frac{2}{5bd(b\tan(c+dx))^{5/2}}+\frac{2}{b^3d\sqrt{b\tan(c+dx)}} + \frac{\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{7/2}d} + \frac{\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{7/2}d} + \frac{\text{Subst}\left(\int\frac{1}{b-\sqrt{2}\sqrt{b}x+x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2b^3d} + \frac{\text{Subst}\left(\int\frac{1}{b+\sqrt{2}\sqrt{b}x+x^2}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{2b^3d}$$

$$= \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{7/2}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{7/2}d} - \frac{2}{5bd(b\tan(c+dx))^{5/2}} + \frac{2}{b^3d\sqrt{b\tan(c+dx)}} + \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d} - \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{7/2}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{7/2}d} - \frac{2}{5bd(b\tan(c+dx))^{5/2}}+\frac{2}{b^3d\sqrt{b\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.41

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \frac{10-2\cot^2(c+dx)+5\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan^2(c+dx)}-5\text{arctanh}}{5b^3d\sqrt{b\tan(c+dx)}}$$

[In] Integrate[(b*Tan[c + d*x])^(-7/2),x]

[Out] (10 - 2*Cot[c + d*x]^2 + 5*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4) - 5*ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4))/(5*b^3*d*Sqrt[b*Tan[c + d*x]])

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{2b\left(\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^2\right)^{\frac{1}{4}}}+1\right)\right)}{8b^4\left(b^2\right)^{\frac{1}{4}}}-}{d}$
default	$\frac{2b\left(\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^2\right)^{\frac{1}{4}}}+1\right)\right)}{8b^4\left(b^2\right)^{\frac{1}{4}}}-}{d}$

[In] int(1/(b*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)

[Out] 2/d*b*(1/8/b^4/(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))-1/5/b^2/(b*tan(d*x+c))^(5/2)+1/b^4/(b*tan(d*x+c))^(1/2))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.09

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \frac{5\,b^4d\left(-\frac{1}{b^{14}d^4}\right)^{\frac{1}{4}}\log\left(b^{11}d^3\left(-\frac{1}{b^{14}d^4}\right)^{\frac{3}{4}}+\sqrt{b\tan\left(dx+c\right)}\right)\tan\left(dx+c\right)^3-5i\,b^4d\left(-\frac{1}{b^{14}d^4}\right)^{\frac{1}{4}}\cdots}{\cdots}$$

```
[In] integrate(1/(b*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/10*(5*b^4*d*(-1/(b^14*d^4))^(1/4)*log(b^11*d^3*(-1/(b^14*d^4))^(3/4) + sq
rt(b*tan(d*x + c)))*tan(d*x + c)^3 - 5*I*b^4*d*(-1/(b^14*d^4))^(1/4)*log(I*
b^11*d^3*(-1/(b^14*d^4))^(3/4) + sqrt(b*tan(d*x + c)))*tan(d*x + c)^3 + 5*I
*b^4*d*(-1/(b^14*d^4))^(1/4)*log(-I*b^11*d^3*(-1/(b^14*d^4))^(3/4) + sqrt(b
*tan(d*x + c)))*tan(d*x + c)^3 - 5*b^4*d*(-1/(b^14*d^4))^(1/4)*log(-b^11*d^
3*(-1/(b^14*d^4))^(3/4) + sqrt(b*tan(d*x + c)))*tan(d*x + c)^3 + 4*sqrt(b*t
an(d*x + c))*(5*tan(d*x + c)^2 - 1))/(b^4*d*tan(d*x + c)^3)
```

Sympy [F]

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.83

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \frac{\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}-\frac{\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\cdots\right)}{\sqrt{b}}\cdots\right)}{b^2}\cdots}{20\,bd}$$

```
[In] integrate(1/(b*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/20*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x +
 c)))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) - 2
*sqrt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c) + sqrt
(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(d*x + c)
- sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b))/b^2 + 8*(5*b^2*tan(d*x
 + c)^2 - b^2)/((b*tan(d*x + c))^(5/2)*b^2))/(b*d)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.39

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}\,d} - \frac{\frac{2}{5\,b}-\frac{2\tan(c+d\,x)^2}{b}}{d\,(b\tan(c+d\,x))^{5/2}} - \frac{(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{b^{7/2}\,d}$$

```
[In] int(1/(b*tan(c + d*x))^(7/2),x)
```

```
[Out] ((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(7/2)*d)
- (2/(5*b) - (2*tan(c + d*x)^2)/b)/(d*(b*tan(c + d*x))^(5/2)) - ((-1)^(1/4)
*atanh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(7/2)*d)
```

3.17 $\int (b\tan(c+dx))^{4/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 243

$$\int (b\tan(c+dx))^{4/3}\,dx = -\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{b^{4/3}\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d} - \frac{b^{4/3}\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d} + \frac{\sqrt{3}b^{4/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} - \frac{\sqrt{3}b^{4/3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} + \frac{3b\sqrt[3]{b\tan(c+dx)}}{d}$$

```
[Out] -b^(4/3)*arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/d-1/2*b^(4/3)*arctan(-3^(1/2)
+2*(b*tan(d*x+c))^(1/3)/b^(1/3))/d-1/2*b^(4/3)*arctan(3^(1/2)+2*(b*tan(d*x+
c))^(1/3)/b^(1/3))/d+1/4*b^(4/3)*ln(b^(2/3)-b^(1/3)*3^(1/2)*(b*tan(d*x+c))^
(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/d-1/4*b^(4/3)*ln(b^(2/3)+b^(1/3)*3^(1/2
)*(b*tan(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/d+3*b*(b*tan(d*x+c))^(
1/3)/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3554, 3557, 335, 215, 648, 632, 210, 642, 209}

$$\int (b\tan(c+dx))^{4/3}\,dx = -\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{b^{4/3}\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d} - \frac{b^{4/3}\arctan\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2d} + \frac{\sqrt{3}b^{4/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} - \frac{\sqrt{3}b^{4/3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} + \frac{3b\sqrt[3]{b\tan(c+dx)}}{d}$$

```
[In] Int[(b*Tan[c + d*x])^(4/3),x]
```

```
[Out] -((b^(4/3)*ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)])/d) + (b^(4/3)*ArcTan[Sqr
t[3] - (2*(b*Tan[c + d*x])^(1/3))/b^(1/3)])/(2*d) - (b^(4/3)*ArcTan[Sqrt[3]
 + (2*(b*Tan[c + d*x])^(1/3))/b^(1/3)])/(2*d) + (Sqrt[3]*b^(4/3)*Log[b^(2/3
) - Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*d)
 - (Sqrt[3]*b^(4/3)*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) +
(b*Tan[c + d*x])^(2/3)])/(4*d) + (3*b*(b*Tan[c + d*x])^(1/3))/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
```

```
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{3b\sqrt[3]{b\tan(c+dx)}}{d} - b^2\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx$$

$$= \frac{3b\sqrt[3]{b\tan(c+dx)}}{d} - \frac{b^3\text{Subst}\left(\int \frac{1}{x^{2/3}(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{3b\sqrt[3]{b\tan(c+dx)}}{d} - \frac{(3b^3)\,\text{Subst}\left(\int \frac{1}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}$$

$$= \frac{3b\sqrt[3]{b\tan(c+dx)}}{d} - \frac{b^{4/3}\text{Subst}\left(\int \frac{\sqrt[3]{b}-\frac{\sqrt{3}x}{2}}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}$$
$$- \frac{b^{4/3}\text{Subst}\left(\int \frac{\sqrt[3]{b}+\frac{\sqrt{3}x}{2}}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}$$
$$- \frac{b^{5/3}\text{Subst}\left(\int \frac{1}{b^{2/3}+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}$$

$$= -\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{3b\sqrt[3]{b\tan(c+dx)}}{d}$$
$$+ \frac{\left(\sqrt{3}b^{4/3}\right)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[3]{b}+2x}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4d}$$
$$- \frac{\left(\sqrt{3}b^{4/3}\right)\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[3]{b}+2x}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4d}$$
$$- \frac{b^{5/3}\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4d}$$
$$- \frac{b^{5/3}\text{Subst}\left(\int \frac{1}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4d}$$

$$= -\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{\sqrt{3}b^{4/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} - \frac{\sqrt{3}b^{4/3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} + \frac{3b\sqrt[3]{b\tan(c+dx)}}{d} - \frac{b^{4/3}\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt{3}d} + \frac{b^{4/3}\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt{3}d}$$

$$= -\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{b^{4/3}\arctan\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2d} - \frac{b^{4/3}\arctan\left(\frac{1}{3}\left(3\sqrt{3}+\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2d} + \frac{\sqrt{3}b^{4/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} - \frac{\sqrt{3}b^{4/3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} + \frac{3b\sqrt[3]{b\tan(c+dx)}}{d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.84

$$\int (b\tan(c+dx))^{4/3}\,dx = \frac{b\sqrt[3]{b\tan(c+dx)}\left(-i\log\left(1-i\sqrt[6]{\tan^2(c+dx)}\right)+i\log\left(1+i\sqrt[6]{\tan^2(c+dx)}\right)-(-1)^{5/6}\log\right.}{}$$

```
[In] Integrate[(b*Tan[c + d*x])^(4/3),x]
```

```
[Out] (b*(b*Tan[c + d*x])^(1/3)*((-I)*Log[1 - I*(Tan[c + d*x]^2)^(1/6)] + I*Log[1
 + I*(Tan[c + d*x]^2)^(1/6)] - (-1)^(5/6)*Log[1 - (-1)^(1/6)*(Tan[c + d*x]^
2)^(1/6)] + (-1)^(5/6)*Log[1 + (-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)] - (-1)^(1
/6)*Log[1 - (-1)^(5/6)*(Tan[c + d*x]^2)^(1/6)] + (-1)^(1/6)*Log[1 + (-1)^(5
/6)*(Tan[c + d*x]^2)^(1/6)] + 6*(Tan[c + d*x]^2)^(1/6)))/(2*d*(Tan[c + d*x]
^2)^(1/6))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{3b(b\tan(dx+c))^{\frac{1}{3}}}{d}+\frac{b\sqrt{3}\,(b^2)^{\frac{1}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}-\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{4d}-\frac{b(b^2)^{\frac{1}{6}}\arctan\left(\frac{2(b\tan(dx+}{(b^2)^{\frac{1}{6}}}\right.}{2d}$
default	$\frac{3b(b\tan(dx+c))^{\frac{1}{3}}}{d}+\frac{b\sqrt{3}\,(b^2)^{\frac{1}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}-\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{4d}-\frac{b(b^2)^{\frac{1}{6}}\arctan\left(\frac{2(b\tan(dx+}{(b^2)^{\frac{1}{6}}}\right.}{2d}$

```
[In] int((b*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*b*(b*tan(d*x+c))^(1/3)/d+1/4/d*b*3^(1/2)*(b^2)^(1/6)*ln((b*tan(d*x+c))^(2
/3)-3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))-1/2/d*b*(b^2)^(1/
6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)-3^(1/2))-1/4/d*b*3^(1/2)*(b^2)
^(1/6)*ln((b*tan(d*x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^
2)^(1/3))-1/2/d*b*(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)+3^(
1/2))-1/d*b*(b^2)^(1/6)*arctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.28

$$\int (b\tan(c+dx))^{4/3}\,dx =$$

$$-\frac{\left(-\frac{b^8}{d^6}\right)^{\frac{1}{6}}\left(\sqrt{-3}d+d\right)\log\left((b\tan(dx+c))^{\frac{1}{3}}b+\frac{1}{2}\left(-\frac{b^8}{d^6}\right)^{\frac{1}{6}}\left(\sqrt{-3}d+d\right)\right)-\left(-\frac{b^8}{d^6}\right)^{\frac{1}{6}}\left(\sqrt{-3}d+d\right)\log\left((b\,\mathrm{ta}\right.}{}$$

```
[In] integrate((b*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] -1/4*((-b^8/d^6)^(1/6)*(sqrt(-3)*d + d)*log((b*tan(d*x + c))^(1/3)*b + 1/2*
(-b^8/d^6)^(1/6)*(sqrt(-3)*d + d)) - (-b^8/d^6)^(1/6)*(sqrt(-3)*d + d)*log(
(b*tan(d*x + c))^(1/3)*b - 1/2*(-b^8/d^6)^(1/6)*(sqrt(-3)*d + d)) + (-b^8/d
^6)^(1/6)*(sqrt(-3)*d - d)*log((b*tan(d*x + c))^(1/3)*b + 1/2*(-b^8/d^6)^(1
```

```
/6)*(sqrt(-3)*d - d)) - (-b^8/d^6)^(1/6)*(sqrt(-3)*d - d)*log((b*tan(d*x +
c))^(1/3)*b - 1/2*(-b^8/d^6)^(1/6)*(sqrt(-3)*d - d)) + 2*(-b^8/d^6)^(1/6)*d
*log((b*tan(d*x + c))^(1/3)*b + (-b^8/d^6)^(1/6)*d) - 2*(-b^8/d^6)^(1/6)*d*
log((b*tan(d*x + c))^(1/3)*b - (-b^8/d^6)^(1/6)*d) - 12*(b*tan(d*x + c))^(1
/3)*b)/d
```

Sympy [F]

$$\int (b\tan(c+dx))^{4/3}\,dx = \int (b\tan{(c+dx)})^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(4/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.76

$$\int (b\tan(c+dx))^{4/3}\,dx =$$

$$-\frac{\sqrt{3}b^{\frac{7}{3}}\log\left(\sqrt{3}(b\tan{(dx+c)})^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan{(dx+c)})^{\frac{2}{3}}+b^{\frac{2}{3}}\right)-\sqrt{3}b^{\frac{7}{3}}\log\left(-\sqrt{3}(b\tan{(dx+c)})^{\frac{1}{3}}b^{\frac{1}{3}}+(b\,\mathrm{ta}\right.}{}$$

```
[In] integrate((b*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] -1/4*(sqrt(3)*b^(7/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(d
*x + c))^(2/3) + b^(2/3)) - sqrt(3)*b^(7/3)*log(-sqrt(3)*(b*tan(d*x + c))^(
1/3)*b^(1/3) + (b*tan(d*x + c))^(2/3) + b^(2/3)) + 2*b^(7/3)*arctan((sqrt(3
)*b^(1/3) + 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 2*b^(7/3)*arctan(-(sqrt(3)
*b^(1/3) - 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 4*b^(7/3)*arctan((b*tan(d*x
 + c))^(1/3)/b^(1/3)) - 12*(b*tan(d*x + c))^(1/3)*b^2)/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.86

$$\int (b\tan(c+dx))^{4/3}\,dx =$$

$$-\frac{1}{4}\,b\left(\frac{\sqrt{3}|b|^{\frac{1}{3}}\log\left(\sqrt{3}(b\tan\left(dx+c\right))^{\frac{1}{3}}\,|b|^{\frac{1}{3}}+(b\tan\left(dx+c\right))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{d}-\frac{\sqrt{3}|b|^{\frac{1}{3}}\log\left(-\sqrt{3}(b\tan\left(dx+c\right))^{\frac{1}{3}}\right.}{d}\right.$$

```
[In] integrate((b*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] -1/4*b*(sqrt(3)*abs(b)^(1/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3
) + (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/d - sqrt(3)*abs(b)^(1/3)*log(-sq
rt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3) + abs(b)
^(2/3))/d + 2*abs(b)^(1/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*(b*tan(d*x + c)
)^(1/3))/abs(b)^(1/3))/d + 2*abs(b)^(1/3)*arctan(-(sqrt(3)*abs(b)^(1/3) - 2
*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/d + 4*abs(b)^(1/3)*arctan((b*tan(d*x
 + c))^(1/3)/abs(b)^(1/3))/d - 12*(b*tan(d*x + c))^(1/3)/d)
```

Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.02

$$\int (b\tan(c+dx))^{4/3}\,dx = \frac{3\,b\,(b\tan(c+d\,x))^{1/3}}{d}-\frac{(-1)^{1/6}\,b^{4/3}\,\mathrm{atan}\left(\frac{(-1)^{5/6}\,(b\tan(c+d\,x))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)\,1\mathrm{i}}{d}$$
$$-\frac{(-1)^{1/6}\,b^{4/3}\,\ln\left((-1)^{1/6}\,b^{1/3}+2\,(b\tan(c+d\,x))^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}$$
$$-\frac{(-1)^{1/6}\,b^{4/3}\,\ln\left(2\,(b\tan(c+d\,x))^{1/3}-(-1)^{1/6}\,b^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}$$
$$+\frac{(-1)^{1/6}\,b^{4/3}\,\ln\left((-1)^{1/6}\,b^{1/3}-2\,(b\tan(c+d\,x))^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$
$$+\frac{(-1)^{1/6}\,b^{4/3}\,\ln\left((-1)^{1/6}\,b^{1/3}+2\,(b\tan(c+d\,x))^{1/3}-(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

```
[In] int((b*tan(c + d*x))^(4/3),x)
```

```
[Out] (3*b*(b*tan(c + d*x))^(1/3))/d - ((-1)^(1/6)*b^(4/3)*atan(((-1)^(5/6)*(b*ta
n(c + d*x))^(1/3)*1i)/b^(1/3))*1i)/d - ((-1)^(1/6)*b^(4/3)*log((-1)^(1/6)*b
^(1/3) + 2*(b*tan(c + d*x))^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1
```

```
i)/2 + 1/2))/(2*d) - ((-1)^(1/6)*b^(4/3)*log(2*(b*tan(c + d*x))^(1/3) - (-1
)^(1/6)*b^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*d)
 + ((-1)^(1/6)*b^(4/3)*log((-1)^(1/6)*b^(1/3) - 2*(b*tan(c + d*x))^(1/3) +
(-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/4 + 1/4))/d + ((-1)^(1/6)*b^(4/3)
*log((-1)^(1/6)*b^(1/3) + 2*(b*tan(c + d*x))^(1/3) - (-1)^(2/3)*3^(1/2)*b^(
1/3))*((3^(1/2)*1i)/4 - 1/4))/d
```

3.18 $\int (b\tan(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 224

$$\int (b\tan(c+dx))^{2/3}\,dx = \frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d} + \frac{b^{2/3}\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d} + \frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} - \frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d}$$

```
[Out] b^(2/3)*arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/d+1/2*b^(2/3)*arctan(-3^(1/2)+
2*(b*tan(d*x+c))^(1/3)/b^(1/3))/d+1/2*b^(2/3)*arctan(3^(1/2)+2*(b*tan(d*x+c
))^(1/3)/b^(1/3))/d+1/4*b^(2/3)*ln(b^(2/3)-b^(1/3)*3^(1/2)*(b*tan(d*x+c))^(
1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/d-1/4*b^(2/3)*ln(b^(2/3)+b^(1/3)*3^(1/2)
*(b*tan(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/d
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3557, 335, 301, 648, 632, 210, 642, 209}

$$\int (b\tan(c+dx))^{2/3}\,dx = \frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d} + \frac{b^{2/3}\arctan\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2d} + \frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} - \frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d}$$

```
[In] Int[(b*Tan[c + d*x])^(2/3),x]
```

```
[Out] (b^(2/3)*ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)])/d - (b^(2/3)*ArcTan[Sqrt[3
] - (2*(b*Tan[c + d*x])^(1/3))/b^(1/3)])/(2*d) + (b^(2/3)*ArcTan[Sqrt[3] +
(2*(b*Tan[c + d*x])^(1/3))/b^(1/3)])/(2*d) + (Sqrt[3]*b^(2/3)*Log[b^(2/3) -
 Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*d) -
(Sqrt[3]*b^(2/3)*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*
Tan[c + d*x])^(2/3)])/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
```

```
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\text{Subst}\left(\int \frac{x^{2/3}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}\\ &= \frac{(3b)\text{Subst}\left(\int \frac{x^4}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}\end{aligned}$$

$$= \frac{b^{2/3}\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}+\frac{\sqrt{3}x}{2}}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d} + \frac{b^{2/3}\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}-\frac{\sqrt{3}x}{2}}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d} + \frac{b\text{Subst}\left(\int \frac{1}{b^{2/3}+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}$$

$$= \frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{\left(\sqrt{3}b^{2/3}\right)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[3]{b}+2x}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4d} - \frac{\left(\sqrt{3}b^{2/3}\right)\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[3]{b}+2x}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4d} + \frac{b\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4d} + \frac{b\text{Subst}\left(\int \frac{1}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4d}$$

$$= \frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} - \frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} + \frac{b^{2/3}\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt{3}d} - \frac{b^{2/3}\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt{3}d}$$

$$= \frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\arctan\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2d}$$
$$+ \frac{b^{2/3}\arctan\left(\frac{1}{3}\left(3\sqrt{3}+\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2d}$$
$$+ \frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d}$$
$$- \frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.83

$$\int (b\tan(c+dx))^{2/3}\,dx = \frac{\left(i\log\left(1-i\sqrt[6]{\tan^2(c+dx)}\right)-i\log\left(1+i\sqrt[6]{\tan^2(c+dx)}\right)+\sqrt[6]{-1}\left(\log\left(1-\sqrt[6]{-1}\sqrt[6]{\tan^2(c\right.\right.\right.}{}$$

```
[In] Integrate[(b*Tan[c + d*x])^(2/3),x]
```

```
[Out] ((I*Log[1 - I*(Tan[c + d*x]^2)^(1/6)] - I*Log[1 + I*(Tan[c + d*x]^2)^(1/6)]
 + (-1)^(1/6)*(Log[1 - (-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)] - Log[1 + (-1)^(1
/6)*(Tan[c + d*x]^2)^(1/6)] + (-1)^(2/3)*(Log[1 - (-1)^(5/6)*(Tan[c + d*x]^
2)^(1/6)] - Log[1 + (-1)^(5/6)*(Tan[c + d*x]^2)^(1/6)])))*(b*Tan[c + d*x])^
(5/3))/(2*b*d*(Tan[c + d*x]^2)^(5/6))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{3b\left(\frac{\sqrt{3}\,(b^2)^{\frac{5}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}-\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{12b^2}+\frac{\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{(b^2)^{\frac{1}{6}}}-\sqrt{3}\right)}{6(b^2)^{\frac{1}{6}}}-\frac{\sqrt{3}\,(b^2)^{\frac{5}{6}}\ln\left((b\tan(d\right.}{}\right.}{d}$
default	$\frac{3b\left(\frac{\sqrt{3}\,(b^2)^{\frac{5}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}-\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{12b^2}+\frac{\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{(b^2)^{\frac{1}{6}}}-\sqrt{3}\right)}{6(b^2)^{\frac{1}{6}}}-\frac{\sqrt{3}\,(b^2)^{\frac{5}{6}}\ln\left((b\tan(d\right.}{}\right.}{d}$

```
[In] int((b*tan(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*b*(1/12/b^2*3^(1/2)*(b^2)^(5/6)*ln((b*tan(d*x+c))^(2/3)-3^(1/2)*(b^2)^(
1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/6/(b^2)^(1/6)*arctan(2*(b*tan(d*x+
c))^(1/3)/(b^2)^(1/6)-3^(1/2))-1/12/b^2*3^(1/2)*(b^2)^(5/6)*ln((b*tan(d*x+c
))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/6/(b^2)^(1
/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)+3^(1/2))+1/3/(b^2)^(1/6)*arct
an((b*tan(d*x+c))^(1/3)/(b^2)^(1/6)))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.40

$$\begin{aligned}
&\int (b\tan(c+dx))^{2/3}\,dx = \\
&-\frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b^3+\frac{1}{2}\left(\sqrt{-3}d^5+d^5\right)\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}\right) \\
&+\frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b^3-\frac{1}{2}\left(\sqrt{-3}d^5+d^5\right)\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}\right) \\
&-\frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b^3+\frac{1}{2}\left(\sqrt{-3}d^5-d^5\right)\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}\right) \\
&+\frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b^3-\frac{1}{2}\left(\sqrt{-3}d^5-d^5\right)\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}\right) \\
&+\frac{1}{2}\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left(d^5\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}+(b\tan(dx+c))^{\frac{1}{3}}b^3\right) \\
&-\frac{1}{2}\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left(-d^5\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}+(b\tan(dx+c))^{\frac{1}{3}}b^3\right)
\end{aligned}$$

```
[In] integrate((b*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-3) - 1)*(-b^4/d^6)^(1/6)*log((b*tan(d*x + c))^(1/3)*b^3 + 1/2*(
sqrt(-3)*d^5 + d^5)*(-b^4/d^6)^(5/6)) + 1/4*(sqrt(-3) - 1)*(-b^4/d^6)^(1/6)
*log((b*tan(d*x + c))^(1/3)*b^3 - 1/2*(sqrt(-3)*d^5 + d^5)*(-b^4/d^6)^(5/6)
) - 1/4*(sqrt(-3) + 1)*(-b^4/d^6)^(1/6)*log((b*tan(d*x + c))^(1/3)*b^3 + 1/
2*(sqrt(-3)*d^5 - d^5)*(-b^4/d^6)^(5/6)) + 1/4*(sqrt(-3) + 1)*(-b^4/d^6)^(1
/6)*log((b*tan(d*x + c))^(1/3)*b^3 - 1/2*(sqrt(-3)*d^5 - d^5)*(-b^4/d^6)^(5
/6)) + 1/2*(-b^4/d^6)^(1/6)*log(d^5*(-b^4/d^6)^(5/6) + (b*tan(d*x + c))^(1/
3)*b^3) - 1/2*(-b^4/d^6)^(1/6)*log(-d^5*(-b^4/d^6)^(5/6) + (b*tan(d*x + c))
^(1/3)*b^3)
```

Sympy [F]

$$\int (b\tan(c+dx))^{2/3}\,dx = \int (b\tan{(c+dx)})^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*tan(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(2/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.75

$$\int (b\tan(c+dx))^{2/3}\,dx = -\frac{\left(\frac{\sqrt{3}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}} - \frac{\sqrt{3}\log\left(-\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}} - \frac{2\arctan\left(\frac{\sqrt{3}b^{\frac{1}{3}}+2\,(b\tan(dx}{b^{\frac{1}{3}}}\right.}{b^{\frac{1}{3}}}\right.}{4\,d}$$

```
[In] integrate((b*tan(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] -1/4*(sqrt(3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(d*x + c))
^(2/3) + b^(2/3))/b^(1/3) - sqrt(3)*log(-sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(
1/3) + (b*tan(d*x + c))^(2/3) + b^(2/3))/b^(1/3) - 2*arctan((sqrt(3)*b^(1/3
) + 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 2*arctan(-(sqrt(3)*b^(1/3)
 - 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 4*arctan((b*tan(d*x + c))^(
1/3)/b^(1/3))/b^(1/3))*b/d
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.92

$$\int (b\tan(c+dx))^{2/3}\,dx = -\frac{\sqrt{3}|b|^{\frac{5}{3}}\log\left(\sqrt{3}(b\tan\left(dx+c\right))^{\frac{1}{3}}|b|^{\frac{1}{3}}+(b\tan\left(dx+c\right))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{4\,bd} + \frac{\sqrt{3}|b|^{\frac{5}{3}}\log\left(-\sqrt{3}(b\tan\left(dx+c\right))^{\frac{1}{3}}|b|^{\frac{1}{3}}+(b\tan\left(dx+c\right))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{4\,bd} + \frac{|b|^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}|b|^{\frac{1}{3}}+2\,(b\tan(dx+c))^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{2\,bd} + \frac{|b|^{\frac{5}{3}}\arctan\left(-\frac{\sqrt{3}|b|^{\frac{1}{3}}-2\,(b\tan(dx+c))^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{2\,bd} + \frac{|b|^{\frac{5}{3}}\arctan\left(\frac{(b\tan(dx+c))^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{bd}$$

```
[In] integrate((b*tan(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*abs(b)^(5/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) +
 (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b*d) + 1/4*sqrt(3)*abs(b)^(5/3)*lo
g(-sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3) + a
bs(b)^(2/3))/(b*d) + 1/2*abs(b)^(5/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*(b*t
an(d*x + c))^(1/3))/abs(b)^(1/3))/(b*d) + 1/2*abs(b)^(5/3)*arctan(-(sqrt(3)
*abs(b)^(1/3) - 2*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b*d) + abs(b)^(5/3
)*arctan((b*tan(d*x + c))^(1/3)/abs(b)^(1/3))/(b*d)
```

Mupad [B] (verification not implemented)

Time = 3.78 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.16

$$\int (b\tan(c+dx))^{2/3}\,dx = \frac{(-1)^{1/6}\,b^{2/3}\,\mathrm{atan}\left(\frac{(-1)^{2/3}\,(b\tan(c+d\,x))^{1/3}}{b^{1/3}}\right)\,1\mathrm{i}}{d}$$
$$-\frac{(-1)^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}+\frac{486\,(-1)^{1/6}\,b^{26/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,(b\tan(c+d\,x))^{1/3}}{d^3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}$$
$$-\frac{(-1)^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}+\frac{486\,(-1)^{1/6}\,b^{26/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,(b\tan(c+d\,x))^{1/3}}{d^3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}$$
$$+\frac{(-1)^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}-\frac{486\,(-1)^{1/6}\,b^{26/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,(b\tan(c+d\,x))^{1/3}}{d^3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$
$$+\frac{(-1)^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}-\frac{486\,(-1)^{1/6}\,b^{26/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,(b\tan(c+d\,x))^{1/3}}{d^3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

```
[In] int((b*tan(c + d*x))^(2/3),x)
```

```
[Out] ((-1)^(1/6)*b^(2/3)*atan(((-1)^(2/3)*(b*tan(c + d*x))^(1/3))/b^(1/3))*1i)/d
 - ((-1)^(1/6)*b^(2/3)*log((972*b^9)/d^3 + (486*(-1)^(1/6)*b^(26/3)*(3^(1/2
)*1i - 1)*(b*tan(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/2 - 1/2))/(2*d) - ((-1
)^(1/6)*b^(2/3)*log((972*b^9)/d^3 + (486*(-1)^(1/6)*b^(26/3)*(3^(1/2)*1i +
1)*(b*tan(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/2 + 1/2))/(2*d) + ((-1)^(1/6)
*b^(2/3)*log((972*b^9)/d^3 - (486*(-1)^(1/6)*b^(26/3)*(3^(1/2)*1i - 1)*(b*t
an(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/4 - 1/4))/d + ((-1)^(1/6)*b^(2/3)*lo
g((972*b^9)/d^3 - (486*(-1)^(1/6)*b^(26/3)*(3^(1/2)*1i + 1)*(b*tan(c + d*x)
)^(1/3))/d^3)*((3^(1/2)*1i)/4 + 1/4))/d
```

3.19 $\int \sqrt[3]{b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 131

$$\int \sqrt[3]{b\tan(c+dx)}\,dx = -\frac{\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2d} - \frac{\sqrt[3]{b}\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}+(b\tan(c+dx))^{4/3}\right)}{4d}$$

```
[Out] -1/2*b^(1/3)*ln(b^(2/3)+(b*tan(d*x+c))^(2/3))/d+1/4*b^(1/3)*ln(b^(4/3)-b^(2
/3)*(b*tan(d*x+c))^(2/3)+(b*tan(d*x+c))^(4/3))/d-1/2*b^(1/3)*arctan(1/3*(b^
(2/3)-2*(b*tan(d*x+c))^(2/3))/b^(2/3)*3^(1/2))*3^(1/2)/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3557, 335, 281, 298, 31, 648, 631, 210, 642}

$$\int \sqrt[3]{b\tan(c+dx)}\,dx = -\frac{\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2d} - \frac{\sqrt[3]{b}\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\log\left(-b^{2/3}(b\tan(c+dx))^{2/3}+b^{4/3}+(b\tan(c+dx))^{4/3}\right)}{4d}$$

```
[In] Int[(b*Tan[c + d*x])^(1/3),x]
```

```
[Out] -1/2*(Sqrt[3]*b^(1/3)*ArcTan[(b^(2/3) - 2*(b*Tan[c + d*x])^(2/3))/(Sqrt[3]*
b^(2/3))])/d - (b^(1/3)*Log[b^(2/3) + (b*Tan[c + d*x])^(2/3)])/(2*d) + (b^(
1/3)*Log[b^(4/3) - b^(2/3)*(b*Tan[c + d*x])^(2/3) + (b*Tan[c + d*x])^(4/3)]
)/(4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{\sqrt[3]{x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d} \\
&= \frac{(3b)\text{Subst}\left(\int \frac{x^3}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d} \\
&= \frac{(3b)\text{Subst}\left(\int \frac{x}{b^2+x^3}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2d} \\
&= -\frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{1}{b^{2/3}+x}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2d} \\
&\quad + \frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{b^{2/3}+x}{b^{4/3}-b^{2/3}x+x^2}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2d} \\
&= -\frac{\sqrt[3]{b}\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d} \\
&\quad + \frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{-b^{2/3}+2x}{b^{4/3}-b^{2/3}x+x^2}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{4d} \\
&\quad + \frac{(3b)\text{Subst}\left(\int \frac{1}{b^{4/3}-b^{2/3}x+x^2}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{4d} \\
&= -\frac{\sqrt[3]{b}\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d} \\
&\quad + \frac{\sqrt[3]{b}\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}+(b\tan(c+dx))^{4/3}\right)}{4d} \\
&\quad + \frac{\left(3\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2(b\tan(c+dx))^{2/3}}{b^{2/3}}\right)}{2d}
\end{aligned}
$$

$$
=-\frac{\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2(b\tan(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2d}-\frac{\sqrt[3]{b}\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d}
+\frac{\sqrt[3]{b}\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}+(b\tan(c+dx))^{4/3}\right)}{4d}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

$$
\int\sqrt[3]{b\tan(c+dx)}\,dx=
-\frac{\left(\log\left(1+\sqrt[3]{\tan^2(c+dx)}\right)-\sqrt[3]{-1}\log\left(1-\sqrt[3]{-1}\sqrt[3]{\tan^2(c+dx)}\right)+(-1)^{2/3}\log\left(1+(-1)^{2/3}\sqrt[3]{\tan^2(c+dx)}\right.\right.}{2bd\tan^2(c+dx)^{2/3}}
$$

```
[In] Integrate[(b*Tan[c + d*x])^(1/3),x]
```

```
[Out] -1/2*((Log[1 + (Tan[c + d*x]^2)^(1/3)] - (-1)^(1/3)*Log[1 - (-1)^(1/3)*(Tan
[c + d*x]^2)^(1/3)] + (-1)^(2/3)*Log[1 + (-1)^(2/3)*(Tan[c + d*x]^2)^(1/3)]
)*(b*Tan[c + d*x])^(4/3))/(b*d*(Tan[c + d*x]^2)^(2/3))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{3b\left(-\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{1}{3}}}+\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}(b^2)^{\frac{1}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c))^{\frac{2}{3}}}{(b^2)^{\frac{1}{3}}}-\right.}{3}\right.}{6(b^2)^{\frac{1}{3}}}\right.}{d}$
default	$\frac{3b\left(-\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{1}{3}}}+\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}(b^2)^{\frac{1}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c))^{\frac{2}{3}}}{(b^2)^{\frac{1}{3}}}-\right.}{3}\right.}{6(b^2)^{\frac{1}{3}}}\right.}{d}$

```
[In] int((b*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*b*(-1/6/(b^2)^(1/3)*ln((b*tan(d*x+c))^(2/3)+(b^2)^(1/3))+1/12/(b^2)^(1/
3)*ln((b*tan(d*x+c))^(4/3)-(b*tan(d*x+c))^(2/3)*(b^2)^(1/3)+(b^2)^(2/3))+1/
6*3^(1/2)/(b^2)^(1/3)*arctan(1/3*3^(1/2)*(2*(b*tan(d*x+c))^(2/3)/(b^2)^(1/3
)-1)))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

$$\int \sqrt[3]{b\tan(c+dx)}\,dx$$
$$=\frac{2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(b\tan(dx+c))^{\frac{2}{3}}(-b)^{\frac{1}{3}}+\sqrt{3}b}{3b}\right)-(-b)^{\frac{1}{3}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b\tan(dx+c)-(b\tan(dx+\right.}{4d}$$

```
[In] integrate((b*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*tan(d*x + c))^(2/3)*(-b)
^(1/3) + sqrt(3)*b)/b) - (-b)^(1/3)*log((b*tan(d*x + c))^(1/3)*b*tan(d*x +
c) - (b*tan(d*x + c))^(2/3)*(-b)^(2/3) - (-b)^(1/3)*b) + 2*(-b)^(1/3)*log((
b*tan(d*x + c))^(2/3) + (-b)^(2/3)))/d
```

Sympy [F]

$$\int \sqrt[3]{b\tan(c+dx)}\,dx = \int \sqrt[3]{b\tan{(c+dx)}}\,dx$$

```
[In] integrate((b*tan(d*x+c))**(1/3),x)
[Out] Integral((b*tan(c + d*x))**(1/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int \sqrt[3]{b\tan(c+dx)}\,dx = \frac{2\sqrt{3}b^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-b^{\frac{2}{3}}\right)}{3\,b^{\frac{2}{3}}}\right)+b^{\frac{4}{3}}\log\left((b\tan{(dx+c)})^{\frac{4}{3}}-(b\tan{(dx+c)})^{\frac{2}{3}}\,b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)-2\,b^{\frac{4}{3}}\log\left(\right.}{4\,bd}$$

```
[In] integrate((b*tan(d*x+c))^(1/3),x, algorithm="maxima")
[Out] 1/4*(2*sqrt(3)*b^(4/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3) - b^(2/
3))/b^(2/3)) + b^(4/3)*log((b*tan(d*x + c))^(4/3) - (b*tan(d*x + c))^(2/3)*
b^(2/3) + b^(4/3)) - 2*b^(4/3)*log((b*tan(d*x + c))^(2/3) + b^(2/3)))/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97

$$\int \sqrt[3]{b\tan(c+dx)}\,dx = \frac{1}{4}\,b\left(\frac{2\sqrt{3}|b|^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-|b|^{\frac{2}{3}}\right)}{3\,|b|^{\frac{2}{3}}}\right)}{b^2d}+\frac{|b|^{\frac{4}{3}}\log\left((b\tan{(dx+c)})^{\frac{1}{3}}\,b\tan{(dx+c)}-(b\tan{(dx+c)}\right.}{b^2d}\right.$$

```
[In] integrate((b*tan(d*x+c))^(1/3),x, algorithm="giac")
[Out] 1/4*b*(2*sqrt(3)*abs(b)^(4/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3)
- abs(b)^(2/3))/abs(b)^(2/3))/(b^2*d) + abs(b)^(4/3)*log((b*tan(d*x + c))^(
1/3)*b*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*abs(b)^(2/3) + abs(b)^(4/3))/(
b^2*d) - 2*abs(b)^(4/3)*log((b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b^2*d))
```

Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

$$\int \sqrt[3]{b\tan(c+dx)}\,dx$$
$$= \frac{(-b)^{1/3}\ln\left(81\,(-b)^{16/3}\,(b\tan(c+d\,x))^{2/3}+81\,b^6\right)}{2\,d} - \frac{(-b)^{1/3}\ln\left(\frac{81\,b^6}{d^4}-\frac{81\,(-b)^{16/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(b\tan(c+d\,x))^{2/3}}{d^4}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} + \frac{(-b)^{1/3}\ln\left(\frac{81\,b^6}{d^4}+\frac{162\,(-b)^{16/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)(b\tan(c+d\,x))^{2/3}}{d^4}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

```
[In] int((b*tan(c + d*x))^(1/3),x)
```

```
[Out] ((-b)^(1/3)*log(81*(-b)^(16/3)*(b*tan(c + d*x))^(2/3) + 81*b^6))/(2*d) - ((
-b)^(1/3)*log((81*b^6)/d^4 - (81*(-b)^(16/3)*((3^(1/2)*1i)/2 + 1/2)*(b*tan(
c + d*x))^(2/3))/d^4)*((3^(1/2)*1i)/2 + 1/2))/(2*d) + ((-b)^(1/3)*log((81*b
^6)/d^4 + (162*(-b)^(16/3)*((3^(1/2)*1i)/4 - 1/4)*(b*tan(c + d*x))^(2/3))/d
^4)*((3^(1/2)*1i)/4 - 1/4))/d
```

3.20 $\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 131

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2\sqrt[3]{b}d} + \frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} - \frac{\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}+(b\tan(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}$$

```
[Out] 1/2*ln(b^(2/3)+(b*tan(d*x+c))^(2/3))/b^(1/3)/d-1/4*ln(b^(4/3)-b^(2/3)*(b*ta
n(d*x+c))^(2/3)+(b*tan(d*x+c))^(4/3))/b^(1/3)/d-1/2*arctan(1/3*(b^(2/3)-2*(
b*tan(d*x+c))^(2/3))/b^(2/3)*3^(1/2))*3^(1/2)/b^(1/3)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3557, 335, 281, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2\sqrt[3]{b}d} + \frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} - \frac{\log\left(-b^{2/3}(b\tan(c+dx))^{2/3}+b^{4/3}+(b\tan(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}$$

```
[In] Int[(b*Tan[c + d*x])^(-1/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(b^(2/3) - 2*(b*Tan[c + d*x])^(2/3))/(Sqrt[3]*b^(2/3))
])/(b^(1/3)*d) + Log[b^(2/3) + (b*Tan[c + d*x])^(2/3)]/(2*b^(1/3)*d) - Log[
```

```
b^(4/3) - b^(2/3)*(b*Tan[c + d*x])^(2/3) + (b*Tan[c + d*x])^(4/3)]/(4*b^(1/
3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}\\
&= \frac{(3b)\text{Subst}\left(\int \frac{x}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}\\
&= \frac{(3b)\text{Subst}\left(\int \frac{1}{b^2+x^3}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{b^{2/3}+x}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} + \frac{\text{Subst}\left(\int \frac{2b^{2/3}-x}{b^{4/3}-b^{2/3}x+x^2}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d}\\
&= \frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} - \frac{\text{Subst}\left(\int \frac{-b^{2/3}+2x}{b^{4/3}-b^{2/3}x+x^2}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{4\sqrt[3]{b}d}\\
&\quad + \frac{\left(3\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{b^{4/3}-b^{2/3}x+x^2}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{4d}\\
&= \frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d}\\
&\quad - \frac{\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}+(b\tan(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}\\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2(b\tan(c+dx))^{2/3}}{b^{2/3}}\right)}{2\sqrt[3]{b}d}\\
&= -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2(b\tan(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2\sqrt[3]{b}d} + \frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d}\\
&\quad - \frac{\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}+(b\tan(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = \frac{\left(2\sqrt{3}\arctan\left(\frac{-1+2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)+2\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)-\log\left(1-\tan^{\frac{2}{3}}(c+dx)+\tan^{\frac{4}{3}}(c+dx)\right)\right)\sqrt[3]{}}{4d\sqrt[3]{b\tan(c+dx)}}$$

```
[In] Integrate[(b*Tan[c + d*x])^(-1/3),x]
```

```
[Out] ((2*Sqrt[3]*ArcTan[(-1 + 2*Tan[c + d*x]^(2/3))/Sqrt[3]] + 2*Log[1 + Tan[c +
 d*x]^(2/3)] - Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])*Tan[c + d*
x]^(1/3))/(4*d*(b*Tan[c + d*x])^(1/3))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{3b\left(\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{1}{3}}\right)}{6\left(b^2\right)^{\frac{2}{3}}}-\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}\left(b^2\right)^{\frac{1}{3}}+\left(b^2\right)^{\frac{2}{3}}\right)}{12\left(b^2\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c))^{\frac{2}{3}}}{\left(b^2\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{6\left(b^2\right)^{\frac{2}{3}}}\right.}{d}$
default	$\frac{3b\left(\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{1}{3}}\right)}{6\left(b^2\right)^{\frac{2}{3}}}-\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}\left(b^2\right)^{\frac{1}{3}}+\left(b^2\right)^{\frac{2}{3}}\right)}{12\left(b^2\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c))^{\frac{2}{3}}}{\left(b^2\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{6\left(b^2\right)^{\frac{2}{3}}}\right.}{d}$

```
[In] int(1/(b*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*b*(1/6/(b^2)^(2/3)*ln((b*tan(d*x+c))^(2/3)+(b^2)^(1/3))-1/12/(b^2)^(2/3
)*ln((b*tan(d*x+c))^(4/3)-(b*tan(d*x+c))^(2/3)*(b^2)^(1/3)+(b^2)^(2/3))+1/6
```

```
/(b^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*tan(d*x+c))^(2/3)/(b^2)^(1/3)
-1)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.28

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx$$
$$= \left[\frac{\sqrt{3}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\tan(dx+c)+2b\tan(dx+c)^2-\sqrt{3}b^{\frac{4}{3}}\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+(b\tan(dx+c))^{\frac{2}{3}}\left(\sqrt{3}b^{\frac{2}{3}}\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-3b^{\frac{1}{3}}\right)-b}{\tan(dx+c)^2+1}\right) -}{4b}\right.$$

```
[In] integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*sqrt(-1/b^(2/3))*log((2*sqrt(3)*(b*tan(d*x + c))^(1/3)*b*sq
rt(-1/b^(2/3))*tan(d*x + c) + 2*b*tan(d*x + c)^2 - sqrt(3)*b^(4/3)*sqrt(-1/
b^(2/3)) + (b*tan(d*x + c))^(2/3)*(sqrt(3)*b^(2/3)*sqrt(-1/b^(2/3)) - 3*b^(
1/3)) - b)/(tan(d*x + c)^2 + 1)) - b^(2/3)*log((b*tan(d*x + c))^(1/3)*b*tan
(d*x + c) - (b*tan(d*x + c))^(2/3)*b^(2/3) + b^(4/3)) + 2*b^(2/3)*log((b*ta
n(d*x + c))^(2/3) + b^(2/3)))/(b*d), 1/4*(2*sqrt(3)*b^(2/3)*arctan(1/3*sqrt
(3)*(2*(b*tan(d*x + c))^(2/3)*b^(2/3) - b^(4/3))/b^(4/3)) - b^(2/3)*log((b*
tan(d*x + c))^(1/3)*b*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*b^(2/3) + b^(4/
3)) + 2*b^(2/3)*log((b*tan(d*x + c))^(2/3) + b^(2/3)))/(b*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b\tan{(c+dx)}}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-1/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = \frac{2\sqrt{3}b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-b^{\frac{2}{3}}\right)}{3\,b^{\frac{2}{3}}}\right)-b^{\frac{2}{3}}\log\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)+2\,b^{\frac{2}{3}}\log}{4\,bd}$$

```
[In] integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(3)*b^(2/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3) - b^(2/
3))/b^(2/3)) - b^(2/3)*log((b*tan(d*x + c))^(4/3) - (b*tan(d*x + c))^(2/3)*
b^(2/3) + b^(4/3)) + 2*b^(2/3)*log((b*tan(d*x + c))^(2/3) + b^(2/3)))/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx &= \frac{\sqrt{3}|b|^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-|b|^{\frac{2}{3}}\right)}{3\,|b|^{\frac{2}{3}}}\right)}{2\,bd} \\ &\quad - \frac{|b|^{\frac{2}{3}}\log\left((b\tan(dx+c))^{\frac{1}{3}}\,b\tan(dx+c)-(b\tan(dx+c))^{\frac{2}{3}}\,|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}\right)}{4\,bd} \\ &\quad + \frac{|b|^{\frac{2}{3}}\log\left((b\tan(dx+c))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{2\,bd}\end{aligned}$$

```
[In] integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*abs(b)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3) - abs
(b)^(2/3))/abs(b)^(2/3))/(b*d) - 1/4*abs(b)^(2/3)*log((b*tan(d*x + c))^(1/3
)*b*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*abs(b)^(2/3) + abs(b)^(4/3))/(b*d
) + 1/2*abs(b)^(2/3)*log((b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b*d)
```

Mupad [B] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = \frac{\ln\left(\left(b\tan(c+d\,x)\right)^{2/3}+b^{2/3}\right)}{2\,b^{1/3}\,d} + \frac{\ln\left(\frac{81\,b^{11/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{d^3}+\frac{162\,b^3\,\left(b\tan(c+d\,x)\right)^{2/3}}{d^3}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4\,b^{1/3}\,d} - \frac{\ln\left(\frac{81\,b^{11/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{d^3}-\frac{162\,b^3\,\left(b\tan(c+d\,x)\right)^{2/3}}{d^3}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4\,b^{1/3}\,d}$$

```
[In] int(1/(b*tan(c + d*x))^(1/3),x)
```

```
[Out] log((b*tan(c + d*x))^(2/3) + b^(2/3))/(2*b^(1/3)*d) + (log((81*b^(11/3)*(3^
(1/2)*1i - 1))/d^3 + (162*b^3*(b*tan(c + d*x))^(2/3))/d^3)*(3^(1/2)*1i - 1)
)/(4*b^(1/3)*d) - (log((81*b^(11/3)*(3^(1/2)*1i + 1))/d^3 - (162*b^3*(b*tan
(c + d*x))^(2/3))/d^3)*(3^(1/2)*1i + 1))/(4*b^(1/3)*d)
```

3.21 $\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 224

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{2/3}d} + \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{2/3}d} - \frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{2/3}d} + \frac{\sqrt{3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{2/3}d}$$

```
[Out] arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/b^(2/3)/d+1/2*arctan(-3^(1/2)+2*(b*tan
(d*x+c))^(1/3)/b^(1/3))/b^(2/3)/d+1/2*arctan(3^(1/2)+2*(b*tan(d*x+c))^(1/3)
/b^(1/3))/b^(2/3)/d-1/4*ln(b^(2/3)-b^(1/3)*3^(1/2)*(b*tan(d*x+c))^(1/3)+(b*
tan(d*x+c))^(2/3))*3^(1/2)/b^(2/3)/d+1/4*ln(b^(2/3)+b^(1/3)*3^(1/2)*(b*tan(
d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/b^(2/3)/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3557, 335, 215, 648, 632, 210, 642, 209}

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{2/3}d} + \frac{\arctan\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2b^{2/3}d} - \frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{2/3}d} + \frac{\sqrt{3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{2/3}d}$$

```
[In] Int[(b*Tan[c + d*x])^(-2/3),x]
```

```
[Out] ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)]/(b^(2/3)*d) - ArcTan[Sqrt[3] - (2*(b
*Tan[c + d*x])^(1/3))/b^(1/3)]/(2*b^(2/3)*d) + ArcTan[Sqrt[3] + (2*(b*Tan[c
 + d*x])^(1/3))/b^(1/3)]/(2*b^(2/3)*d) - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*b^(
1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*b^(2/3)*d) + (Sqr
t[3]*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x]
)^(2/3)])/(4*b^(2/3)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
```

```
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\text{Subst}\left(\int \frac{1}{x^{2/3}(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}\\ &= \frac{(3b)\text{Subst}\left(\int \frac{1}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}\end{aligned}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}-\frac{\sqrt{3}x}{2}}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{b^{2/3}d}$$
$$+ \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+\frac{\sqrt{3}x}{2}}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{b^{2/3}d}$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{b^{2/3}+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{\sqrt[3]{b}d}$$

$$= \frac{\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\sqrt{3}\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[3]{b}+2x}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4b^{2/3}d}$$
$$+ \frac{\sqrt{3}\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[3]{b}+2x}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4b^{2/3}d}$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4\sqrt[3]{b}d}$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4\sqrt[3]{b}d}$$

$$= \frac{\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d}$$
$$- \frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{2/3}d}$$
$$+ \frac{\sqrt{3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{2/3}d}$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt{3}b^{2/3}d}$$
$$- \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt{3}b^{2/3}d}$$

$$= \frac{\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\arctan\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2b^{2/3}d}$$
$$+ \frac{\arctan\left(\frac{1}{3}\left(3\sqrt{3}+\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2b^{2/3}d}$$
$$- \frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{2/3}d}$$
$$+ \frac{\sqrt{3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{2/3}d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.84

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{\left(i\log\left(1-i\sqrt[6]{\tan^2(c+dx)}\right)-i\log\left(1+i\sqrt[6]{\tan^2(c+dx)}\right)+\sqrt[6]{-1}\left((-1)^{2/3}\mathrm{lo}\right.\right.}{}$$

```
[In] Integrate[(b*Tan[c + d*x])^(-2/3),x]
```

```
[Out] ((I*Log[1 - I*(Tan[c + d*x]^2)^(1/6)] - I*Log[1 + I*(Tan[c + d*x]^2)^(1/6)]
 + (-1)^(1/6)*((-1)^(2/3)*Log[1 - (-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)] - (-1)
^(2/3)*Log[1 + (-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)] + Log[1 - (-1)^(5/6)*(Tan
[c + d*x]^2)^(1/6)] - Log[1 + (-1)^(5/6)*(Tan[c + d*x]^2)^(1/6)]))*(b*Tan[c
 + d*x])^(1/3))/(2*b*d*(Tan[c + d*x]^2)^(1/6))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{3b\left(-\frac{\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\ln\left(-\left(b\tan(dx+c)\right)^{\frac{2}{3}}+\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\left(b\tan(dx+c)\right)^{\frac{1}{3}}-\left(b^2\right)^{\frac{1}{3}}\right)}{12b^2}+\frac{\left(b^2\right)^{\frac{1}{6}}\arctan\left(\frac{2\left(b\tan(dx+c)\right)^{\frac{1}{3}}}{\left(b^2\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b^2}+\frac{\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\ln}{}\right.}{}$
default	$\frac{3b\left(-\frac{\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\ln\left(-\left(b\tan(dx+c)\right)^{\frac{2}{3}}+\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\left(b\tan(dx+c)\right)^{\frac{1}{3}}-\left(b^2\right)^{\frac{1}{3}}\right)}{12b^2}+\frac{\left(b^2\right)^{\frac{1}{6}}\arctan\left(\frac{2\left(b\tan(dx+c)\right)^{\frac{1}{3}}}{\left(b^2\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b^2}+\frac{\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\ln}{}\right.}{}$

```
[In] int(1/(b*tan(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*b*(-1/12/b^2*3^(1/2)*(b^2)^(1/6)*ln(-(b*tan(d*x+c))^(2/3)+3^(1/2)*(b^2)
^(1/6)*(b*tan(d*x+c))^(1/3)-(b^2)^(1/3))+1/6/b^2*(b^2)^(1/6)*arctan(2*(b*ta
n(d*x+c))^(1/3)/(b^2)^(1/6)-3^(1/2))+1/12/b^2*3^(1/2)*(b^2)^(1/6)*ln((b*tan
(d*x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/6/b^
2*(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)+3^(1/2))+1/3/b^2*(b
^2)^(1/6)*arctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6)))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.26

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(\frac{1}{2}\left(\sqrt{-3}bd+bd\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right) - \frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(-\frac{1}{2}\left(\sqrt{-3}bd+bd\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right) + \frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(\frac{1}{2}\left(\sqrt{-3}bd-bd\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right) - \frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(-\frac{1}{2}\left(\sqrt{-3}bd-bd\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right) + \frac{1}{2}\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(bd\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right) - \frac{1}{2}\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(-bd\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right)$$

```
[In] integrate(1/(b*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(-3) + 1)*(-1/(b^4*d^6))^(1/6)*log(1/2*(sqrt(-3)*b*d + b*d)*(-1/(b
^4*d^6))^(1/6) + (b*tan(d*x + c))^(1/3)) - 1/4*(sqrt(-3) + 1)*(-1/(b^4*d^6)
)^(1/6)*log(-1/2*(sqrt(-3)*b*d + b*d)*(-1/(b^4*d^6))^(1/6) + (b*tan(d*x + c
))^(1/3)) + 1/4*(sqrt(-3) - 1)*(-1/(b^4*d^6))^(1/6)*log(1/2*(sqrt(-3)*b*d -
 b*d)*(-1/(b^4*d^6))^(1/6) + (b*tan(d*x + c))^(1/3)) - 1/4*(sqrt(-3) - 1)*(
-1/(b^4*d^6))^(1/6)*log(-1/2*(sqrt(-3)*b*d - b*d)*(-1/(b^4*d^6))^(1/6) + (b
*tan(d*x + c))^(1/3)) + 1/2*(-1/(b^4*d^6))^(1/6)*log(b*d*(-1/(b^4*d^6))^(1/
6) + (b*tan(d*x + c))^(1/3)) - 1/2*(-1/(b^4*d^6))^(1/6)*log(-b*d*(-1/(b^4*d
^6))^(1/6) + (b*tan(d*x + c))^(1/3))
```

Sympy [F]

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\tan(c+dx))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-2/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.76

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{\sqrt{3}b^{\frac{1}{3}}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)-\sqrt{3}b^{\frac{1}{3}}\log\left(-\sqrt{3}(b\tan\right.}{}$$

```
[In] integrate(1/(b*tan(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(3)*b^(1/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(d*
x + c))^(2/3) + b^(2/3)) - sqrt(3)*b^(1/3)*log(-sqrt(3)*(b*tan(d*x + c))^(1
/3)*b^(1/3) + (b*tan(d*x + c))^(2/3) + b^(2/3)) + 2*b^(1/3)*arctan((sqrt(3)
*b^(1/3) + 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 2*b^(1/3)*arctan(-(sqrt(3)*
b^(1/3) - 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 4*b^(1/3)*arctan((b*tan(d*x
+ c))^(1/3)/b^(1/3)))/(b*d)
```

Giac [F]

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\tan(dx+c))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c))^(-2/3), x)
```

Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx &= \frac{(-1)^{1/6}\,\mathrm{atan}\left(\frac{(-1)^{5/6}\,(b\tan(c+d\,x))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)\,1\mathrm{i}}{b^{2/3}\,d}\\
&- \frac{(-1)^{1/6}\,\ln\left((-1)^{1/6}\,b^{1/3}-2\,(b\tan(c+d\,x))^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{2/3}\,d}\\
&- \frac{(-1)^{1/6}\,\ln\left((-1)^{1/6}\,b^{1/3}+2\,(b\tan(c+d\,x))^{1/3}-(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{2/3}\,d}\\
&+ \frac{(-1)^{1/6}\,\ln\left((-1)^{1/6}\,b^{1/3}+2\,(b\tan(c+d\,x))^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{2/3}\,d}\\
&+ \frac{(-1)^{1/6}\,\ln\left(2\,(b\tan(c+d\,x))^{1/3}-(-1)^{1/6}\,b^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{2/3}\,d}\end{aligned}$$

```
[In] int(1/(b*tan(c + d*x))^(2/3),x)
```

```
[Out] ((-1)^(1/6)*atan(((-1)^(5/6)*(b*tan(c + d*x))^(1/3)*1i)/b^(1/3))*1i)/(b^(2/
3)*d) - ((-1)^(1/6)*log((-1)^(1/6)*b^(1/3) - 2*(b*tan(c + d*x))^(1/3) + (-1
)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(2*b^(2/3)*d) - ((-1)^(1/6
)*log((-1)^(1/6)*b^(1/3) + 2*(b*tan(c + d*x))^(1/3) - (-1)^(2/3)*3^(1/2)*b^
(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*b^(2/3)*d) + ((-1)^(1/6)*log((-1)^(1/6)*b
^(1/3) + 2*(b*tan(c + d*x))^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1
i)/4 + 1/4))/(b^(2/3)*d) + ((-1)^(1/6)*log(2*(b*tan(c + d*x))^(1/3) - (-1)^
(1/6)*b^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/4 - 1/4))/(b^(2/3
)*d)
```

3.22 $\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 245

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} + \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{4/3}d} - \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{4/3}d} - \frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{4/3}d} + \frac{\sqrt{3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

```
[Out] -arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/b^(4/3)/d-1/2*arctan(-3^(1/2)+2*(b*ta
n(d*x+c))^(1/3)/b^(1/3))/b^(4/3)/d-1/2*arctan(3^(1/2)+2*(b*tan(d*x+c))^(1/3
)/b^(1/3))/b^(4/3)/d-1/4*ln(b^(2/3)-b^(1/3)*3^(1/2)*(b*tan(d*x+c))^(1/3)+(b
*tan(d*x+c))^(2/3))*3^(1/2)/b^(4/3)/d+1/4*ln(b^(2/3)+b^(1/3)*3^(1/2)*(b*tan
(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/b^(4/3)/d-3/b/d/(b*tan(d*x+c))
^(1/3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3555, 3557, 335, 301, 648, 632, 210, 642, 209}

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} + \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{4/3}d} - \frac{\arctan\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2b^{4/3}d} - \frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{4/3}d} + \frac{\sqrt{3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

```
[In] Int[(b*Tan[c + d*x])^(-4/3),x]
```

```
[Out] -(ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)]/(b^(4/3)*d)) + ArcTan[Sqrt[3] - (2
*(b*Tan[c + d*x])^(1/3))/b^(1/3)]/(2*b^(4/3)*d) - ArcTan[Sqrt[3] + (2*(b*Ta
n[c + d*x])^(1/3))/b^(1/3)]/(2*b^(4/3)*d) - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*
b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*b^(4/3)*d) + (
Sqrt[3]*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d
*x])^(2/3)])/(4*b^(4/3)*d) - 3/(b*d*(b*Tan[c + d*x])^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
```

```
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{3}{bd\sqrt[3]{b\tan(c+dx)}} - \frac{\int (b\tan(c+dx))^{2/3}\,dx}{b^2}$$

$$= -\frac{3}{bd\sqrt[3]{b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{x^{2/3}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{3}{bd\sqrt[3]{b\tan(c+dx)}} - \frac{3\text{Subst}\left(\int \frac{x^4}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{bd}$$

$$= -\frac{3}{bd\sqrt[3]{b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}+\frac{\sqrt{3}x}{2}}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{b^{4/3}d} - \frac{\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}-\frac{\sqrt{3}x}{2}}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{b^{4/3}d} - \frac{\text{Subst}\left(\int \frac{1}{b^{2/3}+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{bd}$$

$$= -\frac{\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\tan(c+dx)}} - \frac{\sqrt{3}\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[3]{b}+2x}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4b^{4/3}d} + \frac{\sqrt{3}\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[3]{b}+2x}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4b^{4/3}d} - \frac{\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4bd} - \frac{\text{Subst}\left(\int \frac{1}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4bd}$$

$$= -\frac{\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{4/3}d} + \frac{\sqrt{3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt{3}b^{4/3}d} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt{3}b^{4/3}d}$$

$$= -\frac{\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} + \frac{\arctan\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2b^{4/3}d} - \frac{\arctan\left(\frac{1}{3}\left(3\sqrt{3}+\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2b^{4/3}d} - \frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{4/3}d} + \frac{\sqrt{3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.04

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = \frac{-6 - i\log\left(1-i\sqrt[6]{\tan^2(c+dx)}\right)\sqrt[6]{\tan^2(c+dx)} + i\log\left(1+i\sqrt[6]{\tan^2(c+dx)}\right)}{}$$

```
[In] Integrate[(b*Tan[c + d*x])^(-4/3),x]
```

```
[Out] (-6 - I*Log[1 - I*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6) + I*Log[1
+ I*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6) - (-1)^(1/6)*Log[1 - (-1
)^(1/6)*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6) + (-1)^(1/6)*Log[1 +
 (-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6) - (-1)^(5/6)*Log
[1 - (-1)^(5/6)*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6) + (-1)^(5/6)
*Log[1 + (-1)^(5/6)*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6))/(2*b*d*
(b*Tan[c + d*x])^(1/3))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.88

method	result
derivativedivides	$3b\left(-\frac{1}{b^{2}(b\tan(dx+c))^{\frac{1}{3}}}-\frac{\frac{\sqrt{3}\,(b^{2})^{\frac{5}{6}}\ln\left(-(b\tan(dx+c))^{\frac{2}{3}}+\sqrt{3}\,(b^{2})^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}-(b^{2})^{\frac{1}{3}}\right)}{12b^{2}}+\frac{\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{(b^{2})^{\frac{1}{6}}}-\sqrt{3}\right)}{6(b^{2})^{\frac{1}{6}}}}{}\right.$
default	$3b\left(-\frac{1}{b^{2}(b\tan(dx+c))^{\frac{1}{3}}}-\frac{\frac{\sqrt{3}\,(b^{2})^{\frac{5}{6}}\ln\left(-(b\tan(dx+c))^{\frac{2}{3}}+\sqrt{3}\,(b^{2})^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}-(b^{2})^{\frac{1}{3}}\right)}{12b^{2}}+\frac{\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{(b^{2})^{\frac{1}{6}}}-\sqrt{3}\right)}{6(b^{2})^{\frac{1}{6}}}}{}\right.$

```
[In] int(1/(b*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*b*(-1/b^2/(b*tan(d*x+c))^(1/3)-(1/12/b^2*3^(1/2)*(b^2)^(5/6)*ln(-(b*tan
(d*x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)-(b^2)^(1/3))+1/6/(b
^2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)-3^(1/2))-1/12/b^2*3^(1/
2)*(b^2)^(5/6)*ln((b*tan(d*x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(
1/3)+(b^2)^(1/3))+1/6/(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)
+3^(1/2))+1/3/(b^2)^(1/6)*arctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6)))/b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(187) = 374$.

Time = 0.25 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.76

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx =$$

$$-\frac{2\,b^{2}d\left(-\frac{1}{b^{8}d^{6}}\right)^{\frac{1}{6}}\log\left(b^{7}d^{5}\left(-\frac{1}{b^{8}d^{6}}\right)^{\frac{5}{6}}+(b\tan(dx+c))^{\frac{1}{3}}\right)\tan(dx+c)-2\,b^{2}d\left(-\frac{1}{b^{8}d^{6}}\right)^{\frac{1}{6}}\log\left(-b^{7}d^{5}\left(-\frac{1}{b^{8}d^{6}}\right)^{\frac{5}{6}}+\right.}{}$$

```
[In] integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*d*(-1/(b^8*d^6))^(1/6)*log(b^7*d^5*(-1/(b^8*d^6))^(5/6) + (b*ta
n(d*x + c))^(1/3))*tan(d*x + c) - 2*b^2*d*(-1/(b^8*d^6))^(1/6)*log(-b^7*d^5
*(-1/(b^8*d^6))^(5/6) + (b*tan(d*x + c))^(1/3))*tan(d*x + c) - (sqrt(-3)*b^
```

```
2*d - b^2*d)*(-1/(b^8*d^6))^(1/6)*log(1/2*(sqrt(-3)*b^7*d^5 + b^7*d^5)*(-1/
(b^8*d^6))^(5/6) + (b*tan(d*x + c))^(1/3))*tan(d*x + c) + (sqrt(-3)*b^2*d -
 b^2*d)*(-1/(b^8*d^6))^(1/6)*log(-1/2*(sqrt(-3)*b^7*d^5 + b^7*d^5)*(-1/(b^8
*d^6))^(5/6) + (b*tan(d*x + c))^(1/3))*tan(d*x + c) - (sqrt(-3)*b^2*d + b^2
*d)*(-1/(b^8*d^6))^(1/6)*log(1/2*(sqrt(-3)*b^7*d^5 - b^7*d^5)*(-1/(b^8*d^6)
)^(5/6) + (b*tan(d*x + c))^(1/3))*tan(d*x + c) + (sqrt(-3)*b^2*d + b^2*d)*(
-1/(b^8*d^6))^(1/6)*log(-1/2*(sqrt(-3)*b^7*d^5 - b^7*d^5)*(-1/(b^8*d^6))^(5
/6) + (b*tan(d*x + c))^(1/3))*tan(d*x + c) + 12*(b*tan(d*x + c))^(2/3))/(b^
2*d*tan(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\tan{(c+dx)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-4/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.74

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = \frac{\frac{\sqrt{3}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}} - \frac{\sqrt{3}\log\left(-\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}}{}$$

```
[In] integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(d*x + c))^
(2/3) + b^(2/3))/b^(1/3) - sqrt(3)*log(-sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1
/3) + (b*tan(d*x + c))^(2/3) + b^(2/3))/b^(1/3) - 2*arctan((sqrt(3)*b^(1/3)
 + 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 2*arctan(-(sqrt(3)*b^(1/3)
- 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 4*arctan((b*tan(d*x + c))^(1
/3)/b^(1/3))/b^(1/3) - 12/(b*tan(d*x + c))^(1/3))/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.93

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = \frac{1}{4}b\left(\frac{\sqrt{3}|b|^{\frac{5}{3}}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{b^4d}-\frac{\sqrt{3}|b|^{\frac{5}{3}}\mathrm{lo}\right.$$

```
[In] integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] 1/4*b*(sqrt(3)*abs(b)^(5/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3)
 + (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b^4*d) - sqrt(3)*abs(b)^(5/3)*lo
g(-sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3) + a
bs(b)^(2/3))/(b^4*d) - 2*abs(b)^(5/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*(b*t
an(d*x + c))^(1/3))/abs(b)^(1/3))/(b^4*d) - 2*abs(b)^(5/3)*arctan(-(sqrt(3)
*abs(b)^(1/3) - 2*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b^4*d) - 4*abs(b)^
(5/3)*arctan((b*tan(d*x + c))^(1/3)/abs(b)^(1/3))/(b^4*d) - 12/((b*tan(d*x
+ c))^(1/3)*b^2*d))
```

Mupad [B] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx &= -\frac{3}{b\,d\,(b\tan(c+d\,x))^{1/3}}-\frac{(-1)^{1/6}\,\mathrm{atan}\left(\frac{(-1)^{2/3}\,(b\tan(c+d\,x))^{1/3}}{b^{1/3}}\right)\,1\mathrm{i}}{b^{4/3}\,d}\\&-\frac{(-1)^{1/6}\,\ln\left(972\,b^{12}\,d^6-972\,(-1)^{1/6}\,b^{35/3}\,d^6\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(b\tan(c+d\,x))^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{4/3}\,d}\\&-\frac{(-1)^{1/6}\,\ln\left(972\,b^{12}\,d^6-972\,(-1)^{1/6}\,b^{35/3}\,d^6\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(b\tan(c+d\,x))^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{4/3}\,d}\\&+\frac{(-1)^{1/6}\,\ln\left(972\,b^{12}\,d^6+1944\,(-1)^{1/6}\,b^{35/3}\,d^6\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)\,(b\tan(c+d\,x))^{1/3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{4/3}\,d}\\&+\frac{(-1)^{1/6}\,\ln\left(972\,b^{12}\,d^6+1944\,(-1)^{1/6}\,b^{35/3}\,d^6\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)\,(b\tan(c+d\,x))^{1/3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{4/3}\,d}\end{aligned}$$

```
[In] int(1/(b*tan(c + d*x))^(4/3),x)
```

```
[Out] ((-1)^(1/6)*log(972*b^12*d^6 + 1944*(-1)^(1/6)*b^(35/3)*d^6*((3^(1/2)*1i)/4
 - 1/4)*(b*tan(c + d*x))^(1/3))*((3^(1/2)*1i)/4 - 1/4))/(b^(4/3)*d) - ((-1)
^(1/6)*atan(((-1)^(2/3)*(b*tan(c + d*x))^(1/3))/b^(1/3))*1i)/(b^(4/3)*d) -
```

```
((-1)^(1/6)*log(972*b^12*d^6 - 972*(-1)^(1/6)*b^(35/3)*d^6*((3^(1/2)*1i)/2
- 1/2)*(b*tan(c + d*x))^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*b^(4/3)*d) - ((-1
)^(1/6)*log(972*b^12*d^6 - 972*(-1)^(1/6)*b^(35/3)*d^6*((3^(1/2)*1i)/2 + 1/
2)*(b*tan(c + d*x))^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(2*b^(4/3)*d) - 3/(b*d*(
b*tan(c + d*x))^(1/3)) + ((-1)^(1/6)*log(972*b^12*d^6 + 1944*(-1)^(1/6)*b^(
35/3)*d^6*((3^(1/2)*1i)/4 + 1/4)*(b*tan(c + d*x))^(1/3))*((3^(1/2)*1i)/4 +
1/4))/(b^(4/3)*d)
```

3.23 $\int (b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 50

$$\int (b\tan(c+dx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(c+dx)\right)(b\tan(c+dx))^{1+n}}{bd(1+n)}$$

```
[Out] hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(d*x+c)^2)*(b*tan(d*x+c))^(1+n)/b/
d/(1+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3557, 371}

$$\int (b\tan(c+dx))^n\,dx = \frac{(b\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(c+dx)\right)}{bd(n+1)}$$

```
[In] Int[(b*Tan[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[c + d*x]^2]*(b*Tan[c + d*x
])^(1 + n))/(b*d*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\text{Subst}\left(\int \frac{x^n}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}\\ &= \frac{\text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, -\tan^2(c+dx)\right)(b\tan(c+dx))^{1+n}}{bd(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int (b\tan(c+dx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, -\tan^2(c+dx)\right)\tan(c+dx)(b\tan(c+dx))^n}{d(1+n)}$$

```
[In] Integrate[(b*Tan[c + d*x])^n,x]
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[c + d*x]^2]*Tan[c + d*x]*(
b*Tan[c + d*x])^n)/(d*(1 + n))
```

Maple [F]

$$\int (b\tan(dx+c))^n\,dx$$

```
[In] int((b*tan(d*x+c))^n,x)
[Out] int((b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (b\tan(c+dx))^n\,dx = \int (b\tan(dx+c))^n\,dx$$

```
[In] integrate((b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\tan(c+dx))^n \, dx = \int (b\tan{(c+dx)})^n \, dx$$

```
[In] integrate((b*tan(d*x+c))**n,x)
[Out] Integral((b*tan(c + d*x))**n, x)
```

Maxima [F]

$$\int (b\tan(c+dx))^n \, dx = \int (b\tan{(dx+c)})^n \, dx$$

```
[In] integrate((b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c))^n, x)
```

Giac [F]

$$\int (b\tan(c+dx))^n \, dx = \int (b\tan{(dx+c)})^n \, dx$$

```
[In] integrate((b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\tan(c+dx))^n \, dx = \int (b\tan(c+d\,x))^n \, dx$$

```
[In] int((b*tan(c + d*x))^n,x)
[Out] int((b*tan(c + d*x))^n, x)
```

3.24 $\int (b\tan^2(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 98

$$\int (b\tan^2(c+dx))^{5/2}\,dx = -\frac{b^2\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d} - \frac{b^2\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^2(c+dx)}}{4d}$$

```
[Out] -b^2*cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^2)^(1/2)/d-1/2*b^2*(b*tan(d*x+
c)^2)^(1/2)*tan(d*x+c)/d+1/4*b^2*(b*tan(d*x+c)^2)^(1/2)*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 3556}

$$\int (b\tan^2(c+dx))^{5/2}\,dx = -\frac{b^2\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^2(c+dx)}}{4d} - \frac{b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

```
[In] Int[(b*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] -((b^2*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d) - (b^2*Tan
[c + d*x]*Sqrt[b*Tan[c + d*x]^2])/(2*d) + (b^2*Tan[c + d*x]^3*Sqrt[b*Tan[c
+ d*x]^2])/(4*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int\tan^5(c+dx)\,dx\\ &= \frac{b^2\tan^3(c+dx)\sqrt{b\tan^2(c+dx)}}{4d}-\left(b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int\tan^3(c+dx)\,dx\\ &= -\frac{b^2\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d}+\frac{b^2\tan^3(c+dx)\sqrt{b\tan^2(c+dx)}}{4d}\\ &\quad+\left(b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int\tan(c+dx)\,dx\\ &= -\frac{b^2\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d}\\ &\quad-\frac{b^2\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d}+\frac{b^2\tan^3(c+dx)\sqrt{b\tan^2(c+dx)}}{4d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

$$\int\left(b\tan^2(c+dx)\right)^{5/2}\,dx = -\frac{\cot(c+dx)\left(-1+2\cot^2(c+dx)+4\cot^4(c+dx)\log(\cos(c+dx))\right)\left(b\tan^2(c+dx)\right)^{5/2}}{4d}$$

```
[In] Integrate[(b*Tan[c + d*x]^2)^(5/2),x]
[Out] -1/4*(Cot[c + d*x]*(-1 + 2*Cot[c + d*x]^2 + 4*Cot[c + d*x]^4*Log[Cos[c + d*
x]])*(b*Tan[c + d*x]^2)^(5/2))/d
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}\left(\tan^4(dx+c)-2\left(\tan^2(dx+c)\right)+2\ln\left(1+\tan^2(dx+c)\right)\right)}{4d\tan(dx+c)^5}$
default	$\frac{\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}\left(\tan^4(dx+c)-2\left(\tan^2(dx+c)\right)+2\ln\left(1+\tan^2(dx+c)\right)\right)}{4d\tan(dx+c)^5}$
risch	$\frac{b^2\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,x}{e^{2i(dx+c)}-1}-\frac{2b^2\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(dx+c)}{\left(e^{2i(dx+c)}-1\right)d}-\frac{4ib^2\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}}{\left(e^{2i(dx+c)}-\right.}$

```
[In] int((b*tan(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/4/d*(b*tan(d*x+c)^2)^(5/2)*(tan(d*x+c)^4-2*tan(d*x+c)^2+2*ln(1+tan(d*x+c)
^2))/tan(d*x+c)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx=\frac{\left(b^2\tan\left(dx+c\right)^4-2\,b^2\tan\left(dx+c\right)^2-2\,b^2\log\left(\frac{1}{\tan(dx+c)^2+1}\right)-3\,b^2\right)\sqrt{b\tan\left(dx+c\right)^2}}{4\,d\tan\left(dx+c\right)}$$

```
[In] integrate((b*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
[Out] 1/4*(b^2*tan(d*x + c)^4 - 2*b^2*tan(d*x + c)^2 - 2*b^2*log(1/(tan(d*x + c)^
2 + 1)) - 3*b^2)*sqrt(b*tan(d*x + c)^2)/(d*tan(d*x + c))
```

Sympy [F]

$$\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx=\int \left(b\tan^2\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*tan(d*x+c)**2)**(5/2),x)
[Out] Integral((b*tan(c + d*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.48

$$\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx = \frac{b^{\frac{5}{2}}\tan\left(dx+c\right)^4 - 2\,b^{\frac{5}{2}}\tan\left(dx+c\right)^2 + 2\,b^{\frac{5}{2}}\log\left(\tan\left(dx+c\right)^2+1\right)}{4\,d}$$

```
[In] integrate((b*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b^(5/2)*tan(d*x + c)^4 - 2*b^(5/2)*tan(d*x + c)^2 + 2*b^(5/2)*log(tan(
d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. 2(88) = 176.

Time = 1.18 (sec) , antiderivative size = 646, normalized size of antiderivative = 6.59

$$\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*b^2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*sgn(tan(d*x + c))*tan(d*x)^4*tan(c)
^4 + 3*b^2*sgn(tan(d*x + c))*tan(d*x)^4*tan(c)^4 - 8*b^2*log(4*(tan(d*x)^2*
tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c
)^2 + 1))*sgn(tan(d*x + c))*tan(d*x)^3*tan(c)^3 + 2*b^2*sgn(tan(d*x + c))*t
an(d*x)^4*tan(c)^2 - 8*b^2*sgn(tan(d*x + c))*tan(d*x)^3*tan(c)^3 + 2*b^2*sg
n(tan(d*x + c))*tan(d*x)^2*tan(c)^4 + 12*b^2*log(4*(tan(d*x)^2*tan(c)^2 - 2
*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*sg
n(tan(d*x + c))*tan(d*x)^2*tan(c)^2 - b^2*sgn(tan(d*x + c))*tan(d*x)^4 - 8*
b^2*sgn(tan(d*x + c))*tan(d*x)^3*tan(c) + 4*b^2*sgn(tan(d*x + c))*tan(d*x)^
2*tan(c)^2 - 8*b^2*sgn(tan(d*x + c))*tan(d*x)*tan(c)^3 - b^2*sgn(tan(d*x +
c))*tan(c)^4 - 8*b^2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*sgn(tan(d*x + c))*tan(d*x)
*tan(c) + 2*b^2*sgn(tan(d*x + c))*tan(d*x)^2 - 8*b^2*sgn(tan(d*x + c))*tan(
d*x)*tan(c) + 2*b^2*sgn(tan(d*x + c))*tan(c)^2 + 2*b^2*log(4*(tan(d*x)^2*ta
n(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^
2 + 1))*sgn(tan(d*x + c)) + 3*b^2*sgn(tan(d*x + c)))*sqrt(b)/(d*tan(d*x)^4*
tan(c)^4 - 4*d*tan(d*x)^3*tan(c)^3 + 6*d*tan(d*x)^2*tan(c)^2 - 4*d*tan(d*x)
*tan(c) + d)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx = \int \left(b\tan(c+d\,x)^2\right)^{5/2}\,dx$$

```
[In] int((b*tan(c + d*x)^2)^(5/2),x)
[Out] int((b*tan(c + d*x)^2)^(5/2), x)
```

3.25 $\int \left(b \tan^2(c+dx)\right)^{3/2} dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \left(b \tan^2(c+dx)\right)^{3/2} dx = \frac{b \cot(c+dx) \log(\cos(c+dx)) \sqrt{b \tan^2(c+dx)}}{d} + \frac{b \tan(c+dx) \sqrt{b \tan^2(c+dx)}}{2d}$$

```
[Out] b*cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^2)^(1/2)/d+1/2*b*(b*tan(d*x+c)^2)
^(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 3556}

$$\int \left(b \tan^2(c+dx)\right)^{3/2} dx = \frac{b \tan(c+dx) \sqrt{b \tan^2(c+dx)}}{2d} + \frac{b \cot(c+dx) \sqrt{b \tan^2(c+dx)} \log(\cos(c+dx))}{d}$$

```
[In] Int[(b*Tan[c + d*x]^2)^(3/2),x]
```

```
[Out] (b*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d + (b*Tan[c + d*
x]*Sqrt[b*Tan[c + d*x]^2])/(2*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int\tan^3(c+dx)\,dx\\
&= \frac{b\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d}-\left(b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int\tan(c+dx)\,dx\\
&= \frac{b\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d}+\frac{b\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int\left(b\tan^2(c+dx)\right)^{3/2}\,dx=\frac{\cot^3(c+dx)\left(b\tan^2(c+dx)\right)^{3/2}\left(2\log(\cos(c+dx))+\tan^2(c+dx)\right)}{2d}$$

```
[In] Integrate[(b*Tan[c + d*x]^2)^(3/2),x]
```

```
[Out] (Cot[c + d*x]^3*(b*Tan[c + d*x]^2)^(3/2)*(2*Log[Cos[c + d*x]] + Tan[c + d*x
]^2))/(2*d)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{\left(b \left(\tan^{2}\left(d x+c\right)\right)\right)^{\frac{3}{2}} \left(-\left(\tan^{2}\left(d x+c\right)\right)+\ln\left(1+\tan^{2}\left(d x+c\right)\right)\right)}{2 d \tan\left(d x+c\right)^{3}}$
default	$-\frac{\left(b \left(\tan^{2}\left(d x+c\right)\right)\right)^{\frac{3}{2}} \left(-\left(\tan^{2}\left(d x+c\right)\right)+\ln\left(1+\tan^{2}\left(d x+c\right)\right)\right)}{2 d \tan\left(d x+c\right)^{3}}$
risch	$\frac{b\sqrt{-\frac{b\left(\mathrm{e}^{2i(dx+c)}-1\right)^{2}}{\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}}\left(i\mathrm{e}^{4i(dx+c)}\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)+\mathrm{e}^{4i(dx+c)}dx+2\,\mathrm{e}^{4i(dx+c)}c+2i\mathrm{e}^{2i(dx+c)}\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)+2\,\mathrm{e}^{2i(dx+c}}{\left(\mathrm{e}^{2i(dx+c)}-1\right)\left(\mathrm{e}^{2i(dx+c)}+1\right)d}$

```
[In] int((b*tan(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2/d*(b*tan(d*x+c)^2)^(3/2)*(-tan(d*x+c)^2+ln(1+tan(d*x+c)^2))/tan(d*x+c)
^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx=\frac{\left(b\tan\left(dx+c\right)^{2}+b\log\left(\frac{1}{\tan\left(dx+c\right)^{2}+1}\right)+b\right)\sqrt{b\tan\left(dx+c\right)^{2}}}{2\,d\tan\left(dx+c\right)}$$

```
[In] integrate((b*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
[Out] 1/2*(b*tan(d*x + c)^2 + b*log(1/(tan(d*x + c)^2 + 1)) + b)*sqrt(b*tan(d*x +
 c)^2)/(d*tan(d*x + c))
```

Sympy [F]

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx=\int \left(b\tan^{2}\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*tan(d*x+c)**2)**(3/2),x)
[Out] Integral((b*tan(c + d*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx = \frac{b^{\frac{3}{2}}\tan\left(dx+c\right)^2 - b^{\frac{3}{2}}\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

```
[In] integrate((b*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
[Out] 1/2*(b^(3/2)*tan(d*x + c)^2 - b^(3/2)*log(tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(55) = 110.

Time = 0.61 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.70

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx = \frac{\left(\log\left(\frac{4\left(\tan(dx)^2\tan(c)^2-2\tan(dx)\tan(c)+1\right)}{\tan(dx)^2\tan(c)^2+\tan(dx)^2+\tan(c)^2+1}\right)\tan\left(dx\right)^2\tan\left(c\right)^2+\tan\left(dx\right)^2\tan\left(c\right)^2-2\log\left(\frac{4\left(\tan(d\right.}{\tan(dx}\right.}{2\left(d\,t\right.}$$

```
[In] integrate((b*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
[Out] 1/2*(log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)
^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + tan(d*x)^2*tan(c)^2
- 2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^
2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(c) + tan(d*x)^2 + tan(c)^2 + l
og(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 + tan(c)^2 + 1)) + 1)*b^(3/2)*sgn(tan(d*x + c))/(d*tan(d*x)^2*tan
(c)^2 - 2*d*tan(d*x)*tan(c) + d)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx = \int {\left(b\,{\mathrm{tan}\left(c+d\,x\right)}^2\right)}^{3/2} \,d x$$

```
[In] int((b*tan(c + d*x)^2)^(3/2),x)
[Out] int((b*tan(c + d*x)^2)^(3/2), x)
```

3.26 $\int \sqrt{b \tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \sqrt{b \tan^2(c+dx)}\,dx = -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d}$$

```
[Out] -cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^2)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3739, 3556}

$$\int \sqrt{b \tan^2(c+dx)}\,dx = -\frac{\cot(c+dx)\sqrt{b\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

```
[In] Int[Sqrt[b*Tan[c + d*x]^2],x]
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \left(\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int \tan(c+dx)\,dx$$
$$= -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \sqrt{b\tan^2(c+dx)}\,dx = -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d}$$

```
[In] Integrate[Sqrt[b*Tan[c + d*x]^2],x]
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\sqrt{b\left(\tan^2(dx+c)\right)}\,\ln\left(1+\tan^2(dx+c)\right)}{2d\tan(dx+c)}$
default	$\frac{\sqrt{b\left(\tan^2(dx+c)\right)}\,\ln\left(1+\tan^2(dx+c)\right)}{2d\tan(dx+c)}$
risch	$\frac{\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,\left(e^{2i(dx+c)}+1\right)x}{e^{2i(dx+c)}-1} - \frac{2\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,\left(e^{2i(dx+c)}+1\right)(dx+c)}{\left(e^{2i(dx+c)}-1\right)d} - \frac{i\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,\left(e^{2i(dx+\right.}{\left(e^{2i(dx+c)}\right.}$

```
[In] int((b*tan(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/d*(b*tan(d*x+c)^2)^(1/2)/tan(d*x+c)*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \sqrt{b \tan^2(c+dx)}\, dx = -\frac{\sqrt{b \tan{(dx+c)}^2} \log{\left(\frac{1}{\tan{(dx+c)}^2+1}\right)}}{2\, d \tan{(dx+c)}}$$

```
[In] integrate((b*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(b*tan(d*x + c)^2)*log(1/(tan(d*x + c)^2 + 1))/(d*tan(d*x + c))
```

Sympy [F]

$$\int \sqrt{b \tan^2(c+dx)}\, dx = \int \sqrt{b \tan^2{(c+dx)}}\, dx$$

```
[In] integrate((b*tan(d*x+c)**2)**(1/2),x)
[Out] Integral(sqrt(b*tan(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \sqrt{b \tan^2(c+dx)}\, dx = \frac{\sqrt{b} \log{\left(\tan{(dx+c)}^2+1\right)}}{2\, d}$$

```
[In] integrate((b*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(b)*log(tan(d*x + c)^2 + 1)/d
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \sqrt{b \tan^2(c+dx)}\, dx = -\frac{\sqrt{b} \log{(|\cos{(dx+c)}|)} \operatorname{sgn}(\tan{(dx+c)})}{d}$$

```
[In] integrate((b*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
[Out] -sqrt(b)*log(abs(cos(d*x + c)))*sgn(tan(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\tan^2(c+dx)}\,dx = \int \sqrt{b\tan{(c+d\,x)}^2}\,dx$$

```
[In] int((b*tan(c + d*x)^2)^(1/2),x)
[Out] int((b*tan(c + d*x)^2)^(1/2), x)
```

3.27 $\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{b\tan^2(c+dx)}}$$

```
[Out] ln(sin(d*x+c))*tan(d*x+c)/d/(b*tan(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3739, 3556}

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \frac{\tan(c+dx)\log(\sin(c+dx))}{d\sqrt{b\tan^2(c+dx)}}$$

```
[In] Int[1/Sqrt[b*Tan[c + d*x]^2],x]
```

```
[Out] (Log[Sin[c + d*x]]*Tan[c + d*x])/(d*Sqrt[b*Tan[c + d*x]^2])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\tan(c+dx)\int\cot(c+dx)\,dx}{\sqrt{b\tan^2(c+dx)}}\\ &= \frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{b\tan^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int\frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \frac{(\log(\cos(c+dx))+\log(\tan(c+dx)))\tan(c+dx)}{d\sqrt{b\tan^2(c+dx)}}$$

```
[In] Integrate[1/Sqrt[b*Tan[c + d*x]^2],x]
[Out] ((Log[Cos[c + d*x]] + Log[Tan[c + d*x]])*Tan[c + d*x])/(d*Sqrt[b*Tan[c + d*
x]^2])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

method	result
derivativedivides	$-\frac{\tan(dx+c)\left(\ln\left(1+\tan^2(dx+c)\right)-2\ln(\tan(dx+c))\right)}{2d\sqrt{b\left(\tan^2(dx+c)\right)}}$
default	$-\frac{\tan(dx+c)\left(\ln\left(1+\tan^2(dx+c)\right)-2\ln(\tan(dx+c))\right)}{2d\sqrt{b\left(\tan^2(dx+c)\right)}}$
risch	$\frac{\left(e^{2i(dx+c)}-1\right)x}{\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)}-\frac{2\left(e^{2i(dx+c)}-1\right)(dx+c)}{\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)d}-\frac{i\left(e^{2i(dx+c)}-1\right)\ln\left(e^{2i(dx+c)}-1\right)}{\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)d}$

```
[In] int(1/(b*tan(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/d*tan(d*x+c)*(ln(1+tan(d*x+c)^2)-2*ln(tan(d*x+c)))/(b*tan(d*x+c)^2)^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \frac{\sqrt{b\tan\left(dx+c\right)^2}\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)}{2\,bd\tan\left(dx+c\right)}$$

```
[In] integrate(1/(b*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(b*tan(d*x + c)^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))/(b*d*ta
n(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan^2\left(c+dx\right)}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c)**2)**(1/2),x)
[Out] Integral(1/sqrt(b*tan(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = -\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{b}} - \frac{2\,\log(\tan(dx+c))}{\sqrt{b}}}{2\,d}$$

```
[In] integrate(1/(b*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*(log(tan(d*x + c)^2 + 1)/sqrt(b) - 2*log(tan(d*x + c))/sqrt(b))/d
```

Giac [F]

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan{(dx+c)}^2}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*tan(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{-b}\tan(c+dx)}{\sqrt{b\tan(c+dx)^2}}\right)}{\sqrt{-b}\,d}$$

```
[In] int(1/(b*tan(c + d*x)^2)^(1/2),x)
[Out] atan(((-b)^(1/2)*tan(c + d*x))/(b*tan(c + d*x)^2)^(1/2))/((-b)^(1/2)*d)
```

3.28 $\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 66

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = -\frac{\cot(c+dx)}{2bd\sqrt{b\tan^2(c+dx)}} - \frac{\log(\sin(c+dx))\tan(c+dx)}{bd\sqrt{b\tan^2(c+dx)}}$$

```
[Out] -1/2*cot(d*x+c)/b/d/(b*tan(d*x+c)^2)^(1/2)-ln(sin(d*x+c))*tan(d*x+c)/b/d/(b
*tan(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 3556}

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = -\frac{\cot(c+dx)}{2bd\sqrt{b\tan^2(c+dx)}} - \frac{\tan(c+dx)\log(\sin(c+dx))}{bd\sqrt{b\tan^2(c+dx)}}$$

```
[In] Int[(b*Tan[c + d*x]^2)^(-3/2),x]
```

```
[Out] -1/2*Cot[c + d*x]/(b*d*Sqrt[b*Tan[c + d*x]^2]) - (Log[Sin[c + d*x]]*Tan[c +
 d*x])/(b*d*Sqrt[b*Tan[c + d*x]^2])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\tan(c+dx)\int\cot^3(c+dx)\,dx}{b\sqrt{b\tan^2(c+dx)}}\\ &= -\frac{\cot(c+dx)}{2bd\sqrt{b\tan^2(c+dx)}}-\frac{\tan(c+dx)\int\cot(c+dx)\,dx}{b\sqrt{b\tan^2(c+dx)}}\\ &= -\frac{\cot(c+dx)}{2bd\sqrt{b\tan^2(c+dx)}}-\frac{\log(\sin(c+dx))\tan(c+dx)}{bd\sqrt{b\tan^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int\frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = -\frac{\left(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx))\right)\tan^3(c+dx)}{2d\left(b\tan^2(c+dx)\right)^{3/2}}$$

```
[In] Integrate[(b*Tan[c + d*x]^2)^(-3/2),x]
```

```
[Out] -1/2*((Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]])*Tan[c +
d*x]^3)/(d*(b*Tan[c + d*x]^2)^(3/2))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\tan(dx+c)\left(\ln\left(1+\tan^2(dx+c)\right)\left(\tan^2(dx+c)\right)-2\ln(\tan(dx+c))\left(\tan^2(dx+c)\right)-1\right)}{2d\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}$
default	$\frac{\tan(dx+c)\left(\ln\left(1+\tan^2(dx+c)\right)\left(\tan^2(dx+c)\right)-2\ln(\tan(dx+c))\left(\tan^2(dx+c)\right)-1\right)}{2d\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}$
risch	$\frac{i e^{4i(dx+c)}\ln\left(e^{2i(dx+c)}-1\right)+e^{4i(dx+c)}dx+2e^{4i(dx+c)}c-2ie^{2i(dx+c)}\ln\left(e^{2i(dx+c)}-1\right)-2e^{2i(dx+c)}dx-2ie^{2i(dx+c)}-4e^{2i(}}{b\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,d}$

[In] int(1/(b*tan(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2/d*tan(d*x+c)*(ln(1+tan(d*x+c)^2)*tan(d*x+c)^2-2*ln(tan(d*x+c))*tan(d*x+c)^2-1)/(b*tan(d*x+c)^2)^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = -\frac{\sqrt{b\tan(dx+c)^2}\left(\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2+\tan(dx+c)^2+1\right)}{2b^2d\tan(dx+c)^3}$$

[In] integrate(1/(b*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")

[Out] -1/2*sqrt(b*tan(d*x + c)^2)*(log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 + tan(d*x + c)^2 + 1)/(b^2*d*tan(d*x + c)^3)

Sympy [F]

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan^2(c+dx)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(b*tan(d*x+c)**2)**(3/2),x)

[Out] Integral((b*tan(c + d*x)**2)**(-3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70

$$\int \frac{1}{(b\tan^2(c+dx))^{3/2}}\,dx = \frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{b^{\frac{3}{2}}} - \frac{2\log(\tan(dx+c))}{b^{\frac{3}{2}}} - \frac{1}{b^{\frac{3}{2}}\tan(dx+c)^2}}{2\,d}$$

```
[In] integrate(1/(b*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
[Out] 1/2*(log(tan(d*x + c)^2 + 1)/b^(3/2) - 2*log(tan(d*x + c))/b^(3/2) - 1/(b^(
3/2)*tan(d*x + c)^2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(60) = 120.

Time = 0.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.56

$$\int \frac{1}{(b\tan^2(c+dx))^{3/2}}\,dx = \frac{\frac{\left(\frac{4(\cos(dx+c)-1)}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{\sqrt{b}(\cos(dx+c)-1)\mathrm{sgn}(\tan(dx+c))} - \frac{4\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{\sqrt{b}\mathrm{sgn}(\tan(dx+c))} + \frac{8\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{\sqrt{b}\mathrm{sgn}(\tan(dx+c))} + \frac{c}{\sqrt{b}(\cos(dx-}}{8\,bd}$$

```
[In] integrate(1/(b*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
[Out] 1/8*((4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(sqrt
(b)*(cos(d*x + c) - 1)*sgn(tan(d*x + c))) - 4*log(abs(-cos(d*x + c) + 1)/ab
s(cos(d*x + c) + 1))/(sqrt(b)*sgn(tan(d*x + c))) + 8*log(abs(-(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 1))/(sqrt(b)*sgn(tan(d*x + c))) + (cos(d*x + c)
- 1)/(sqrt(b)*(cos(d*x + c) + 1)*sgn(tan(d*x + c))))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan^2(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+d\,x\right)^2\right)^{3/2}}\,dx$$

```
[In] int(1/(b*tan(c + d*x)^2)^(3/2),x)
[Out] int(1/(b*tan(c + d*x)^2)^(3/2), x)
```

3.29 $\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 97

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx = \frac{\cot(c+dx)}{2b^2d\sqrt{b\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}} + \frac{\log(\sin(c+dx))\tan(c+dx)}{b^2d\sqrt{b\tan^2(c+dx)}}$$

```
[Out] 1/2*cot(d*x+c)/b^2/d/(b*tan(d*x+c)^2)^(1/2)-1/4*cot(d*x+c)^3/b^2/d/(b*tan(d
*x+c)^2)^(1/2)+ln(sin(d*x+c))*tan(d*x+c)/b^2/d/(b*tan(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 3556}

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx = -\frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}} + \frac{\cot(c+dx)}{2b^2d\sqrt{b\tan^2(c+dx)}} + \frac{\tan(c+dx)\log(\sin(c+dx))}{b^2d\sqrt{b\tan^2(c+dx)}}$$

```
[In] Int[(b*Tan[c + d*x]^2)^(-5/2),x]
```

```
[Out] Cot[c + d*x]/(2*b^2*d*Sqrt[b*Tan[c + d*x]^2]) - Cot[c + d*x]^3/(4*b^2*d*Sqr
t[b*Tan[c + d*x]^2]) + (Log[Sin[c + d*x]]*Tan[c + d*x])/(b^2*d*Sqrt[b*Tan[c
 + d*x]^2])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tan(c+dx)\int \cot^5(c+dx)\,dx}{b^2\sqrt{b\tan^2(c+dx)}}\\
&= -\frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}} - \frac{\tan(c+dx)\int \cot^3(c+dx)\,dx}{b^2\sqrt{b\tan^2(c+dx)}}\\
&= \frac{\cot(c+dx)}{2b^2d\sqrt{b\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}} + \frac{\tan(c+dx)\int \cot(c+dx)\,dx}{b^2\sqrt{b\tan^2(c+dx)}}\\
&= \frac{\cot(c+dx)}{2b^2d\sqrt{b\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}} + \frac{\log(\sin(c+dx))\tan(c+dx)}{b^2d\sqrt{b\tan^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

$$
\int \frac{1}{(b\tan^2(c+dx))^{5/2}}\,dx = \frac{2\cot(c+dx) - \cot^3(c+dx) + 4(\log(\cos(c+dx)) + \log(\tan(c+dx)))\tan(c+dx}{4b^2d\sqrt{b\tan^2(c+dx)}}
$$

```
[In] Integrate[(b*Tan[c + d*x]^2)^(-5/2),x]

[Out] (2*Cot[c + d*x] - Cot[c + d*x]^3 + 4*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]
)*Tan[c + d*x])/(4*b^2*d*Sqrt[b*Tan[c + d*x]^2])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

method	result
derivativedivides	$-\frac{\tan(dx+c)\left(2\ln\left(1+\tan^2(dx+c)\right)\left(\tan^4(dx+c)\right)-4\ln(\tan(dx+c))\left(\tan^4(dx+c)\right)-2\left(\tan^2(dx+c)\right)+1\right)}{4d\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}}$
default	$-\frac{\tan(dx+c)\left(2\ln\left(1+\tan^2(dx+c)\right)\left(\tan^4(dx+c)\right)-4\ln(\tan(dx+c))\left(\tan^4(dx+c)\right)-2\left(\tan^2(dx+c)\right)+1\right)}{4d\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}}$
risch	$\frac{\left(e^{2i(dx+c)}-1\right)x}{b^2\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}}-\frac{2\left(e^{2i(dx+c)}-1\right)(dx+c)}{b^2\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,d}+\frac{4i\left(e^{6i(dx+c)}-e^{4i(dx+c)}\right.}{b^2\left(e^{2i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)}$

[In] int(1/(b*tan(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/4/d*tan(d*x+c)*(2*ln(1+tan(d*x+c)^2)*tan(d*x+c)^4-4*ln(tan(d*x+c))*tan(d
*x+c)^4-2*tan(d*x+c)^2+1)/(b*tan(d*x+c)^2)^(5/2)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx = \frac{\left(2\,\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^4+3\,\tan\left(dx+c\right)^4+2\,\tan\left(dx+c\right)^2-1\right)\sqrt{b}}{4\,b^3d\tan\left(dx+c\right)^5}$$

[In] integrate(1/(b*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")

[Out] 1/4*(2*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^4 + 3*tan(d*x
+ c)^4 + 2*tan(d*x + c)^2 - 1)*sqrt(b*tan(d*x + c)^2)/(b^3*d*tan(d*x + c)^5
)

Sympy [F]

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan^2\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(1/(b*tan(d*x+c)**2)**(5/2),x)

[Out] Integral((b*tan(c + d*x)**2)**(-5/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx = -\frac{\frac{2\,\log\left(\tan(dx+c)^2+1\right)}{b^{\frac{5}{2}}} - \frac{4\,\log(\tan(dx+c))}{b^{\frac{5}{2}}} - \frac{2\,\sqrt{b}\tan(dx+c)^2-\sqrt{b}}{b^3\tan(dx+c)^4}}{4\,d}$$

```
[In] integrate(1/(b*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*log(tan(d*x + c)^2 + 1)/b^(5/2) - 4*log(tan(d*x + c))/b^(5/2) - (2*
sqrt(b)*tan(d*x + c)^2 - sqrt(b))/(b^3*tan(d*x + c)^4))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(87) = 174.

Time = 0.44 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.32

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx =$$

$$-\frac{\frac{\left(\frac{12\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^2}{b^{\frac{5}{2}}(\cos(dx+c)-1)^2\mathrm{sgn}(\tan(dx+c))} - \frac{32\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{b^{\frac{5}{2}}\,\mathrm{sgn}(\tan(dx+c))} + \frac{64\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{b^{\frac{5}{2}}\,\mathrm{sgn}(\tan(dx+c))} + \frac{12\,b^{\frac{5}{2}}\,(\cos(dx+c)-1)\mathrm{sgn}(}{\cos(dx+c)+1}}{64\,d}$$

```
[In] integrate(1/(b*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/64*((12*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(b^(5/2)*(cos(d*x + c) - 1)^
2*sgn(tan(d*x + c))) - 32*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))
/(b^(5/2)*sgn(tan(d*x + c))) + 64*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 1))/(b^(5/2)*sgn(tan(d*x + c))) + (12*b^(5/2)*(cos(d*x + c) - 1)*sg
n(tan(d*x + c))/(cos(d*x + c) + 1) + b^(5/2)*(cos(d*x + c) - 1)^2*sgn(tan(d
*x + c))/(cos(d*x + c) + 1)^2)/b^5)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+d\,x\right)^2\right)^{5/2}}\,dx$$

```
[In] int(1/(b*tan(c + d*x)^2)^(5/2),x)
```

```
[Out] int(1/(b*tan(c + d*x)^2)^(5/2), x)
```

3.30 $\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 364

$$\begin{aligned}\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx &= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d}\\
&-\frac{b^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}\\
&+\frac{b^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}\\
&-\frac{b^2\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}\\
&+\frac{b^2\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}\\
&+\frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d}-\frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d}\\
&+\frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d}\end{aligned}$$

```
[Out] -2*b^2*cot(d*x+c)*(b*tan(d*x+c)^3)^(1/2)/d+1/2*b^2*arctan(-1+2^(1/2)*tan(d*
x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+1/2*b^2*arcta
n(1+2^(1/2)*tan(d*x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(
3/2)-1/4*b^2*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(b*tan(d*x+c)^3)^(1/
2)/d*2^(1/2)/tan(d*x+c)^(3/2)+1/4*b^2*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x
+c))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+2/5*b^2*(b*tan(d*x+c
)^3)^(1/2)*tan(d*x+c)/d-2/9*b^2*(b*tan(d*x+c)^3)^(1/2)*tan(d*x+c)^3/d+2/13*
b^2*(b*tan(d*x+c)^3)^(1/2)*tan(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3739, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx = -\frac{b^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{b^2\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} + \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d} - \frac{b^2\sqrt{b\tan^3(c+dx)}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{b^2\sqrt{b\tan^3(c+dx)}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d}$$

```
[In] Int[(b*Tan[c + d*x]^3)^(5/2),x]
```

```
[Out] (-2*b^2*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^3])/d - (b^2*ArcTan[1 - Sqrt[2]*Sq
rt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x]^(3/2)) +
(b^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2
]*d*Tan[c + d*x]^(3/2)) - (b^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]^(3/2)) + (b^2*Log[
1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*S
qrt[2]*d*Tan[c + d*x]^(3/2)) + (2*b^2*Tan[c + d*x]*Sqrt[b*Tan[c + d*x]^3])/
(5*d) - (2*b^2*Tan[c + d*x]^3*Sqrt[b*Tan[c + d*x]^3])/(9*d) + (2*b^2*Tan[c
+ d*x]^5*Sqrt[b*Tan[c + d*x]^3])/(13*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\int \tan^{\frac{15}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d} - \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\int \tan^{\frac{11}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)} \\
&= -\frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d} + \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d} \\
&\quad + \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\int \tan^{\frac{7}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - \frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d} \\
&\quad + \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d} - \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\int \tan^{\frac{3}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)} \\
&= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} \\
&\quad - \frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d} + \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d} \\
&\quad + \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{\tan^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - \frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d} + \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d} + \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - \frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d} + \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d} + \frac{\left(2b^2\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - \frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d} + \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d} + \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\tan^{\frac{3}{2}}(c+dx)} + \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - \frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d} + \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d} + \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d\tan^{\frac{3}{2}}(c+dx)} + \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d\tan^{\frac{3}{2}}(c+dx)} - \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{b^2\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{b^2\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - \frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d} + \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d} + \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{b^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{b^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{b^2\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{b^2\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - \frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d} + \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.56

$$\int \left(b\tan^3(c+dx)\right)^{5/2} dx = \frac{\left(b\tan^3(c+dx)\right)^{5/2}\left(-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(\right.}{2\sqrt{2}}\right.}{}$$

```
[In] Integrate[(b*Tan[c + d*x]^3)^(5/2),x]
```

```
[Out] ((b*Tan[c + d*x]^3)^(5/2)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]
) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]]/(2*Sqrt[2]) - 2*Sqrt[Tan[c + d*x]] + (2*Tan[c + d*x]^(5/
2))/5 - (2*Tan[c + d*x]^(9/2))/9 + (2*Tan[c + d*x]^(13/2))/13))/(d*Tan[c +
d*x]^(15/2))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\left(b\left(\tan^3(dx+c)\right)\right)^{\frac{5}{2}}\left(360\left(b\tan(dx+c)\right)^{\frac{13}{2}}-520b^2\left(b\tan(dx+c)\right)^{\frac{9}{2}}+585b^6\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{}}{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{}}\right.\right.}{}$
default	$\frac{\left(b\left(\tan^3(dx+c)\right)\right)^{\frac{5}{2}}\left(360\left(b\tan(dx+c)\right)^{\frac{13}{2}}-520b^2\left(b\tan(dx+c)\right)^{\frac{9}{2}}+585b^6\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{}}{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{}}\right.\right.}{}$

```
[In] int((b*tan(d*x+c)^3)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2340/d*(b*tan(d*x+c)^3)^(5/2)*(360*(b*tan(d*x+c))^(13/2)-520*b^2*(b*tan(d
*x+c))^(9/2)+585*b^6*(b^2)^(1/4)*2^(1/2)*ln((b*tan(d*x+c)+(b^2)^(1/4)*(b*ta
n(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c
))^(1/2)*2^(1/2)+(b^2)^(1/2)))+1170*b^6*(b^2)^(1/4)*2^(1/2)*arctan((2^(1/2)
*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+1170*b^6*(b^2)^(1/4)*2^(1/2
)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))+936*b^4*(b
*tan(d*x+c))^(5/2)-4680*b^6*(b*tan(d*x+c))^(1/2))/tan(d*x+c)^5/(b*tan(d*x+c
))^(5/2)/b^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.91

$$\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx = \frac{585\left(-\frac{b^{10}}{d^4}\right)^{\frac{1}{4}} d\log\left(\frac{\sqrt{b\tan(dx+c)^3}b^2+\left(-\frac{b^{10}}{d^4}\right)^{\frac{1}{4}} d\tan(dx+c)}{\tan(dx+c)}\right)\tan(dx+c)+585i\left(-\frac{b^{10}}{d^4}\right)^{\frac{1}{4}} d\log\left(\frac{\sqrt{b\tan}}{}\right.}{}$$

```
[In] integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1170*(585*(-b^10/d^4)^(1/4)*d*log((sqrt(b*tan(d*x + c)^3)*b^2 + (-b^10/d^
4)^(1/4)*d*tan(d*x + c))/tan(d*x + c))*tan(d*x + c) + 585*I*(-b^10/d^4)^(1/
4)*d*log((sqrt(b*tan(d*x + c)^3)*b^2 + I*(-b^10/d^4)^(1/4)*d*tan(d*x + c))/
tan(d*x + c))*tan(d*x + c) - 585*I*(-b^10/d^4)^(1/4)*d*log((sqrt(b*tan(d*x
+ c)^3)*b^2 - I*(-b^10/d^4)^(1/4)*d*tan(d*x + c))/tan(d*x + c))*tan(d*x + c
) - 585*(-b^10/d^4)^(1/4)*d*log((sqrt(b*tan(d*x + c)^3)*b^2 - (-b^10/d^4)^(
1/4)*d*tan(d*x + c))/tan(d*x + c))*tan(d*x + c) + 4*(45*b^2*tan(d*x + c)^6
- 65*b^2*tan(d*x + c)^4 + 117*b^2*tan(d*x + c)^2 - 585*b^2)*sqrt(b*tan(d*x
+ c)^3))/(d*tan(d*x + c))
```

Sympy [F]

$$\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx = \int \left(b\tan^3\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*tan(d*x+c)**3)**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**3)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.49

$$\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx = \frac{360\,b^{\frac{5}{2}}\tan\left(dx+c\right)^{\frac{13}{2}}-520\,b^{\frac{5}{2}}\tan\left(dx+c\right)^{\frac{9}{2}}+936\,b^{\frac{5}{2}}\tan\left(dx+c\right)^{\frac{5}{2}}+585\left(2\sqrt{2}\sqrt{b}\arctan\left(\frac{1}{2}\right.\right.}{}$$

```
[In] integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2340*(360*b^(5/2)*tan(d*x + c)^(13/2) - 520*b^(5/2)*tan(d*x + c)^(9/2) +
936*b^(5/2)*tan(d*x + c)^(5/2) + 585*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*
(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(
sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan(d*x
 + c)) + tan(d*x + c) + 1) - sqrt(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(d*x + c)
) + tan(d*x + c) + 1))*b^2 - 4680*b^(5/2)*sqrt(tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.80

$$\int (b\tan^3(c+dx))^{5/2}\,dx = \frac{1}{2340}\left(\frac{1170\sqrt{2}b\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d} + \frac{1170\sqrt{2}b\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|\cdots}\right.}{\cdots}\right.}{d}\cdots\right.$$

```
[In] integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2340*(1170*sqrt(2)*b*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)
) + 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d + 1170*sqrt(2)*b*sqrt(abs(b))*a
rctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs
(b)))/d + 585*sqrt(2)*b*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*ta
n(d*x + c))*sqrt(abs(b)) + abs(b))/d - 585*sqrt(2)*b*sqrt(abs(b))*log(b*tan
(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/d + 8*(45*s
qrt(b*tan(d*x + c))*b^66*d^12*tan(d*x + c)^6 - 65*sqrt(b*tan(d*x + c))*b^66
*d^12*tan(d*x + c)^4 + 117*sqrt(b*tan(d*x + c))*b^66*d^12*tan(d*x + c)^2 -
585*sqrt(b*tan(d*x + c))*b^66*d^12)/(b^65*d^13))*b*sgn(tan(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int (b\tan^3(c+dx))^{5/2}\,dx = \int \left(b\tan(c+d\,x)^3\right)^{5/2}\,dx$$

```
[In] int((b*tan(c + d*x)^3)^(5/2),x)
```

```
[Out] int((b*tan(c + d*x)^3)^(5/2), x)
```

3.31 $\int (b\tan^3(c+dx))^{3/2}\, dx$

Optimal result

Integrand size = 14, antiderivative size = 286

$$\begin{aligned}\int (b\tan^3(c+dx))^{3/2}\, dx = &-\frac{2b\sqrt{b\tan^3(c+dx)}}{3d}\\ &-\frac{b\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}\\ &+\frac{b\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}\\ &+\frac{b\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}\\ &-\frac{b\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}\\ &+\frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d}\end{aligned}$$

```
[Out] -2/3*b*(b*tan(d*x+c)^3)^(1/2)/d+1/2*b*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(
b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+1/2*b*arctan(1+2^(1/2)*tan
(d*x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+1/4*b*ln(1
-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(
d*x+c)^(3/2)-1/4*b*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(b*tan(d*x+c)^
3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+2/7*b*(b*tan(d*x+c)^3)^(1/2)*tan(d*x+c)
^2/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3739, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx = -\frac{b\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{b\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} + \frac{b\sqrt{b\tan^3(c+dx)}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{b\sqrt{b\tan^3(c+dx)}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}$$

```
[In] Int[(b*Tan[c + d*x]^3)^(3/2),x]
```

```
[Out] (-2*b*Sqrt[b*Tan[c + d*x]^3])/(3*d) - (b*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x]^(3/2)) + (b*ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x
]^(3/2)) + (b*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan
[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]^(3/2)) - (b*Log[1 + Sqrt[2]*Sqrt[Ta
n[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*
x]^(3/2)) + (2*b*Tan[c + d*x]^2*Sqrt[b*Tan[c + d*x]^3])/(7*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\int \tan^{\frac{9}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} - \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\int \tan^{\frac{5}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\int \sqrt{\tan(c+dx)}\,dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} + \frac{\left(2b\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} - \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\tan^{\frac{3}{2}}(c+dx)} + \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d\tan^{\frac{3}{2}}(c+dx)} + \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d\tan^{\frac{3}{2}}(c+dx)} + \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{b\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{b\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} - \frac{b\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{b\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{b\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{b\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.40

$$\int (b\tan^3(c+dx))^{3/2}\,dx = \frac{b\sqrt{b\tan^3(c+dx)}\left(21\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan(c+dx)} - 21\text{arctanh}\left(\sqrt[4]{-\tan^2(c+dx)}\right)\ldots\right.}{21d\tan^{\frac{7}{4}}(c+dx)}$$

[In] Integrate[(b*Tan[c + d*x]^3)^(3/2),x]

[Out] (b*Sqrt[b*Tan[c + d*x]^3]*(21*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x])^(1/4) - 21*ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x])^(1/4) + 2*Tan[c + d*x]^(7/4)*(-7 + 3*Tan[c + d*x]^2)))/(21*d*Tan[c + d*x]^(7/4))

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{(b(\tan^3(dx+c)))^{\frac{3}{2}}\left(24(b\tan(dx+c))^{\frac{7}{2}}(b^2)^{\frac{1}{4}}+21b^4\sqrt{2}\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+42b^4\sqrt{2}\arctan\ldots\right.}{84d\tan(dx+c)^3(b\tan(dx+\ldots}$
default	$\frac{(b(\tan^3(dx+c)))^{\frac{3}{2}}\left(24(b\tan(dx+c))^{\frac{7}{2}}(b^2)^{\frac{1}{4}}+21b^4\sqrt{2}\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+42b^4\sqrt{2}\arctan\ldots\right.}{84d\tan(dx+c)^3(b\tan(dx+\ldots}$

[In] int((b*tan(d*x+c)^3)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/84/d*(b*tan(d*x+c)^3)^(3/2)*(24*(b*tan(d*x+c))^(7/2)*(b^2)^(1/4)+21*b^4*2^(1/2)*ln(-((b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+42*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+42*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))-56*b^2*(b*tan(d*x+c))^(3/2)*(b^2)^(1/4))/tan(d*x+c)^3/(b*tan(d*x+c))^(3/2)/b^2/(b^2)^(1/4)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.98

$$\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx = \frac{21\left(-\frac{b^6}{d^4}\right)^{\frac{1}{4}}d\log\left(\frac{\left(-\frac{b^6}{d^4}\right)^{\frac{3}{4}}d^3\tan(dx+c)+\sqrt{b\tan(dx+c)^3}b^4}{\tan(dx+c)}\right)-21\left(-\frac{b^6}{d^4}\right)^{\frac{1}{4}}d\log\left(-\frac{\left(-\frac{b^6}{d^4}\right)^{\frac{3}{4}}d^3\tan(dx+c)-\sqrt{}}{\tan(dx+c)}\right.}{}$$

```
[In] integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/42*(21*(-b^6/d^4)^(1/4)*d*log(((-b^6/d^4)^(3/4)*d^3*tan(d*x + c) + sqrt(b
*tan(d*x + c)^3)*b^4)/tan(d*x + c)) - 21*(-b^6/d^4)^(1/4)*d*log(-((-b^6/d^4
)^(3/4)*d^3*tan(d*x + c) - sqrt(b*tan(d*x + c)^3)*b^4)/tan(d*x + c)) - 21*I
*(-b^6/d^4)^(1/4)*d*log((I*(-b^6/d^4)^(3/4)*d^3*tan(d*x + c) + sqrt(b*tan(d
*x + c)^3)*b^4)/tan(d*x + c)) + 21*I*(-b^6/d^4)^(1/4)*d*log((-I*(-b^6/d^4)^
(3/4)*d^3*tan(d*x + c) + sqrt(b*tan(d*x + c)^3)*b^4)/tan(d*x + c)) + 4*sqrt
(b*tan(d*x + c)^3)*(3*b*tan(d*x + c)^2 - 7*b))/d
```

Sympy [F]

$$\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx = \int \left(b\tan^3\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*tan(d*x+c)**3)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**3)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.49

$$\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx = \frac{24\,b^{\frac{3}{2}}\tan\left(dx+c\right)^{\frac{7}{2}}-56\,b^{\frac{3}{2}}\tan\left(dx+c\right)^{\frac{3}{2}}+21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)\right.}{}$$

```
[In] integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/84*(24*b^(3/2)*tan(d*x + c)^(7/2) - 56*b^(3/2)*tan(d*x + c)^(3/2) + 21*(2
*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*log(sqrt(2)*
sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(tan(d*x
+ c)) + tan(d*x + c) + 1))*b^(3/2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.88

$$\int (b\tan^3(c+dx))^{3/2}\,dx = \frac{1}{84}b\left(\frac{42\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{bd} + \frac{42\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(d}\right.}{2\sqrt{|b|}}\right.}{bd}\right.$$

```
[In] integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/84*b*(42*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) +
2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b*d) + 42*sqrt(2)*abs(b)^(3/2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b))
)/(b*d) - 21*sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d
*x + c))*sqrt(abs(b)) + abs(b))/(b*d) + 21*sqrt(2)*abs(b)^(3/2)*log(b*tan(d
*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b*d) + 8*(3*
sqrt(b*tan(d*x + c))*b^21*d^6*tan(d*x + c)^3 - 7*sqrt(b*tan(d*x + c))*b^21*
d^6*tan(d*x + c))/(b^21*d^7))*sgn(tan(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int (b\tan^3(c+dx))^{3/2}\,dx = \int {\left(b\,{\mathrm{tan}\left(c+d\,x\right)}^3\right)}^{3/2}\,dx$$

```
[In] int((b*tan(c + d*x)^3)^(3/2),x)
```

```
[Out] int((b*tan(c + d*x)^3)^(3/2), x)
```

3.32 $\int \sqrt{b\tan^3(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 255

$$\int \sqrt{b\tan^3(c+dx)}\,dx = \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2*cot(d*x+c)*(b*tan(d*x+c)^3)^(1/2)/d-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2
))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)-1/2*arctan(1+2^(1/2)*t
an(d*x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+1/4*ln(1
-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(
d*x+c)^(3/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(b*tan(d*x+c)^3)
^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int \sqrt{b\tan^3(c+dx)}\,dx = {} & \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} \\ & - \frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} \\ & + \frac{\sqrt{b\tan^3(c+dx)}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} \\ & - \frac{\sqrt{b\tan^3(c+dx)}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} \\ & + \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d}\end{aligned}$$

```
[In] Int[Sqrt[b*Tan[c + d*x]^3],x]
```

```
[Out] (2*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^3])/d + (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x]^(3/2)) - (ArcTan[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d
*x]^(3/2)) + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan
[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]^(3/2)) - (Log[1 + Sqrt[2]*Sqrt[Tan[
c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]
^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{b\tan^3(c+dx)}\int \tan^{\frac{3}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{\sqrt{b\tan^3(c+dx)}\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{\sqrt{b\tan^3(c+dx)}\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{\left(2\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} \\
&\quad - \frac{\sqrt{b\tan^3(c+dx)}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\tan^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{\sqrt{b\tan^3(c+dx)}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} \\
&\quad - \frac{\sqrt{b\tan^3(c+dx)}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d\tan^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{\sqrt{b\tan^3(c+dx)}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d\tan^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{\sqrt{b\tan^3(c+dx)}\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{\sqrt{b\tan^3(c+dx)}\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

$$= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{\sqrt{b\tan^3(c+dx)}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{\sqrt{b\tan^3(c+dx)}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.64

$$\int\sqrt{b\tan^3(c+dx)}\,dx = \frac{\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx\right.}{2\sqrt{2}}\right.}{d\tan^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[Sqrt[b*Tan[c + d*x]^3],x]
```

```
[Out] ((ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[
Tan[c + d*x]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]
/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2
]) + 2*Sqrt[Tan[c + d*x]])*Sqrt[b*Tan[c + d*x]^3])/(d*Tan[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{\sqrt{b(\tan^3(dx+c))}\left((b^2)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2(b^2)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}+(b}{(b^2)^{\frac{1}{4}}}\right.\right.}{4d\tan(dx+c)\sqrt{b\tan(dx+c)}}$
default	$-\frac{\sqrt{b(\tan^3(dx+c))}\left((b^2)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2(b^2)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}+(b}{(b^2)^{\frac{1}{4}}}\right.\right.}{4d\tan(dx+c)\sqrt{b\tan(dx+c)}}$

```
[In] int((b*tan(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(b*tan(d*x+c)^3)^(1/2)*((b^2)^(1/4)*2^(1/2)*ln((b*tan(d*x+c)+(b^2)^(
1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b
*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*(b^2)^(1/4)*2^(1/2)*arctan((2^(1
/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+2*(b^2)^(1/4)*2^(1/2)*ar
ctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))-8*(b*tan(d*x+c
))^(1/2))/tan(d*x+c)/(b*tan(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.07

$$\int\sqrt{b\tan^3(c+dx)}\,dx = -\frac{d\left(-\frac{b^2}{d^4}\right)^{\frac{1}{4}}\log\left(\frac{d\left(-\frac{b^2}{d^4}\right)^{\frac{1}{4}}\tan(dx+c)+\sqrt{b\tan(dx+c)^3}}{\tan(dx+c)}\right)\tan\left(dx+c\right)-d\left(-\frac{b^2}{d^4}\right)^{\frac{1}{4}}\log\left(-\frac{d\left(-\frac{b^2}{d^4}\right)^{\frac{1}{4}}\tan(dx+c)-\sqrt{b\tan(d}}{\tan(dx+c)}\right.}{}$$

```
[In] integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(d*(-b^2/d^4)^(1/4)*log((d*(-b^2/d^4)^(1/4)*tan(d*x + c) + sqrt(b*tan(
d*x + c)^3))/tan(d*x + c))*tan(d*x + c) - d*(-b^2/d^4)^(1/4)*log(-(d*(-b^2/
d^4)^(1/4)*tan(d*x + c) - sqrt(b*tan(d*x + c)^3))/tan(d*x + c))*tan(d*x + c
) + I*d*(-b^2/d^4)^(1/4)*log((I*d*(-b^2/d^4)^(1/4)*tan(d*x + c) + sqrt(b*ta
n(d*x + c)^3))/tan(d*x + c))*tan(d*x + c) - I*d*(-b^2/d^4)^(1/4)*log((-I*d*
(-b^2/d^4)^(1/4)*tan(d*x + c) + sqrt(b*tan(d*x + c)^3))/tan(d*x + c))*tan(d
*x + c) - 4*sqrt(b*tan(d*x + c)^3))/(d*tan(d*x + c))
```

Sympy [F]

$$\int \sqrt{b\tan^3(c+dx)}\,dx = \int \sqrt{b\tan^3{(c+dx)}}\,dx$$

```
[In] integrate((b*tan(d*x+c)**3)**(1/2),x)
[Out] Integral(sqrt(b*tan(c + d*x)**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.52

$$\int \sqrt{b\tan^3(c+dx)}\,dx = -\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan{(dx+c)}}\right)\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan{(dx+c)}}\right)\right)+$$

```
[In] integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="maxima")
[Out] -1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))
) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))))
 + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqr
t(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 8*sqrt(b
)*sqrt(tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.76

$$\int \sqrt{b\tan^3(c+dx)}\,dx = -\frac{1}{4}\left(\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d}+\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d}+\frac{\sqrt{2}}{}\right.$$

$$+c))$$

```
[In] integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*s
qrt(b*tan(d*x + c)))/sqrt(abs(b)))/d + 2*sqrt(2)*sqrt(abs(b))*arctan(-1/2*s
qrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d + sq
rt(2)*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(a
bs(b)) + abs(b))/d - sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) - sqrt(2)*sqrt
(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/d - 8*sqrt(b*tan(d*x + c))/d)*sgn(t
an(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\tan^3(c+dx)}\,dx = \int \sqrt{b\tan{(c+d\,x)}^3}\,dx$$

```
[In] int((b*tan(c + d*x)^3)^(1/2),x)
[Out] int((b*tan(c + d*x)^3)^(1/2), x)
```

3.33 $\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 255

$$\begin{aligned}\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = &-\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}}+\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}}\\&-\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}}\\&-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d\sqrt{b\tan^3(c+dx)}}\\&+\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d\sqrt{b\tan^3(c+dx)}}\end{aligned}$$

```
[Out] -2*tan(d*x+c)/d/(b*tan(d*x+c)^3)^(1/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/
2))*tan(d*x+c)^(3/2)/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)-1/2*arctan(1+2^(1/2)*
tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)-1/4*ln(
1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/d*2^(1/2)/(b*tan(d*
x+c)^3)^(1/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2
)/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3739, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = \frac{\tan^{\frac{3}{2}}(c+dx)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}} - \frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\sqrt{b\tan^3(c+dx)}}$$

```
[In] Int[1/Sqrt[b*Tan[c + d*x]^3],x]
```

```
[Out] (-2*Tan[c + d*x])/(d*Sqrt[b*Tan[c + d*x]^3]) + (ArcTan[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sqrt[2]*d*Sqrt[b*Tan[c + d*x]^3]) - (ArcTa
n[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sqrt[2]*d*Sqrt[b*Tan
[c + d*x]^3]) - (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c +
 d*x]^(3/2))/(2*Sqrt[2]*d*Sqrt[b*Tan[c + d*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d*Sqrt[b*Tan[c
+ d*x]^3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\int \sqrt{\tan(c+dx)}\,dx}{\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\left(2\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d\sqrt{b\tan^3(c+dx)}}$$
$$+ \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d\sqrt{b\tan^3(c+dx)}}$$
$$- \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}}$$
$$+ \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} + \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}}$$
$$- \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}}$$
$$- \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d\sqrt{b\tan^3(c+dx)}}$$
$$+ \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d\sqrt{b\tan^3(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx$$
$$= \frac{\tan(c+dx)\left(-2-\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan^2(c+dx)}+\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan^2(c+dx)}\right)}{d\sqrt{b\tan^3(c+dx)}}$$

```
[In] Integrate[1/Sqrt[b*Tan[c + d*x]^3],x]
```

```
[Out] (Tan[c + d*x]*(-2 - ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4)
 + ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4)))/(d*Sqrt[b*Tan
[c + d*x]^3])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{\tan(dx+c)\left(\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{b}\cdots}{}\right.}{4d\sqrt{b(\tan^3(dx+c))}\,(b^2)^{\frac{1}{4}}}$
default	$-\frac{\tan(dx+c)\left(\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{b}\cdots}{}\right.}{4d\sqrt{b(\tan^3(dx+c))}\,(b^2)^{\frac{1}{4}}}$

```
[In] int(1/(b*tan(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*tan(d*x+c)*(2^(1/2)*(b*tan(d*x+c))^(1/2)*ln(-((b^2)^(1/4)*(b*tan(d*x
+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*t
an(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*2^(1/2)*(b*tan(d*x+c))^(1/2)*arcta
n((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+2*2^(1/2)*(b*tan(
d*x+c))^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4)
)+8*(b^2)^(1/4))/(b*tan(d*x+c)^3)^(1/2)/(b^2)^(1/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx =$$

$$-\frac{bd\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2d^3\left(-\frac{1}{b^2d^4}\right)^{\frac{3}{4}}\tan(dx+c)+\sqrt{b\tan(dx+c)^3}}{\tan(dx+c)}\right)\tan(dx+c)^2-bd\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\log\left(-\frac{b^2d^3\left(-\frac{1}{b^2d^4}\right)^{\frac{3}{4}}\tan(\cdots}{\tan(\cdots}\right.}{}$$

```
[In] integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*d*(-1/(b^2*d^4))^(1/4)*log((b^2*d^3*(-1/(b^2*d^4))^(3/4)*tan(d*x +
c) + sqrt(b*tan(d*x + c)^3))/tan(d*x + c))*tan(d*x + c)^2 - b*d*(-1/(b^2*d^
4))^(1/4)*log(-(b^2*d^3*(-1/(b^2*d^4))^(3/4)*tan(d*x + c) - sqrt(b*tan(d*x
+ c)^3))/tan(d*x + c))*tan(d*x + c)^2 - I*b*d*(-1/(b^2*d^4))^(1/4)*log((I*b
^2*d^3*(-1/(b^2*d^4))^(3/4)*tan(d*x + c) + sqrt(b*tan(d*x + c)^3))/tan(d*x
+ c))*tan(d*x + c)^2 + I*b*d*(-1/(b^2*d^4))^(1/4)*log((-I*b^2*d^3*(-1/(b^2*
d^4))^(3/4)*tan(d*x + c) + sqrt(b*tan(d*x + c)^3))/tan(d*x + c))*tan(d*x +
c)^2 + 4*sqrt(b*tan(d*x + c)^3))/(b*d*tan(d*x + c)^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan^3{(c+dx)}}}\,dx$$

[In] integrate(1/(b*tan(d*x+c)**3)**(1/2),x)

[Out] Integral(1/sqrt(b*tan(c + d*x)**3), x)

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}\log\left(-\sqrt{\cdots}\right.}{\sqrt{b}}}{4d}$$

[In] integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="maxima")

[Out] -1/4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/sqrt(b) + 8/(sqrt(b)*sqrt(tan(d*x + c))))/d

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = -\frac{1}{4}b^2\left(\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^4d\,\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}+\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^4d\,\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}-\frac{\sqrt{2}|\cdots}{\cdots}\right.$$

[In] integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="giac")

```
[Out] -1/4*b^2*(2*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) +
 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^4*d*sgn(tan(d*x + c))) + 2*sqrt(2
)*abs(b)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x
 + c)))/sqrt(abs(b)))/(b^4*d*sgn(tan(d*x + c))) - sqrt(2)*abs(b)^(3/2)*log(
b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^4*d
*sgn(tan(d*x + c))) + sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) - sqrt(2)*sqr
t(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^4*d*sgn(tan(d*x + c))) + 8/(sqr
t(b*tan(d*x + c))*b^2*d*sgn(tan(d*x + c))))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\tan\left(c+d\,x\right)^3}}\,dx$$

```
[In] int(1/(b*tan(c + d*x)^3)^(1/2),x)
```

```
[Out] int(1/(b*tan(c + d*x)^3)^(1/2), x)
```

3.34 $\int \frac{1}{(b\tan^3(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 298

$$\begin{aligned}\int \frac{1}{(b\tan^3(c+dx))^{3/2}}\,dx &= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}}\\ &\quad - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}\\ &\quad + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}\\ &\quad - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}\\ &\quad + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}\end{aligned}$$

```
[Out] 2/3/b/d/(b*tan(d*x+c)^3)^(1/2)-2/7*cot(d*x+c)^2/b/d/(b*tan(d*x+c)^3)^(1/2)+
1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)/b/d*2^(1/2)/(b*tan
(d*x+c)^3)^(1/2)+1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)/b/
d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+
c))*tan(d*x+c)^(3/2)/b/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)+1/4*ln(1+2^(1/2)*ta
n(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/b/d*2^(1/2)/(b*tan(d*x+c)^3)^(1
/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int \frac{1}{(b\tan^3(c+dx))^{3/2}}\,dx = &-\frac{\tan^{\frac{3}{2}}(c+dx)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}} \\ &+\frac{\tan^{\frac{3}{2}}(c+dx)\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}+\frac{2}{3bd\sqrt{b\tan^3(c+dx)}} \\ &-\frac{\tan^{\frac{3}{2}}(c+dx)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}bd\sqrt{b\tan^3(c+dx)}} \\ &+\frac{\tan^{\frac{3}{2}}(c+dx)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}-\frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}}\end{aligned}$$

```
[In] Int[(b*Tan[c + d*x]^3)^(-3/2),x]
```

```
[Out] 2/(3*b*d*Sqrt[b*Tan[c + d*x]^3]) - (2*Cot[c + d*x]^2)/(7*b*d*Sqrt[b*Tan[c +
 d*x]^3]) - (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sq
rt[2]*b*d*Sqrt[b*Tan[c + d*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
*Tan[c + d*x]^(3/2))/(Sqrt[2]*b*d*Sqrt[b*Tan[c + d*x]^3]) - (Log[1 - Sqrt[2
]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*b*d*Sqr
t[b*Tan[c + d*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*
Tan[c + d*x]^(3/2))/(2*Sqrt[2]*b*d*Sqrt[b*Tan[c + d*x]^3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
```

```
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\tan^{\frac{3}{2}}(c+dx)\int\frac{1}{\tan^{\frac{9}{2}}(c+dx)}\,dx}{b\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\int\frac{1}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{b\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\int\frac{1}{\sqrt{\tan(c+dx)}}\,dx}{b\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int\frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(c+dx)\right)}{bd\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} + \frac{\left(2\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{1}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{bd\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{bd\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{bd\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2bd\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2bd\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}bd\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}bd\sqrt{b\tan^3(c+dx)}} + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}bd\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}bd\sqrt{b\tan^3(c+dx)}} + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.33

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx = \frac{14-6\cot^2(c+dx)-21\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\left(-\tan^2(c+dx)\right)^{3/4}-21\mathrm{ar}}{21bd\sqrt{b\tan^3(c+dx)}}$$

[In] Integrate[(b*Tan[c + d*x]^3)^(-3/2),x]

[Out] (14 - 6*Cot[c + d*x]^2 - 21*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(3/4) - 21*ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(3/4))/(21*b*d*Sqrt[b*Tan[c + d*x]^3])

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\tan(dx+c)\left(21\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\left(b\tan(dx+c)\right)^{\frac{7}{2}}\ln\left(\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+42\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\left(b\tan(dx+c)\right)^{\frac{7}{2}}\mathrm{ar}}{84d\,b^4(b(}$
default	$\frac{\tan(dx+c)\left(21\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\left(b\tan(dx+c)\right)^{\frac{7}{2}}\ln\left(\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+42\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\left(b\tan(dx+c)\right)^{\frac{7}{2}}\mathrm{ar}}{84d\,b^4(b(}$

[In] int(1/(b*tan(d*x+c)^3)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/84/d*tan(d*x+c)/b^4*(21*(b^2)^(1/4)*2^(1/2)*(b*tan(d*x+c))^(7/2)*ln((b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+42*(b^2)^(1/4)*2^(1/2)*(b*tan(d*x+c))^(7/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+42*(b^2)^(1/4)*2^(1/2)*(b*tan(d*x+c))^(7/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))+56*b^4*tan(d*x+c)^2-24*b^4)/(b*tan(d*x+c)^3)^(3/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx = \frac{21\,b^2 d\left(-\frac{1}{b^6 d^4}\right)^{\frac{1}{4}} \log\left(\frac{b^2 d\left(-\frac{1}{b^6 d^4}\right)^{\frac{1}{4}} \tan(dx+c) + \sqrt{b\tan(dx+c)^3}}{\tan(dx+c)}\right) \tan\left(dx+c\right)^5 - 21\,b^2 d\left(-\frac{1}{b^6 \cdots}\right.}{\cdots}$$

```
[In] integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/42*(21*b^2*d*(-1/(b^6*d^4))^(1/4)*log((b^2*d*(-1/(b^6*d^4))^(1/4)*tan(d*x
 + c) + sqrt(b*tan(d*x + c)^3))/tan(d*x + c))*tan(d*x + c)^5 - 21*b^2*d*(-1
/(b^6*d^4))^(1/4)*log(-(b^2*d*(-1/(b^6*d^4))^(1/4)*tan(d*x + c) - sqrt(b*ta
n(d*x + c)^3))/tan(d*x + c))*tan(d*x + c)^5 + 21*I*b^2*d*(-1/(b^6*d^4))^(1/
4)*log((I*b^2*d*(-1/(b^6*d^4))^(1/4)*tan(d*x + c) + sqrt(b*tan(d*x + c)^3))
/tan(d*x + c))*tan(d*x + c)^5 - 21*I*b^2*d*(-1/(b^6*d^4))^(1/4)*log((-I*b^2
*d*(-1/(b^6*d^4))^(1/4)*tan(d*x + c) + sqrt(b*tan(d*x + c)^3))/tan(d*x + c)
)*tan(d*x + c)^5 + 4*sqrt(b*tan(d*x + c)^3)*(7*tan(d*x + c)^2 - 3))/(b^2*d*
tan(d*x + c)^5)
```

Sympy [F]

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan^3\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c)**3)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**3)**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.55

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx = \frac{\frac{21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(\cdots}\right.\right.}{b^{\frac{3}{2}}}\cdots}{\cdots}$$

```
[In] integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/84*(21*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*l
og(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)*sq
rt(tan(d*x + c)) + tan(d*x + c) + 1))/b^(3/2) + 8*(21*sqrt(tan(d*x + c)) +
7/tan(d*x + c)^(3/2) - 3/tan(d*x + c)^(7/2))/b^(3/2) - 168*sqrt(tan(d*x + c
))/b^(3/2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.94

$$\int \frac{1}{(b\tan^3(c+dx))^{3/2}}\,dx = \frac{1}{84}b^4\left(\frac{42\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^6 d\,\mathrm{sgn}\left(\tan\left(dx+c\right)\right)} + \frac{42\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}}{}\right.}{b^6 d\,\mathrm{sgn}\left(\tan\right.}\right.$$

```
[In] integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/84*b^4*(42*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b))
+ 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^6*d*sgn(tan(d*x + c))) + 42*sqrt
(2)*sqrt(abs(b))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d
*x + c)))/sqrt(abs(b)))/(b^6*d*sgn(tan(d*x + c))) + 21*sqrt(2)*sqrt(abs(b))
*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(
b^6*d*sgn(tan(d*x + c))) - 21*sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) - sqr
t(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^6*d*sgn(tan(d*x + c)))
+ 8*(7*b^2*tan(d*x + c)^2 - 3*b^2)/(sqrt(b*tan(d*x + c))*b^7*d*sgn(tan(d*x
+ c))*tan(d*x + c)^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan^3(c+dx))^{3/2}}\,dx = \int \frac{1}{{\left(b\,{\tan\left(c+d\,x\right)}^3\right)}^{3/2}}\,dx$$

```
[In] int(1/(b*tan(c + d*x)^3)^(3/2),x)
```

```
[Out] int(1/(b*tan(c + d*x)^3)^(3/2), x)
```

3.35 $\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 364

$$\begin{aligned}\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx &= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} \\ &+ \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\tan(c+dx)}{b^2d\sqrt{b\tan^3(c+dx)}} \\ &- \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}} \\ &+ \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}} \\ &+ \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}} \\ &- \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}\end{aligned}$$

```
[Out] -2/5*cot(d*x+c)/b^2/d/(b*tan(d*x+c)^3)^(1/2)+2/9*cot(d*x+c)^3/b^2/d/(b*tan(
d*x+c)^3)^(1/2)-2/13*cot(d*x+c)^5/b^2/d/(b*tan(d*x+c)^3)^(1/2)+2*tan(d*x+c)
/b^2/d/(b*tan(d*x+c)^3)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d
*x+c)^(3/2)/b^2/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)+1/2*arctan(1+2^(1/2)*tan(d
*x+c)^(1/2))*tan(d*x+c)^(3/2)/b^2/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)+1/4*ln(1
-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/b^2/d*2^(1/2)/(b*tan
(d*x+c)^3)^(1/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(
3/2)/b^2/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3739, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx = &-\frac{\tan^{\frac{3}{2}}(c+dx)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}\\&+\frac{\tan^{\frac{3}{2}}(c+dx)\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}+\frac{2\tan(c+dx)}{b^2d\sqrt{b\tan^3(c+dx)}}\\&+\frac{\tan^{\frac{3}{2}}(c+dx)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}\\&-\frac{\tan^{\frac{3}{2}}(c+dx)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}\\&-\frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}}+\frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}}-\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}}\end{aligned}$$

```
[In] Int[(b*Tan[c + d*x]^3)^(-5/2),x]
```

```
[Out] (-2*Cot[c + d*x])/(5*b^2*d*Sqrt[b*Tan[c + d*x]^3]) + (2*Cot[c + d*x]^3)/(9*
b^2*d*Sqrt[b*Tan[c + d*x]^3]) - (2*Cot[c + d*x]^5)/(13*b^2*d*Sqrt[b*Tan[c +
 d*x]^3]) + (2*Tan[c + d*x])/(b^2*d*Sqrt[b*Tan[c + d*x]^3]) - (ArcTan[1 - S
qrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sqrt[2]*b^2*d*Sqrt[b*Tan[c
+ d*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(S
qrt[2]*b^2*d*Sqrt[b*Tan[c + d*x]^3]) + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*b^2*d*Sqrt[b*Tan[c + d*x]^3]
) - (Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))
/(2*Sqrt[2]*b^2*d*Sqrt[b*Tan[c + d*x]^3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{15}{2}}(c+dx)}\,dx}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{11}{2}}(c+dx)}\,dx}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\tan(c+dx)}{b^2d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\int \sqrt{\tan(c+dx)}\,dx}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\tan(c+dx)}{b^2d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{b^2d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\tan(c+dx)}{b^2d\sqrt{b\tan^3(c+dx)}} + \frac{\left(2\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{b^2d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\tan(c+dx)}{b^2d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{b^2d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{b^2d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\tan(c+dx)}{b^2d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2b^2d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2b^2d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\tan(c+dx)}{b^2d\sqrt{b\tan^3(c+dx)}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}}$$
$$+ \frac{2\tan(c+dx)}{b^2d\sqrt{b\tan^3(c+dx)}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}$$
$$+ \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}$$
$$+ \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}$$
$$- \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.38

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx = \frac{-234\cot(c+dx)+130\cot^3(c+dx)-90\cot^5(c+dx)+585\text{arctanh}\left(\sqrt[4]{-\tan^2\right.}}{}$$

```
[In] Integrate[(b*Tan[c + d*x]^3)^(-5/2),x]
```

```
[Out] (-234*Cot[c + d*x] + 130*Cot[c + d*x]^3 - 90*Cot[c + d*x]^5 + 585*ArcTanh[(
-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x])^(5/4)*Tan[c + d*x]^(1/4) + 1170*Tan
[c + d*x] + 585*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x])^(1/4)*Tan[c
 + d*x]^(5/4))/(585*b^2*d*Sqrt[b*Tan[c + d*x]^3])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\tan(dx+c)\left(585\sqrt{2}\,(b\tan(dx+c))^{\frac{13}{2}}\ln\left(-\frac{\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+1170\sqrt{2}\,(b\tan(dx+c))^{\frac{13}{2}}\arctan\right.}{}$
default	$\frac{\tan(dx+c)\left(585\sqrt{2}\,(b\tan(dx+c))^{\frac{13}{2}}\ln\left(-\frac{\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+1170\sqrt{2}\,(b\tan(dx+c))^{\frac{13}{2}}\arctan\right.}{}$

```
[In] int(1/(b*tan(d*x+c)^3)^(5/2),x,method=_RETURNVERBOSE)
```

[Out] 1/2340/d*tan(d*x+c)/b^6*(585*2^(1/2)*(b*tan(d*x+c))^(13/2)*ln(-((b^2)^(1/4)
*(b*tan(d*x+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)
^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+1170*2^(1/2)*(b*tan(d*x+c
))^(13/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+11
70*2^(1/2)*(b*tan(d*x+c))^(13/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)
^(1/4))/(b^2)^(1/4))+4680*(b^2)^(1/4)*b^6*tan(d*x+c)^6-936*b^6*(b^2)^(1/4)*
tan(d*x+c)^4+520*b^6*(b^2)^(1/4)*tan(d*x+c)^2-360*b^6*(b^2)^(1/4))/(b*tan(d
*x+c)^3)^(5/2)/(b^2)^(1/4)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.96

$$\int \frac{1}{(b\tan^3(c+dx))^{5/2}}\,dx = \frac{585\,b^3d\left(-\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}\log\left(\frac{b^8d^3\left(-\frac{1}{b^{10}d^4}\right)^{\frac{3}{4}}\tan(dx+c)+\sqrt{b\tan(dx+c)^3}}{\tan(dx+c)}\right)\tan(dx+c)^8 - 585\,b^3c$$

[In] integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="fricas")

[Out] 1/1170*(585*b^3*d*(-1/(b^10*d^4))^(1/4)*log((b^8*d^3*(-1/(b^10*d^4))^(3/4)*
tan(d*x + c) + sqrt(b*tan(d*x + c)^3))/tan(d*x + c))*tan(d*x + c)^8 - 585*b
^3*d*(-1/(b^10*d^4))^(1/4)*log(-(b^8*d^3*(-1/(b^10*d^4))^(3/4)*tan(d*x + c)
 - sqrt(b*tan(d*x + c)^3))/tan(d*x + c))*tan(d*x + c)^8 - 585*I*b^3*d*(-1/(
b^10*d^4))^(1/4)*log((I*b^8*d^3*(-1/(b^10*d^4))^(3/4)*tan(d*x + c) + sqrt(b
*tan(d*x + c)^3))/tan(d*x + c))*tan(d*x + c)^8 + 585*I*b^3*d*(-1/(b^10*d^4)
)^(1/4)*log((-I*b^8*d^3*(-1/(b^10*d^4))^(3/4)*tan(d*x + c) + sqrt(b*tan(d*x
 + c)^3))/tan(d*x + c))*tan(d*x + c)^8 + 4*(585*tan(d*x + c)^6 - 117*tan(d*
x + c)^4 + 65*tan(d*x + c)^2 - 45)*sqrt(b*tan(d*x + c)^3))/(b^3*d*tan(d*x +
 c)^8)

Sympy [F]

$$\int \frac{1}{(b\tan^3(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\tan^3\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(1/(b*tan(d*x+c)**3)**(5/2),x)

[Out] Integral((b*tan(c + d*x)**3)**(-5/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.47

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx = \frac{585\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\mathrm{ta}\right.\right.}{b^{\frac{5}{2}}}$$

```
[In] integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2340*(585*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2
)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)
*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/b^(5/2) + 8*(585*sqrt(b)/sqrt(tan(
d*x + c)) - 117*sqrt(b)/tan(d*x + c)^(5/2) + 65*sqrt(b)/tan(d*x + c)^(9/2)
- 45*sqrt(b)/tan(d*x + c)^(13/2))/b^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.86 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx = \frac{1}{2340}\,b^6\left(\frac{1170\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^{10}d\,\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}+\frac{1170\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\right.}{b^{10}d\,\mathrm{sgn}}\right.$$

```
[In] integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2340*b^6*(1170*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(
b)) + 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^10*d*sgn(tan(d*x + c))) + 11
70*sqrt(2)*abs(b)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(
b*tan(d*x + c)))/sqrt(abs(b)))/(b^10*d*sgn(tan(d*x + c))) - 585*sqrt(2)*abs
(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) +
abs(b))/(b^10*d*sgn(tan(d*x + c))) + 585*sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x
 + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^10*d*sgn(tan
(d*x + c))) + 8*(585*b^6*tan(d*x + c)^6 - 117*b^6*tan(d*x + c)^4 + 65*b^6*t
an(d*x + c)^2 - 45*b^6)/(sqrt(b*tan(d*x + c))*b^14*d*sgn(tan(d*x + c))*tan(
d*x + c)^6))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+d\,x\right)^3\right)^{5/2}}\,dx$$

```
[In] int(1/(b*tan(c + d*x)^3)^(5/2),x)
[Out] int(1/(b*tan(c + d*x)^3)^(5/2), x)
```

3.36 $\int \left(b \tan^4(c+dx)\right)^{5/2} dx$

Optimal result

Integrand size = 14, antiderivative size = 182

$$\int \left(b \tan^4(c+dx)\right)^{5/2} dx = \frac{b^2 \cot(c+dx)\sqrt{b \tan^4(c+dx)}}{d} - b^2 x \cot^2(c+dx)\sqrt{b \tan^4(c+dx)} - \frac{b^2 \tan(c+dx)\sqrt{b \tan^4(c+dx)}}{3d} + \frac{b^2 \tan^3(c+dx)\sqrt{b \tan^4(c+dx)}}{5d} - \frac{b^2 \tan^5(c+dx)\sqrt{b \tan^4(c+dx)}}{7d} + \frac{b^2 \tan^7(c+dx)\sqrt{b \tan^4(c+dx)}}{9d}$$

```
[Out] b^2*cot(d*x+c)*(tan(d*x+c)^4*b)^(1/2)/d-b^2*x*cot(d*x+c)^2*(tan(d*x+c)^4*b)
^(1/2)-1/3*b^2*(tan(d*x+c)^4*b)^(1/2)*tan(d*x+c)/d+1/5*b^2*(tan(d*x+c)^4*b)
^(1/2)*tan(d*x+c)^3/d-1/7*b^2*(tan(d*x+c)^4*b)^(1/2)*tan(d*x+c)^5/d+1/9*b^2
*(tan(d*x+c)^4*b)^(1/2)*tan(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 8}

$$\int \left(b \tan^4(c+dx)\right)^{5/2} dx = -\frac{b^2 \tan(c+dx)\sqrt{b \tan^4(c+dx)}}{3d} + \frac{b^2 \tan^7(c+dx)\sqrt{b \tan^4(c+dx)}}{9d} - \frac{b^2 \tan^5(c+dx)\sqrt{b \tan^4(c+dx)}}{7d} + \frac{b^2 \tan^3(c+dx)\sqrt{b \tan^4(c+dx)}}{5d} - b^2 x \cot^2(c+dx)\sqrt{b \tan^4(c+dx)} + \frac{b^2 \cot(c+dx)\sqrt{b \tan^4(c+dx)}}{d}$$

```
[In] Int[(b*Tan[c + d*x]^4)^(5/2),x]
```

```
[Out] (b^2*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/d - b^2*x*Cot[c + d*x]^2*Sqrt[b*T
an[c + d*x]^4] - (b^2*Tan[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/(3*d) + (b^2*Tan
[c + d*x]^3*Sqrt[b*Tan[c + d*x]^4])/(5*d) - (b^2*Tan[c + d*x]^5*Sqrt[b*Tan[
c + d*x]^4])/(7*d) + (b^2*Tan[c + d*x]^7*Sqrt[b*Tan[c + d*x]^4])/(9*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int\tan^{10}(c+dx)\,dx \\
&= \frac{b^2\tan^7(c+dx)\sqrt{b\tan^4(c+dx)}}{9d}-\left(b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int\tan^8(c+dx)\,dx \\
&= -\frac{b^2\tan^5(c+dx)\sqrt{b\tan^4(c+dx)}}{7d}+\frac{b^2\tan^7(c+dx)\sqrt{b\tan^4(c+dx)}}{9d} \\
&\quad+\left(b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int\tan^6(c+dx)\,dx \\
&= \frac{b^2\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d}-\frac{b^2\tan^5(c+dx)\sqrt{b\tan^4(c+dx)}}{7d} \\
&\quad+\frac{b^2\tan^7(c+dx)\sqrt{b\tan^4(c+dx)}}{9d} \\
&\quad-\left(b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int\tan^4(c+dx)\,dx
\end{aligned}
$$

$$= -\frac{b^2\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} - \frac{b^2\tan^5(c+dx)\sqrt{b\tan^4(c+dx)}}{7d} + \frac{b^2\tan^7(c+dx)\sqrt{b\tan^4(c+dx)}}{9d} + \left(b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int\tan^2(c+dx)\,dx$$

$$= \frac{b^2\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - \frac{b^2\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} - \frac{b^2\tan^5(c+dx)\sqrt{b\tan^4(c+dx)}}{7d} + \frac{b^2\tan^7(c+dx)\sqrt{b\tan^4(c+dx)}}{9d} - \left(b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int 1\,dx$$

$$= \frac{b^2\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - b^2x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)} - \frac{b^2\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} - \frac{b^2\tan^5(c+dx)\sqrt{b\tan^4(c+dx)}}{7d} + \frac{b^2\tan^7(c+dx)\sqrt{b\tan^4(c+dx)}}{9d}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.47

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx = \frac{\cot(c+dx)\left(35-45\cot^2(c+dx)+63\cot^4(c+dx)-105\cot^6(c+dx)+315\cot^8(c+dx)-\right.}{315d}$$

```
[In] Integrate[(b*Tan[c + d*x]^4)^(5/2),x]
```

```
[Out] (Cot[c + d*x]*(35 - 45*Cot[c + d*x]^2 + 63*Cot[c + d*x]^4 - 105*Cot[c + d*x
]^6 + 315*Cot[c + d*x]^8 - 315*ArcTan[Tan[c + d*x]]*Cot[c + d*x]^9)*(b*Tan[
c + d*x]^4)^(5/2))/(315*d)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.46

method	result
derivativedivides	$-\frac{\left(\left(\tan^{4}(dx+c)\right)b\right)^{\frac{5}{2}}\left(-35\left(\tan^{9}(dx+c)\right)+45\left(\tan^{7}(dx+c)\right)-63\left(\tan^{5}(dx+c)\right)+105\left(\tan^{3}(dx+c)\right)+315\arctan(\tan(dx+c))\right.}{315d\tan(dx+c)^{10}}$
default	$-\frac{\left(\left(\tan^{4}(dx+c)\right)b\right)^{\frac{5}{2}}\left(-35\left(\tan^{9}(dx+c)\right)+45\left(\tan^{7}(dx+c)\right)-63\left(\tan^{5}(dx+c)\right)+105\left(\tan^{3}(dx+c)\right)+315\arctan(\tan(dx+c))\right.}{315d\tan(dx+c)^{10}}$
risch	$\frac{b^{2}\left(e^{2i(dx+c)}+1\right)^{2}\sqrt{\frac{\left(e^{2i(dx+c)}-1\right)^{4}b}{\left(e^{2i(dx+c)}+1\right)^{4}}}\,x}{\left(e^{2i(dx+c)}-1\right)^{2}}-\frac{2ib^{2}\sqrt{\frac{\left(e^{2i(dx+c)}-1\right)^{4}b}{\left(e^{2i(dx+c)}+1\right)^{4}}}\left(1575\,e^{16i(dx+c)}+6300\,e^{14i(dx+c)}+21000\,e^{12i(dx+c)}+\right.}{315\left(e^{2i(dx-\right.}}$

```
[In] int((tan(d*x+c)^4*b)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/315/d*(tan(d*x+c)^4*b)^(5/2)*(-35*tan(d*x+c)^9+45*tan(d*x+c)^7-63*tan(d*
x+c)^5+105*tan(d*x+c)^3+315*arctan(tan(d*x+c))-315*tan(d*x+c))/tan(d*x+c)^1
0
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.53

$$\int \left(b\tan^{4}(c+dx)\right)^{5/2}\,dx=\frac{\left(35\,b^{2}\tan\left(dx+c\right)^{9}-45\,b^{2}\tan\left(dx+c\right)^{7}+63\,b^{2}\tan\left(dx+c\right)^{5}-105\,b^{2}\tan\left(dx+c\right)^{3}-315\,b^{2}d\right.}{315\,d\tan\left(dx+c\right)^{2}}$$

```
[In] integrate((tan(d*x+c)^4*b)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*tan(d*x + c)^9 - 45*b^2*tan(d*x + c)^7 + 63*b^2*tan(d*x + c)^
5 - 105*b^2*tan(d*x + c)^3 - 315*b^2*d*x + 315*b^2*tan(d*x + c))*sqrt(b*tan
(d*x + c)^4)/(d*tan(d*x + c)^2)
```

Sympy [F]

$$\int \left(b\tan^{4}(c+dx)\right)^{5/2}\,dx=\int \left(b\tan^{4}\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((tan(d*x+c)**4*b)**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**4)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.43

$$\int (b\tan^4(c+dx))^{5/2}\,dx = \frac{35\,b^{\frac{5}{2}}\tan(dx+c)^9 - 45\,b^{\frac{5}{2}}\tan(dx+c)^7 + 63\,b^{\frac{5}{2}}\tan(dx+c)^5 - 105\,b^{\frac{5}{2}}\tan(dx+c)^3 - 315\,(}{315\,d}$$

```
[In] integrate((tan(d*x+c)^4*b)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^(5/2)*tan(d*x + c)^9 - 45*b^(5/2)*tan(d*x + c)^7 + 63*b^(5/2)*t
an(d*x + c)^5 - 105*b^(5/2)*tan(d*x + c)^3 - 315*(d*x + c)*b^(5/2) + 315*b^
(5/2)*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 960 vs. $2(162) = 324$.

Time = 6.98 (sec) , antiderivative size = 960, normalized size of antiderivative = 5.27

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((tan(d*x+c)^4*b)^(5/2),x, algorithm="giac")
```

```
[Out] -1/315*(315*b^2*d*x*tan(d*x)^9*tan(c)^9 - 2835*b^2*d*x*tan(d*x)^8*tan(c)^8
+ 315*b^2*tan(d*x)^9*tan(c)^8 + 315*b^2*tan(d*x)^8*tan(c)^9 + 11340*b^2*d*x
*tan(d*x)^7*tan(c)^7 - 105*b^2*tan(d*x)^9*tan(c)^6 - 2835*b^2*tan(d*x)^8*ta
n(c)^7 - 2835*b^2*tan(d*x)^7*tan(c)^8 - 105*b^2*tan(d*x)^6*tan(c)^9 - 26460
*b^2*d*x*tan(d*x)^6*tan(c)^6 + 63*b^2*tan(d*x)^9*tan(c)^4 + 945*b^2*tan(d*x
)^8*tan(c)^5 + 11340*b^2*tan(d*x)^7*tan(c)^6 + 11340*b^2*tan(d*x)^6*tan(c)^
7 + 945*b^2*tan(d*x)^5*tan(c)^8 + 63*b^2*tan(d*x)^4*tan(c)^9 + 39690*b^2*d*
x*tan(d*x)^5*tan(c)^5 - 45*b^2*tan(d*x)^9*tan(c)^2 - 567*b^2*tan(d*x)^8*tan
(c)^3 - 3780*b^2*tan(d*x)^7*tan(c)^4 - 26460*b^2*tan(d*x)^6*tan(c)^5 - 2646
0*b^2*tan(d*x)^5*tan(c)^6 - 3780*b^2*tan(d*x)^4*tan(c)^7 - 567*b^2*tan(d*x)
^3*tan(c)^8 - 45*b^2*tan(d*x)^2*tan(c)^9 - 39690*b^2*d*x*tan(d*x)^4*tan(c)^
4 + 35*b^2*tan(d*x)^9 + 405*b^2*tan(d*x)^8*tan(c) + 2268*b^2*tan(d*x)^7*tan
(c)^2 + 8820*b^2*tan(d*x)^6*tan(c)^3 + 39690*b^2*tan(d*x)^5*tan(c)^4 + 3969
0*b^2*tan(d*x)^4*tan(c)^5 + 8820*b^2*tan(d*x)^3*tan(c)^6 + 2268*b^2*tan(d*x
)^2*tan(c)^7 + 405*b^2*tan(d*x)*tan(c)^8 + 35*b^2*tan(c)^9 + 26460*b^2*d*x*
tan(d*x)^3*tan(c)^3 - 45*b^2*tan(d*x)^7 - 567*b^2*tan(d*x)^6*tan(c) - 3780*
b^2*tan(d*x)^5*tan(c)^2 - 26460*b^2*tan(d*x)^4*tan(c)^3 - 26460*b^2*tan(d*x
)^3*tan(c)^4 - 3780*b^2*tan(d*x)^2*tan(c)^5 - 567*b^2*tan(d*x)*tan(c)^6 - 4
```

```
5*b^2*tan(c)^7 - 11340*b^2*d*x*tan(d*x)^2*tan(c)^2 + 63*b^2*tan(d*x)^5 + 94
5*b^2*tan(d*x)^4*tan(c) + 11340*b^2*tan(d*x)^3*tan(c)^2 + 11340*b^2*tan(d*x
)^2*tan(c)^3 + 945*b^2*tan(d*x)*tan(c)^4 + 63*b^2*tan(c)^5 + 2835*b^2*d*x*t
an(d*x)*tan(c) - 105*b^2*tan(d*x)^3 - 2835*b^2*tan(d*x)^2*tan(c) - 2835*b^2
*tan(d*x)*tan(c)^2 - 105*b^2*tan(c)^3 - 315*b^2*d*x + 315*b^2*tan(d*x) + 31
5*b^2*tan(c))*sqrt(b)/(d*tan(d*x)^9*tan(c)^9 - 9*d*tan(d*x)^8*tan(c)^8 + 36
*d*tan(d*x)^7*tan(c)^7 - 84*d*tan(d*x)^6*tan(c)^6 + 126*d*tan(d*x)^5*tan(c)
^5 - 126*d*tan(d*x)^4*tan(c)^4 + 84*d*tan(d*x)^3*tan(c)^3 - 36*d*tan(d*x)^2
*tan(c)^2 + 9*d*tan(d*x)*tan(c) - d)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx=\int \left(b\tan(c+d\,x)^4\right)^{5/2}\,dx$$

```
[In] int((b*tan(c + d*x)^4)^(5/2),x)
[Out] int((b*tan(c + d*x)^4)^(5/2), x)
```

3.37 $\int (b\tan^4(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int (b\tan^4(c+dx))^{3/2}\,dx = \frac{b\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - bx\cot^2(c+dx)\sqrt{b\tan^4(c+dx)} - \frac{b\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d}$$

```
[Out] b*cot(d*x+c)*(tan(d*x+c)^4*b)^(1/2)/d-b*x*cot(d*x+c)^2*(tan(d*x+c)^4*b)^(1/
2)-1/3*b*(tan(d*x+c)^4*b)^(1/2)*tan(d*x+c)/d+1/5*b*(tan(d*x+c)^4*b)^(1/2)*t
an(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$\int (b\tan^4(c+dx))^{3/2}\,dx = -\frac{b\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} - bx\cot^2(c+dx)\sqrt{b\tan^4(c+dx)} + \frac{b\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d}$$

```
[In] Int[(b*Tan[c + d*x]^4)^(3/2),x]
```

```
[Out] (b*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/d - b*x*Cot[c + d*x]^2*Sqrt[b*Tan[c
 + d*x]^4] - (b*Tan[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/(3*d) + (b*Tan[c + d*x
]^3*Sqrt[b*Tan[c + d*x]^4])/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int\tan^6(c+dx)\,dx\\
&= \frac{b\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d}-\left(b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int\tan^4(c+dx)\,dx\\
&= -\frac{b\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d}+\frac{b\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d}\\
&\quad+\left(b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int\tan^2(c+dx)\,dx\\
&= \frac{b\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d}-\frac{b\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d}\\
&\quad+\frac{b\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d}-\left(b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int 1\,dx\\
&= \frac{b\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d}-bx\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\\
&\quad-\frac{b\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d}+\frac{b\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.60

$$\int (b\tan^4(c+dx))^{3/2}\,dx = \frac{\cot(c+dx)\left(3-5\cot^2(c+dx)+15\cot^4(c+dx)-15\arctan(\tan(c+dx))\cot^5(c+dx)\right)(b\tan^4(c+dx))^{3/2}}{15d}$$

```
[In] Integrate[(b*Tan[c + d*x]^4)^(3/2),x]
[Out] (Cot[c + d*x]*(3 - 5*Cot[c + d*x]^2 + 15*Cot[c + d*x]^4 - 15*ArcTan[Tan[c +
 d*x]]*Cot[c + d*x]^5)*(b*Tan[c + d*x]^4)^(3/2))/(15*d)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

method	result
derivativedivides	$-\frac{\left(\left(\tan^4(dx+c)\right)b\right)^{\frac{3}{2}}\left(-3\left(\tan^5(dx+c)\right)+5\left(\tan^3(dx+c)\right)+15\arctan(\tan(dx+c))-15\tan(dx+c)\right)}{15d\tan(dx+c)^6}$
default	$-\frac{\left(\left(\tan^4(dx+c)\right)b\right)^{\frac{3}{2}}\left(-3\left(\tan^5(dx+c)\right)+5\left(\tan^3(dx+c)\right)+15\arctan(\tan(dx+c))-15\tan(dx+c)\right)}{15d\tan(dx+c)^6}$
risch	$\frac{b\left(e^{2i(dx+c)}+1\right)^2\sqrt{\frac{\left(e^{2i(dx+c)}-1\right)^4 b}{\left(e^{2i(dx+c)}+1\right)^4}}\,x}{\left(e^{2i(dx+c)}-1\right)^2}-\frac{2ib\sqrt{\frac{\left(e^{2i(dx+c)}-1\right)^4 b}{\left(e^{2i(dx+c)}+1\right)^4}}\left(45\,e^{8i(dx+c)}+90\,e^{6i(dx+c)}+140\,e^{4i(dx+c)}+70\,e^{2i(dx+}\right.}{15\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)^3 d}$

```
[In] int((tan(d*x+c)^4*b)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/15/d*(tan(d*x+c)^4*b)^(3/2)*(-3*tan(d*x+c)^5+5*tan(d*x+c)^3+15*arctan(ta
n(d*x+c))-15*tan(d*x+c))/tan(d*x+c)^6
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.56

$$\int (b\tan^4(c+dx))^{3/2}\,dx = \frac{\left(3\,b\tan(dx+c)^5-5\,b\tan(dx+c)^3-15\,bdx+15\,b\tan(dx+c)\right)\sqrt{b\tan(dx+c)^4}}{15\,d\tan(dx+c)^2}$$

```
[In] integrate((tan(d*x+c)^4*b)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*tan(d*x + c)^5 - 5*b*tan(d*x + c)^3 - 15*b*d*x + 15*b*tan(d*x + c
))*sqrt(b*tan(d*x + c)^4)/(d*tan(d*x + c)^2)
```

Sympy [F]

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx = \int \left(b\tan^4\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((tan(d*x+c)**4*b)**(3/2),x)
[Out] Integral((b*tan(c + d*x)**4)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.48

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx = \frac{3\,b^{\frac{3}{2}}\tan\left(dx+c\right)^5 - 5\,b^{\frac{3}{2}}\tan\left(dx+c\right)^3 - 15\left(dx+c\right)b^{\frac{3}{2}} + 15\,b^{\frac{3}{2}}\tan\left(dx+c\right)}{15\,d}$$

```
[In] integrate((tan(d*x+c)^4*b)^(3/2),x, algorithm="maxima")
[Out] 1/15*(3*b^(3/2)*tan(d*x + c)^5 - 5*b^(3/2)*tan(d*x + c)^3 - 15*(d*x + c)*b^
(3/2) + 15*b^(3/2)*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(98) = 196.

Time = 2.94 (sec) , antiderivative size = 992, normalized size of antiderivative = 9.02

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate((tan(d*x+c)^4*b)^(3/2),x, algorithm="giac")
[Out] 1/60*(15*pi - 60*d*x*tan(d*x)^5*tan(c)^5 - 15*pi*sgn(2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^5 - 15*pi*ta
n(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*ta
n(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*ta
n(d*x)^5*tan(c)^5 + 300*d*x*tan(d*x)^4*tan(c)^4 + 75*pi*sgn(2*tan(d*x)^2*ta
n(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 7
```

```
5*pi*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan
(c)))*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c)
 - 1))*tan(d*x)^4*tan(c)^4 - 60*tan(d*x)^5*tan(c)^4 - 60*tan(d*x)^4*tan(c)^
5 - 600*d*x*tan(d*x)^3*tan(c)^3 - 150*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*
x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 + 20*tan(d*x)^5*ta
n(c)^2 - 150*pi*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x)*tan(c) - 1)/(tan
(d*x) + tan(c)))*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x) + tan(c))/(tan(
d*x)*tan(c) - 1))*tan(d*x)^3*tan(c)^3 + 300*tan(d*x)^4*tan(c)^3 + 300*tan(d
*x)^3*tan(c)^4 + 20*tan(d*x)^2*tan(c)^5 + 600*d*x*tan(d*x)^2*tan(c)^2 + 150
*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*
tan(d*x)^2*tan(c)^2 - 12*tan(d*x)^5 - 100*tan(d*x)^4*tan(c) + 150*pi*tan(d*
x)^2*tan(c)^2 - 300*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d
*x)^2*tan(c)^2 - 300*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(
d*x)^2*tan(c)^2 - 600*tan(d*x)^3*tan(c)^2 - 600*tan(d*x)^2*tan(c)^3 - 100*t
an(d*x)*tan(c)^4 - 12*tan(c)^5 - 300*d*x*tan(d*x)*tan(c) - 75*pi*sgn(2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c
) + 20*tan(d*x)^3 - 75*pi*tan(d*x)*tan(c) + 150*arctan((tan(d*x)*tan(c) - 1
)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 150*arctan((tan(d*x) + tan(c))/(ta
n(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 300*tan(d*x)^2*tan(c) + 300*tan(d*x)*
tan(c)^2 + 20*tan(c)^3 + 60*d*x + 15*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x
)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 30*arctan((tan(d*x)*tan(c) - 1)/(tan(
d*x) + tan(c))) - 30*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) - 60
*tan(d*x) - 60*tan(c))*b^(3/2)/(d*tan(d*x)^5*tan(c)^5 - 5*d*tan(d*x)^4*tan(
c)^4 + 10*d*tan(d*x)^3*tan(c)^3 - 10*d*tan(d*x)^2*tan(c)^2 + 5*d*tan(d*x)*t
an(c) - d)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx=\int {\left(b\,{\mathrm{tan}\left(c+d\,x\right)}^4\right)}^{3/2}\,dx$$

```
[In] int((b*tan(c + d*x)^4)^(3/2),x)
[Out] int((b*tan(c + d*x)^4)^(3/2), x)
```

3.38 $\int \sqrt{b\tan^4(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \sqrt{b\tan^4(c+dx)}\,dx = \frac{\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}$$

```
[Out] cot(d*x+c)*(tan(d*x+c)^4*b)^(1/2)/d-x*cot(d*x+c)^2*(tan(d*x+c)^4*b)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 8}

$$\int \sqrt{b\tan^4(c+dx)}\,dx = \frac{\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}$$

```
[In] Int[Sqrt[b*Tan[c + d*x]^4],x]
```

```
[Out] (Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/d - x*Cot[c + d*x]^2*Sqrt[b*Tan[c + d
*x]^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int \tan^2(c+dx)\,dx\\
&= \frac{\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - \left(\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int 1\,dx\\
&= \frac{\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \sqrt{b\tan^4(c+dx)}\,dx = -\frac{\cot(c+dx)(-1+\arctan(\tan(c+dx))\cot(c+dx))\sqrt{b\tan^4(c+dx)}}{d}$$

```
[In] Integrate[Sqrt[b*Tan[c + d*x]^4],x]
[Out] -((Cot[c + d*x]*(-1 + ArcTan[Tan[c + d*x]]*Cot[c + d*x])*Sqrt[b*Tan[c + d*x
]^4])/d)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{\sqrt{(\tan^4(dx+c))b}\,(-\tan(dx+c)+\arctan(\tan(dx+c)))}{d\tan(dx+c)^2}$	42
default	$-\frac{\sqrt{(\tan^4(dx+c))b}\,(-\tan(dx+c)+\arctan(\tan(dx+c)))}{d\tan(dx+c)^2}$	42
risch	$\frac{\sqrt{\frac{\left(e^{2i(dx+c)}-1\right)^4 b}{\left(e^{2i(dx+c)}+1\right)^4}}\,\left(e^{2i(dx+c)}+1\right)^2 x}{\left(e^{2i(dx+c)}-1\right)^2}-\frac{2i\sqrt{\frac{\left(e^{2i(dx+c)}-1\right)^4 b}{\left(e^{2i(dx+c)}+1\right)^4}}\,\left(e^{2i(dx+c)}+1\right)}{\left(e^{2i(dx+c)}-1\right)^2 d}$	120

```
[In] int((tan(d*x+c)^4*b)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/d*(tan(d*x+c)^4*b)^(1/2)*(-tan(d*x+c)+arctan(tan(d*x+c)))/tan(d*x+c)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int \sqrt{b\tan^4(c+dx)}\,dx = -\frac{\sqrt{b\tan{(dx+c)}^4}(dx-\tan{(dx+c)})}{d\tan{(dx+c)}^2}$$

```
[In] integrate((tan(d*x+c)^4*b)^(1/2),x, algorithm="fricas")
[Out] -sqrt(b*tan(d*x + c)^4)*(d*x - tan(d*x + c))/(d*tan(d*x + c)^2)
```

Sympy [F]

$$\int \sqrt{b\tan^4(c+dx)}\,dx = \int \sqrt{b\tan^4{(c+dx)}}\,dx$$

```
[In] integrate((tan(d*x+c)**4*b)**(1/2),x)
[Out] Integral(sqrt(b*tan(c + d*x)**4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int \sqrt{b\tan^4(c+dx)}\,dx = -\frac{(dx+c)\sqrt{b}-\sqrt{b}\tan(dx+c)}{d}$$

```
[In] integrate((tan(d*x+c)^4*b)^(1/2),x, algorithm="maxima")
[Out] -((d*x + c)*sqrt(b) - sqrt(b)*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(46) = 92$.

Time = 0.46 (sec) , antiderivative size = 229, normalized size of antiderivative = 4.58

$$\int \sqrt{b\tan^4(c+dx)}\,dx$$
$$= \frac{\left(\pi - 4\,dx\tan(dx)\tan(c) - \pi\mathrm{sgn}\left(2\tan(dx)^2\tan(c) + 2\tan(dx)\tan(c)^2 - 2\tan(dx) - 2\tan(c)\right)\tan(\right.}{}$$

```
[In] integrate((tan(d*x+c)^4*b)^(1/2),x, algorithm="giac")
[Out] 1/4*(pi - 4*d*x*tan(d*x)*tan(c) - pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*t
an(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - pi*tan(d*x)*tan(c) + 2*a
rctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 2*arctan
((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 4*d*x + pi*sg
n(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 2*ar
ctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 2*arctan((tan(d*x) + tan(
c))/(tan(d*x)*tan(c) - 1)) - 4*tan(d*x) - 4*tan(c))*sqrt(b)/(d*tan(d*x)*tan
(c) - d)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\tan^4(c+dx)}\,dx = \int \sqrt{b\,{\tan\left(c+d\,x\right)}^4}\,dx$$

```
[In] int((b*tan(c + d*x)^4)^(1/2),x)
[Out] int((b*tan(c + d*x)^4)^(1/2), x)
```

3.39 $\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\frac{\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}} - \frac{x\tan^2(c+dx)}{\sqrt{b\tan^4(c+dx)}}$$

```
[Out] -tan(d*x+c)/d/(tan(d*x+c)^4*b)^(1/2)-x*tan(d*x+c)^2/(tan(d*x+c)^4*b)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\frac{\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}} - \frac{x\tan^2(c+dx)}{\sqrt{b\tan^4(c+dx)}}$$

```
[In] Int[1/Sqrt[b*Tan[c + d*x]^4],x]
```

```
[Out] -(Tan[c + d*x]/(d*Sqrt[b*Tan[c + d*x]^4])) - (x*Tan[c + d*x]^2)/Sqrt[b*Tan[
c + d*x]^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\tan^2(c+dx)\int\cot^2(c+dx)\,dx}{\sqrt{b\tan^4(c+dx)}}\\ &= -\frac{\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}}-\frac{\tan^2(c+dx)\int 1\,dx}{\sqrt{b\tan^4(c+dx)}}\\ &= -\frac{\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}}-\frac{x\tan^2(c+dx)}{\sqrt{b\tan^4(c+dx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int\frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}}$$

```
[In] Integrate[1/Sqrt[b*Tan[c + d*x]^4],x]
[Out] -((Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(d*Sqrt[b
*Tan[c + d*x]^4]))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{\tan(dx+c)(\arctan(\tan(dx+c))\tan(dx+c)+1)}{d\sqrt{(\tan^4(dx+c))b}}$	40
default	$-\frac{\tan(dx+c)(\arctan(\tan(dx+c))\tan(dx+c)+1)}{d\sqrt{(\tan^4(dx+c))b}}$	40
risch	$\frac{\left(e^{2i(dx+c)}-1\right)^2x}{\sqrt{\frac{\left(e^{2i(dx+c)}-1\right)^4b}{\left(e^{2i(dx+c)}+1\right)^4}}\left(e^{2i(dx+c)}+1\right)^2}+\frac{2i\left(e^{2i(dx+c)}-1\right)}{\sqrt{\frac{\left(e^{2i(dx+c)}-1\right)^4b}{\left(e^{2i(dx+c)}+1\right)^4}}\left(e^{2i(dx+c)}+1\right)^2d}$	120

```
[In] int(1/(tan(d*x+c)^4*b)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/d*tan(d*x+c)*(arctan(tan(d*x+c))*tan(d*x+c)+1)/(tan(d*x+c)^4*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\frac{\sqrt{b\tan\left(dx+c\right)^4}(dx\tan\left(dx+c\right)+1)}{bd\tan\left(dx+c\right)^3}$$

```
[In] integrate(1/(tan(d*x+c)^4*b)^(1/2),x, algorithm="fricas")
[Out] -sqrt(b*tan(d*x + c)^4)*(d*x*tan(d*x + c) + 1)/(b*d*tan(d*x + c)^3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan^4\left(c+dx\right)}}\,dx$$

```
[In] integrate(1/(tan(d*x+c)**4*b)**(1/2),x)
[Out] Integral(1/sqrt(b*tan(c + d*x)**4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\frac{\frac{dx+c}{\sqrt{b}} + \frac{1}{\sqrt{b}\tan(dx+c)}}{d}$$

```
[In] integrate(1/(tan(d*x+c)^4*b)^(1/2),x, algorithm="maxima")
[Out] -((d*x + c)/sqrt(b) + 1/(sqrt(b)*tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\frac{\frac{2\,(dx+c)}{\sqrt{b}} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{b}} + \frac{1}{\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

```
[In] integrate(1/(tan(d*x+c)^4*b)^(1/2),x, algorithm="giac")
[Out] -1/2*(2*(d*x + c)/sqrt(b) - tan(1/2*d*x + 1/2*c)/sqrt(b) + 1/(sqrt(b)*tan(1
/2*d*x + 1/2*c)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(c+d\,x\right)^4}}\,dx$$

```
[In] int(1/(b*tan(c + d*x)^4)^(1/2),x)
[Out] int(1/(b*tan(c + d*x)^4)^(1/2), x)
```

3.40 $\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 119

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = \frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}} - \frac{\tan(c+dx)}{bd\sqrt{b\tan^4(c+dx)}} - \frac{x\tan^2(c+dx)}{b\sqrt{b\tan^4(c+dx)}}$$

```
[Out] 1/3*cot(d*x+c)/b/d/(tan(d*x+c)^4*b)^(1/2)-1/5*cot(d*x+c)^3/b/d/(tan(d*x+c)^
4*b)^(1/2)-tan(d*x+c)/b/d/(tan(d*x+c)^4*b)^(1/2)-x*tan(d*x+c)^2/b/(tan(d*x+
c)^4*b)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 8}

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = -\frac{\tan(c+dx)}{bd\sqrt{b\tan^4(c+dx)}} - \frac{x\tan^2(c+dx)}{b\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}} + \frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}}$$

```
[In] Int[(b*Tan[c + d*x]^4)^(-3/2),x]
```

```
[Out] Cot[c + d*x]/(3*b*d*Sqrt[b*Tan[c + d*x]^4]) - Cot[c + d*x]^3/(5*b*d*Sqrt[b*
Tan[c + d*x]^4]) - Tan[c + d*x]/(b*d*Sqrt[b*Tan[c + d*x]^4]) - (x*Tan[c + d
*x]^2)/(b*Sqrt[b*Tan[c + d*x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan^2(c+dx)\int\cot^6(c+dx)\,dx}{b\sqrt{b\tan^4(c+dx)}}\\
&= -\frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}}-\frac{\tan^2(c+dx)\int\cot^4(c+dx)\,dx}{b\sqrt{b\tan^4(c+dx)}}\\
&= \frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}}-\frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}}+\frac{\tan^2(c+dx)\int\cot^2(c+dx)\,dx}{b\sqrt{b\tan^4(c+dx)}}\\
&= \frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}}-\frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}}-\frac{\tan(c+dx)}{bd\sqrt{b\tan^4(c+dx)}}-\frac{\tan^2(c+dx)\int 1\,dx}{b\sqrt{b\tan^4(c+dx)}}\\
&= \frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}}-\frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}}-\frac{\tan(c+dx)}{bd\sqrt{b\tan^4(c+dx)}}-\frac{x\tan^2(c+dx)}{b\sqrt{b\tan^4(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.38

$$\int\frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)\tan(c+dx)}{5d\left(b\tan^4(c+dx)\right)^{3/2}}$$

```
[In] Integrate[(b*Tan[c + d*x]^4)^(-3/2),x]
[Out] -1/5*(Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(d*(b
*Tan[c + d*x]^4)^(3/2))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.53

method	result	size
derivativedivides	$-\frac{\tan(dx+c)(15\arctan(\tan(dx+c))(\tan^5(dx+c))+15(\tan^4(dx+c))-5(\tan^2(dx+c))+3)}{15d((\tan^4(dx+c))b)^{\frac{3}{2}}}$	63
default	$-\frac{\tan(dx+c)(15\arctan(\tan(dx+c))(\tan^5(dx+c))+15(\tan^4(dx+c))-5(\tan^2(dx+c))+3)}{15d((\tan^4(dx+c))b)^{\frac{3}{2}}}$	63
risch	$\frac{(e^{2i(dx+c)}-1)^2 x}{b(e^{2i(dx+c)}+1)^2\sqrt{\frac{(e^{2i(dx+c)}-1)^4 b}{(e^{2i(dx+c)}+1)^4}}}+\frac{2i(45\,e^{8i(dx+c)}-90\,e^{6i(dx+c)}+140\,e^{4i(dx+c)}-70\,e^{2i(dx+c)}+23)}{15b(e^{2i(dx+c)}-1)^3(e^{2i(dx+c)}+1)^2\sqrt{\frac{(e^{2i(dx+c)}-1)^4 b}{(e^{2i(dx+c)}+1)^4}}\,d}$	174

[In] int(1/(tan(d*x+c)^4*b)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/15/d*tan(d*x+c)*(15*arctan(tan(d*x+c))*tan(d*x+c)^5+15*tan(d*x+c)^4-5*ta
n(d*x+c)^2+3)/(tan(d*x+c)^4*b)^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.52

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = -\frac{\left(15\,dx\tan\left(dx+c\right)^5+15\,\tan\left(dx+c\right)^4-5\,\tan\left(dx+c\right)^2+3\right)\sqrt{b\tan\left(dx+c\right)^4}}{15\,b^2d\tan\left(dx+c\right)^7}$$

[In] integrate(1/(tan(d*x+c)^4*b)^(3/2),x, algorithm="fricas")

[Out] -1/15*(15*d*x*tan(d*x + c)^5 + 15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)*sq
rt(b*tan(d*x + c)^4)/(b^2*d*tan(d*x + c)^7)

Sympy [F]

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan^4{\left(c+dx\right)}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(tan(d*x+c)**4*b)**(3/2),x)

[Out] Integral((b*tan(c + d*x)**4)**(-3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = -\frac{\frac{15\,(dx+c)}{b^{\frac{3}{2}}} + \frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{b^{\frac{3}{2}}\,\tan(dx+c)^5}}{15\,d}$$

```
[In] integrate(1/(tan(d*x+c)^4*b)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(d*x + c)/b^(3/2) + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/(b
^(3/2)*tan(d*x + c)^5))/d
```

Giac [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = -\frac{\frac{480\,(dx+c)}{\sqrt{b}} - \frac{3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+330\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{b^{\frac{5}{2}}} + \frac{330\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3}{\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,bd}$$

```
[In] integrate(1/(tan(d*x+c)^4*b)^(3/2),x, algorithm="giac")
```

```
[Out] -1/480*(480*(d*x + c)/sqrt(b) - (3*b^2*tan(1/2*d*x + 1/2*c)^5 - 35*b^2*tan(
1/2*d*x + 1/2*c)^3 + 330*b^2*tan(1/2*d*x + 1/2*c))/b^(5/2) + (330*tan(1/2*d
*x + 1/2*c)^4 - 35*tan(1/2*d*x + 1/2*c)^2 + 3)/(sqrt(b)*tan(1/2*d*x + 1/2*c
)^5))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{{\left(b\,{\mathrm{tan}\left(c+d\,x\right)}^4\right)}^{3/2}} \,d x$$

```
[In] int(1/(b*tan(c + d*x)^4)^(3/2),x)
```

```
[Out] int(1/(b*tan(c + d*x)^4)^(3/2), x)
```

3.41 $\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 183

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx = \frac{\cot(c+dx)}{3b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\tan(c+dx)}{b^2d\sqrt{b\tan^4(c+dx)}} - \frac{x\tan^2(c+dx)}{b^2\sqrt{b\tan^4(c+dx)}}$$

```
[Out] 1/3*cot(d*x+c)/b^2/d/(tan(d*x+c)^4*b)^(1/2)-1/5*cot(d*x+c)^3/b^2/d/(tan(d*x
+c)^4*b)^(1/2)+1/7*cot(d*x+c)^5/b^2/d/(tan(d*x+c)^4*b)^(1/2)-1/9*cot(d*x+c)
^7/b^2/d/(tan(d*x+c)^4*b)^(1/2)-tan(d*x+c)/b^2/d/(tan(d*x+c)^4*b)^(1/2)-x*t
an(d*x+c)^2/b^2/(tan(d*x+c)^4*b)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 8}

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx = -\frac{\tan(c+dx)}{b^2d\sqrt{b\tan^4(c+dx)}} - \frac{x\tan^2(c+dx)}{b^2\sqrt{b\tan^4(c+dx)}} - \frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\cot(c+dx)}{3b^2d\sqrt{b\tan^4(c+dx)}}$$

```
[In] Int[(b*Tan[c + d*x]^4)^(-5/2),x]
```

```
[Out] Cot[c + d*x]/(3*b^2*d*Sqrt[b*Tan[c + d*x]^4]) - Cot[c + d*x]^3/(5*b^2*d*Sqr
t[b*Tan[c + d*x]^4]) + Cot[c + d*x]^5/(7*b^2*d*Sqrt[b*Tan[c + d*x]^4]) - Co
t[c + d*x]^7/(9*b^2*d*Sqrt[b*Tan[c + d*x]^4]) - Tan[c + d*x]/(b^2*d*Sqrt[b*
Tan[c + d*x]^4]) - (x*Tan[c + d*x]^2)/(b^2*Sqrt[b*Tan[c + d*x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan^2(c+dx)\int\cot^{10}(c+dx)\,dx}{b^2\sqrt{b\tan^4(c+dx)}}\\
&= -\frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}}-\frac{\tan^2(c+dx)\int\cot^8(c+dx)\,dx}{b^2\sqrt{b\tan^4(c+dx)}}\\
&= \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}}-\frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}}+\frac{\tan^2(c+dx)\int\cot^6(c+dx)\,dx}{b^2\sqrt{b\tan^4(c+dx)}}\\
&= -\frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}}+\frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}}\\
&\quad-\frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}}-\frac{\tan^2(c+dx)\int\cot^4(c+dx)\,dx}{b^2\sqrt{b\tan^4(c+dx)}}\\
&= \frac{\cot(c+dx)}{3b^2d\sqrt{b\tan^4(c+dx)}}-\frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}}+\frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}}\\
&\quad-\frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}}+\frac{\tan^2(c+dx)\int\cot^2(c+dx)\,dx}{b^2\sqrt{b\tan^4(c+dx)}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\cot(c+dx)}{3b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}} \\
&\quad - \frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\tan(c+dx)}{b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\tan^2(c+dx)\int 1\,dx}{b^2\sqrt{b\tan^4(c+dx)}} \\
&= \frac{\cot(c+dx)}{3b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}} \\
&\quad - \frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\tan(c+dx)}{b^2d\sqrt{b\tan^4(c+dx)}} - \frac{x\tan^2(c+dx)}{b^2\sqrt{b\tan^4(c+dx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.25

$$
\int \frac{1}{(b\tan^4(c+dx))^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(c+dx)\right)\tan(c+dx)}{9d\,(b\tan^4(c+dx))^{5/2}}
$$

```
[In] Integrate[(b*Tan[c + d*x]^4)^(-5/2),x]
```

```
[Out] -1/9*(Hypergeometric2F1[-9/2, 1, -7/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(d*(b
*Tan[c + d*x]^4)^(5/2))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

method	result
derivativedivides	$-\frac{\tan(dx+c)(315\arctan(\tan(dx+c))(\tan^9(dx+c))+315(\tan^8(dx+c))-105(\tan^6(dx+c))+63(\tan^4(dx+c))-45(\tan^2(dx}{315d((\tan^4(dx+c))b)^{\frac{5}{2}}}$
default	$-\frac{\tan(dx+c)(315\arctan(\tan(dx+c))(\tan^9(dx+c))+315(\tan^8(dx+c))-105(\tan^6(dx+c))+63(\tan^4(dx+c))-45(\tan^2(dx}{315d((\tan^4(dx+c))b)^{\frac{5}{2}}}$
risch	$\frac{(e^{2i(dx+c)}-1)^2 x}{b^2(e^{2i(dx+c)}+1)^2\sqrt{\frac{(e^{2i(dx+c)}-1)^4 b}{(e^{2i(dx+c)}+1)^4}}} + \frac{2i(1575\,e^{16i(dx+c)}-6300\,e^{14i(dx+c)}+21000\,e^{12i(dx+c)}-31500\,e^{10i(dx+c)}+3943}{315b^2(e^{2i(dx+c)}-1)^7(e^{2i(dx+c)}+1}$

```
[In] int(1/(tan(d*x+c)^4*b)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/315/d*tan(d*x+c)*(315*arctan(tan(d*x+c))*tan(d*x+c)^9+315*tan(d*x+c)^8-1
05*tan(d*x+c)^6+63*tan(d*x+c)^4-45*tan(d*x+c)^2+35)/(tan(d*x+c)^4*b)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx = -\frac{\left(315\,dx\tan\left(dx+c\right)^9+315\,\tan\left(dx+c\right)^8-105\,\tan\left(dx+c\right)^6+63\,\tan\left(dx+c\right)^4-45\,\tan\left(dx+c\right)^2+35\right)\ldots}{315\,b^3 d\tan\left(dx+c\right)^{11}}$$

[In] integrate(1/(tan(d*x+c)^4*b)^(5/2),x, algorithm="fricas")

[Out] -1/315*(315*d*x*tan(d*x + c)^9 + 315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*tan(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)*sqrt(b*tan(d*x + c)^4)/(b^3*d*tan(d*x + c)^11)

Sympy [F]

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan^4\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(1/(tan(d*x+c)**4*b)**(5/2),x)

[Out] Integral((b*tan(c + d*x)**4)**(-5/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.38

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx = -\frac{\frac{315\,(dx+c)}{b^{\frac{5}{2}}}+\frac{315\,\tan(dx+c)^8-105\,\tan(dx+c)^6+63\,\tan(dx+c)^4-45\,\tan(dx+c)^2+35}{b^{\frac{5}{2}}\tan(dx+c)^9}}{315\,d}$$

[In] integrate(1/(tan(d*x+c)^4*b)^(5/2),x, algorithm="maxima")

[Out] -1/315*(315*(d*x + c)/b^(5/2) + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*tan(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/(b^(5/2)*tan(d*x + c)^9))/d

Giac [A] (verification not implemented)

none

Time = 1.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx = -\frac{\frac{161280\,(dx+c)}{b^{\frac{5}{2}}}+\frac{121590\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-18480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+3528\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-495\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+35}{b^{\frac{5}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}}-\frac{35\,b^{20}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-\cdots}{\cdots}}{161280\,d}$$

```
[In] integrate(1/(tan(d*x+c)^4*b)^(5/2),x, algorithm="giac")
```

```
[Out] -1/161280*(161280*(d*x + c)/b^(5/2) + (121590*tan(1/2*d*x + 1/2*c)^8 - 1848
0*tan(1/2*d*x + 1/2*c)^6 + 3528*tan(1/2*d*x + 1/2*c)^4 - 495*tan(1/2*d*x +
1/2*c)^2 + 35)/(b^(5/2)*tan(1/2*d*x + 1/2*c)^9) - (35*b^20*tan(1/2*d*x + 1/
2*c)^9 - 495*b^20*tan(1/2*d*x + 1/2*c)^7 + 3528*b^20*tan(1/2*d*x + 1/2*c)^5
 - 18480*b^20*tan(1/2*d*x + 1/2*c)^3 + 121590*b^20*tan(1/2*d*x + 1/2*c))/b^
(45/2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{{\left(b\,{\tan\left(c+d\,x\right)}^4\right)}^{5/2}}\,dx$$

```
[In] int(1/(b*tan(c + d*x)^4)^(5/2),x)
```

```
[Out] int(1/(b*tan(c + d*x)^4)^(5/2), x)
```

3.42 $\int (b\tan^p(c+dx))^n \, dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int (b\tan^p(c+dx))^n \, dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^p(c+dx)\right)^n}{d(1+np)}$$

```
[Out] hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(d*x+c)^2)*tan(d*x+c)*(b*tan(d
*x+c)^p)^n/d/(n*p+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3740, 3557, 371}

$$\int (b\tan^p(c+dx))^n \, dx = \frac{\tan(c+dx)\left(b\tan^p(c+dx)\right)^n \text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(c+dx)\right)}{d(np+1)}$$

```
[In] Int[(b*Tan[c + d*x]^p)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^p)^n)/(d*(1 + n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \left(\tan^{-np}(c+dx)\left(b\tan^p(c+dx)\right)^n\right)\int \tan^{np}(c+dx)\,dx$$

$$= \frac{\left(\tan^{-np}(c+dx)\left(b\tan^p(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{np}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+np), \frac{1}{2}(3+np), -\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^p(c+dx)\right)^n}{d(1+np)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \left(b\tan^p(c+dx)\right)^n\,dx$$

$$= \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+np), \frac{1}{2}(3+np), -\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^p(c+dx)\right)^n}{d(1+np)}$$

```
[In] Integrate[(b*Tan[c + d*x]^p)^n,x]
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^p)^n)/(d*(1 + n*p))
```

Maple [F]

$$\int (b(\tan^p(dx+c)))^n dx$$

```
[In] int((b*tan(d*x+c)^p)^n,x)
[Out] int((b*tan(d*x+c)^p)^n,x)
```

Fricas [F]

$$\int (b\tan^p(c+dx))^n\ dx = \int (b\tan(dx+c)^p)^n\ dx$$

```
[In] integrate((b*tan(d*x+c)^p)^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c)^p)^n, x)
```

Sympy [F]

$$\int (b\tan^p(c+dx))^n\ dx = \int (b\tan^p(c+dx))^n\ dx$$

```
[In] integrate((b*tan(d*x+c)**p)**n,x)
[Out] Integral((b*tan(c + d*x)**p)**n, x)
```

Maxima [F]

$$\int (b\tan^p(c+dx))^n\ dx = \int (b\tan(dx+c)^p)^n\ dx$$

```
[In] integrate((b*tan(d*x+c)^p)^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c)^p)^n, x)
```

Giac [F]

$$\int (b\tan^p(c+dx))^n \, dx = \int (b\tan{(dx+c)}^p)^n \, dx$$

```
[In] integrate((b*tan(d*x+c)^p)^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c)^p)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\tan^p(c+dx))^n \, dx = \int (b\tan(c+d\,x)^p)^n \,dx$$

```
[In] int((b*tan(c + d*x)^p)^n,x)
[Out] int((b*tan(c + d*x)^p)^n, x)
```

3.43 $\int (b\tan^2(c+dx))^n \, dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int (b\tan^2(c+dx))^n \, dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+2n), \frac{1}{2}(3+2n), -\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^2(c+dx)\right)^n}{d(1+2n)}$$

```
[Out] hypergeom([1, 1/2+n],[3/2+n],-tan(d*x+c)^2)*tan(d*x+c)*(b*tan(d*x+c)^2)^n/d
/(1+2*n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3739, 3557, 371}

$$\int (b\tan^2(c+dx))^n \, dx = \frac{\tan(c+dx)\left(b\tan^2(c+dx)\right)^n \text{Hypergeometric2F1}\left(1, \frac{1}{2}(2n+1), \frac{1}{2}(2n+3), -\tan^2(c+dx)\right)}{d(2n+1)}$$

```
[In] Int[(b*Tan[c + d*x]^2)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 2*n)/2, (3 + 2*n)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^2)^n)/(d*(1 + 2*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \left(\tan^{-2n}(c+dx)\left(b\tan^2(c+dx)\right)^n\right)\int \tan^{2n}(c+dx)\,dx$$

$$= \frac{\left(\tan^{-2n}(c+dx)\left(b\tan^2(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{2n}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+2n), \frac{1}{2}(3+2n), -\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^2(c+dx)\right)^n}{d(1+2n)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \left(b\tan^2(c+dx)\right)^n\,dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}+n, \frac{3}{2}+n, -\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^2(c+dx)\right)^n}{d(1+2n)}$$

```
[In] Integrate[(b*Tan[c + d*x]^2)^n,x]
[Out] (Hypergeometric2F1[1, 1/2 + n, 3/2 + n, -Tan[c + d*x]^2]*Tan[c + d*x]*(b*Ta
n[c + d*x]^2)^n)/(d*(1 + 2*n))
```

Maple [F]

$$\int \left(b\left(\tan^2\left(dx+c\right)\right)\right)^n dx$$

```
[In] int((b*tan(d*x+c)^2)^n,x)
[Out] int((b*tan(d*x+c)^2)^n,x)
```

Fricas [F]

$$\int \left(b\tan^2(c+dx)\right)^n \, dx = \int \left(b\tan\left(dx+c\right)^2\right)^n \, dx$$

```
[In] integrate((b*tan(d*x+c)^2)^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c)^2)^n, x)
```

Sympy [F]

$$\int \left(b\tan^2(c+dx)\right)^n \, dx = \int \left(b\tan^2\left(c+dx\right)\right)^n \, dx$$

```
[In] integrate((b*tan(d*x+c)**2)**n,x)
[Out] Integral((b*tan(c + d*x)**2)**n, x)
```

Maxima [F]

$$\int \left(b\tan^2(c+dx)\right)^n \, dx = \int \left(b\tan\left(dx+c\right)^2\right)^n \, dx$$

```
[In] integrate((b*tan(d*x+c)^2)^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c)^2)^n, x)
```

Giac [F]

$$\int \left(b\tan^2(c+dx)\right)^n\,dx = \int \left(b\tan\left(dx+c\right)^2\right)^n\,dx$$

```
[In] integrate((b*tan(d*x+c)^2)^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c)^2)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^2(c+dx)\right)^n\,dx = \int \left(b\tan(c+d\,x)^2\right)^n\,dx$$

```
[In] int((b*tan(c + d*x)^2)^n,x)
[Out] int((b*tan(c + d*x)^2)^n, x)
```

3.44 $\int (b\tan^3(c+dx))^n\,dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int (b\tan^3(c+dx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+3n),\frac{3(1+n)}{2},-\tan^2(c+dx)\right)\tan(c+dx)\,(b\tan^3(c+dx))^n}{d(1+3n)}$$

```
[Out] hypergeom([1, 1/2+3/2*n],[3/2+3/2*n],-tan(d*x+c)^2)*tan(d*x+c)*(b*tan(d*x+c
)^3)^n/d/(1+3*n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3739, 3557, 371}

$$\int (b\tan^3(c+dx))^n\,dx = \frac{\tan(c+dx)\,(b\tan^3(c+dx))^n\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(3n+1),\frac{3(n+1)}{2},-\tan^2(c+dx)\right)}{d(3n+1)}$$

```
[In] Int[(b*Tan[c + d*x]^3)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 3*n)/2, (3*(1 + n))/2, -Tan[c + d*x]^2]*Tan[c +
d*x]*(b*Tan[c + d*x]^3)^n)/(d*(1 + 3*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \left(\tan^{-3n}(c+dx)\left(b\tan^3(c+dx)\right)^n\right)\int \tan^{3n}(c+dx)\,dx$$

$$= \frac{\left(\tan^{-3n}(c+dx)\left(b\tan^3(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{3n}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+3n), \frac{3(1+n)}{2}, -\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^3(c+dx)\right)^n}{d(1+3n)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \left(b\tan^3(c+dx)\right)^n\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+3n), \frac{3(1+n)}{2}, -\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^3(c+dx)\right)^n}{d(1+3n)}$$

```
[In] Integrate[(b*Tan[c + d*x]^3)^n,x]
[Out] (Hypergeometric2F1[1, (1 + 3*n)/2, (3*(1 + n))/2, -Tan[c + d*x]^2]*Tan[c +
d*x]*(b*Tan[c + d*x]^3)^n)/(d*(1 + 3*n))
```

Maple [F]

$$\int \left(b \left(\tan^3 \left(dx + c\right)\right)\right)^n dx$$

[In] int((b*tan(d*x+c)^3)^n,x)

[Out] int((b*tan(d*x+c)^3)^n,x)

Fricas [F]

$$\int \left(b \tan^3(c+dx)\right)^n \, dx = \int \left(b \tan \left(dx + c\right)^3\right)^n \, dx$$

[In] integrate((b*tan(d*x+c)^3)^n,x, algorithm="fricas")

[Out] integral((b*tan(d*x + c)^3)^n, x)

Sympy [F]

$$\int \left(b \tan^3(c+dx)\right)^n \, dx = \int \left(b \tan^3 \left(c + dx\right)\right)^n \, dx$$

[In] integrate((b*tan(d*x+c)**3)**n,x)

[Out] Integral((b*tan(c + d*x)**3)**n, x)

Maxima [F]

$$\int \left(b \tan^3(c+dx)\right)^n \, dx = \int \left(b \tan \left(dx + c\right)^3\right)^n \, dx$$

[In] integrate((b*tan(d*x+c)^3)^n,x, algorithm="maxima")

[Out] integrate((b*tan(d*x + c)^3)^n, x)

Giac [F]

$$\int \left(b\tan^3(c+dx)\right)^n\, dx = \int \left(b\tan\left(dx+c\right)^3\right)^n\, dx$$

```
[In] integrate((b*tan(d*x+c)^3)^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c)^3)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^3(c+dx)\right)^n\, dx = \int \left(b\tan(c+d\,x)^3\right)^n\, dx$$

```
[In] int((b*tan(c + d*x)^3)^n,x)
[Out] int((b*tan(c + d*x)^3)^n, x)
```

3.45 $\int (b\tan^4(c+dx))^n \, dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int (b\tan^4(c+dx))^n \, dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+4n),\frac{1}{2}(3+4n),-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^4(c+dx)\right)^n}{d(1+4n)}$$

```
[Out] hypergeom([1, 1/2+2*n],[3/2+2*n],-tan(d*x+c)^2)*tan(d*x+c)*(tan(d*x+c)^4*b)
^n/d/(1+4*n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3739, 3557, 371}

$$\int (b\tan^4(c+dx))^n \, dx = \frac{\tan(c+dx)\left(b\tan^4(c+dx)\right)^n \text{Hypergeometric2F1}\left(1,\frac{1}{2}(4n+1),\frac{1}{2}(4n+3),-\tan^2(c+dx)\right)}{d(4n+1)}$$

```
[In] Int[(b*Tan[c + d*x]^4)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 4*n)/2, (3 + 4*n)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^4)^n)/(d*(1 + 4*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \left(\tan^{-4n}(c+dx)\left(b\tan^4(c+dx)\right)^n\right)\int \tan^{4n}(c+dx)\,dx$$

$$= \frac{\left(\tan^{-4n}(c+dx)\left(b\tan^4(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{4n}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+4n), \frac{1}{2}(3+4n), -\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^4(c+dx)\right)^n}{d(1+4n)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \left(b\tan^4(c+dx)\right)^n\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}+2n, \frac{3}{2}+2n, -\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^4(c+dx)\right)^n}{d(1+4n)}$$

```
[In] Integrate[(b*Tan[c + d*x]^4)^n,x]
[Out] (Hypergeometric2F1[1, 1/2 + 2*n, 3/2 + 2*n, -Tan[c + d*x]^2]*Tan[c + d*x]*(
b*Tan[c + d*x]^4)^n)/(d*(1 + 4*n))
```

Maple [F]

$$\int \left(\left(\tan^{4}\left(dx+c\right)\right)b\right)^{n}dx$$

```
[In] int((tan(d*x+c)^4*b)^n,x)
[Out] int((tan(d*x+c)^4*b)^n,x)
```

Fricas [F]

$$\int \left(b\tan^4(c+dx)\right)^n\,dx=\int \left(b\tan\left(dx+c\right)^4\right)^n\,dx$$

```
[In] integrate((tan(d*x+c)^4*b)^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c)^4)^n, x)
```

Sympy [F]

$$\int \left(b\tan^4(c+dx)\right)^n\,dx=\int \left(b\tan^4\left(c+dx\right)\right)^n\,dx$$

```
[In] integrate((tan(d*x+c)**4*b)**n,x)
[Out] Integral((b*tan(c + d*x)**4)**n, x)
```

Maxima [F]

$$\int \left(b\tan^4(c+dx)\right)^n\,dx=\int \left(b\tan\left(dx+c\right)^4\right)^n\,dx$$

```
[In] integrate((tan(d*x+c)^4*b)^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c)^4)^n, x)
```

Giac [F]

$$\int \left(b\tan^4(c+dx)\right)^n\,dx = \int \left(b\tan\left(dx+c\right)^4\right)^n\,dx$$

```
[In] integrate((tan(d*x+c)^4*b)^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c)^4)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^4(c+dx)\right)^n\,dx = \int \left(b\tan(c+d\,x)^4\right)^n\,dx$$

```
[In] int((b*tan(c + d*x)^4)^n,x)
[Out] int((b*tan(c + d*x)^4)^n, x)
```

3.46 $\int (b\tan^p(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 71

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \frac{2b^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+5p),\frac{1}{4}(6+5p),-\tan^2(c+dx)\right)\tan^{1+2p}(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+5p)}$$

```
[Out] 2*b^2*hypergeom([1, 1/2+5/4*p],[3/2+5/4*p],-tan(d*x+c)^2)*(b*tan(d*x+c)^p)^
(1/2)*tan(d*x+c)^(1+2*p)/d/(2+5*p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3740, 3557, 371}

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \frac{2b^2\tan^{2p+1}(c+dx)\sqrt{b\tan^p(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(5p+2),\frac{1}{4}(5p+6),-\tan^2(c+dx)\right)}{d(5p+2)}$$

```
[In] Int[(b*Tan[c + d*x]^p)^(5/2),x]
```

```
[Out] (2*b^2*Hypergeometric2F1[1, (2 + 5*p)/4, (6 + 5*p)/4, -Tan[c + d*x]^2]*Tan[
c + d*x]^(1 + 2*p)*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + 5*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(b^2\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\int\tan^{\frac{5p}{2}}(c+dx)\,dx\\ &= \frac{\left(b^2\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\operatorname{Subst}\left(\int\frac{x^{5p/2}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{2b^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+5p),\frac{1}{4}(6+5p),-\tan^2(c+dx)\right)\tan^{1+2p}(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+5p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int(b\tan^p(c+dx))^{5/2}\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+5p),\frac{1}{4}(6+5p),-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^p(c+dx)\right)^{5/2}}{d\left(1+\frac{5p}{2}\right)}$$

```
[In] Integrate[(b*Tan[c + d*x]^p)^(5/2),x]
[Out] (Hypergeometric2F1[1, (2 + 5*p)/4, (6 + 5*p)/4, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^p)^(5/2))/(d*(1 + (5*p)/2))
```

Maple [F]

$$\int (b(\tan^p (dx + c)))^{\frac{5}{2}} dx$$

```
[In] int((b*tan(d*x+c)^p)^(5/2),x)
[Out] int((b*tan(d*x+c)^p)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (b\tan^p(c + dx))^{5/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*tan(d*x+c)^p)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (b\tan^p(c + dx))^{5/2} \, dx = \int (b\tan^p (c + dx))^{\frac{5}{2}} \, dx$$

```
[In] integrate((b*tan(d*x+c)**p)**(5/2),x)
[Out] Integral((b*tan(c + d*x)**p)**(5/2), x)
```

Maxima [F]

$$\int (b\tan^p(c + dx))^{5/2} \, dx = \int (b\tan (dx + c)^p)^{\frac{5}{2}} \, dx$$

```
[In] integrate((b*tan(d*x+c)^p)^(5/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c)^p)^(5/2), x)
```

Giac [F]

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \int (b\tan{(dx+c)}^p)^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*tan(d*x+c)^p)^(5/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c)^p)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \int (b\tan(c+d\,x)^p)^{5/2}\,dx$$

```
[In] int((b*tan(c + d*x)^p)^(5/2),x)
[Out] int((b*tan(c + d*x)^p)^(5/2), x)
```

3.47 $\int (b\tan^p(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int (b\tan^p(c+dx))^{3/2}\,dx=\frac{2b\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+3p),\frac{3(2+p)}{4},-\tan^2(c+dx)\right)\tan^{1+p}(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+3p)}$$

```
[Out] 2*b*hypergeom([1, 1/2+3/4*p],[3/2+3/4*p],-tan(d*x+c)^2)*(b*tan(d*x+c)^p)^(1
/2)*tan(d*x+c)^(p+1)/d/(2+3*p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3740, 3557, 371}

$$\int (b\tan^p(c+dx))^{3/2}\,dx=\frac{2b\tan^{p+1}(c+dx)\sqrt{b\tan^p(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(3p+2),\frac{3(p+2)}{4},-\tan^2(c+dx)\right)}{d(3p+2)}$$

```
[In] Int[(b*Tan[c + d*x]^p)^(3/2),x]
```

```
[Out] (2*b*Hypergeometric2F1[1, (2 + 3*p)/4, (3*(2 + p))/4, -Tan[c + d*x]^2]*Tan[
c + d*x]^(1 + p)*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + 3*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \left(b\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\int\tan^{\frac{3p}{2}}(c+dx)\,dx$$

$$= \frac{\left(b\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\text{Subst}\left(\int\frac{x^{3p/2}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}$$

$$= \frac{2b\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+3p),\frac{3(2+p)}{4},-\tan^2(c+dx)\right)\tan^{1+p}(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+3p)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int(b\tan^p(c+dx))^{3/2}\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+3p),\frac{3(2+p)}{4},-\tan^2(c+dx)\right)\tan(c+dx)\,(b\tan^p(c+dx))^{3/2}}{d\left(1+\frac{3p}{2}\right)}$$

```
[In] Integrate[(b*Tan[c + d*x]^p)^(3/2),x]
[Out] (Hypergeometric2F1[1, (2 + 3*p)/4, (3*(2 + p))/4, -Tan[c + d*x]^2]*Tan[c +
d*x]*(b*Tan[c + d*x]^p)^(3/2))/(d*(1 + (3*p)/2))
```

Maple [F]

$$\int (b(\tan^p(dx+c)))^{\frac{3}{2}} dx$$

```
[In] int((b*tan(d*x+c)^p)^(3/2),x)
[Out] int((b*tan(d*x+c)^p)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (b\tan^p(c+dx))^{3/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*tan(d*x+c)^p)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (b\tan^p(c+dx))^{3/2}\, dx = \int (b\tan^p(c+dx))^{\frac{3}{2}}\, dx$$

```
[In] integrate((b*tan(d*x+c)**p)**(3/2),x)
[Out] Integral((b*tan(c + d*x)**p)**(3/2), x)
```

Maxima [F]

$$\int (b\tan^p(c+dx))^{3/2}\, dx = \int (b\tan(dx+c)^p)^{\frac{3}{2}}\, dx$$

```
[In] integrate((b*tan(d*x+c)^p)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c)^p)^(3/2), x)
```

Giac [F]

$$\int (b \tan^p(c+dx))^{3/2} \, dx = \int (b \tan{(dx+c)}^p)^{\frac{3}{2}} \, dx$$

```
[In] integrate((b*tan(d*x+c)^p)^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c)^p)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b \tan^p(c+dx))^{3/2} \, dx = \int (b \tan(c+d\,x)^p)^{3/2} \, dx$$

```
[In] int((b*tan(c + d*x)^p)^(3/2),x)
[Out] int((b*tan(c + d*x)^p)^(3/2), x)
```

3.48 $\int \sqrt{b\tan^p(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 56

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{2+p}{4},\frac{6+p}{4},-\tan^2(c+dx)\right)\tan(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+p)}$$

```
[Out] 2*hypergeom([1, 1/2+1/4*p],[3/2+1/4*p],-tan(d*x+c)^2)*(b*tan(d*x+c)^p)^(1/2
)*tan(d*x+c)/d/(2+p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3740, 3557, 371}

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \frac{2\tan(c+dx)\sqrt{b\tan^p(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{p+2}{4},\frac{p+6}{4},-\tan^2(c+dx)\right)}{d(p+2)}$$

```
[In] Int[Sqrt[b*Tan[c + d*x]^p],x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + p)/4, (6 + p)/4, -Tan[c + d*x]^2]*Tan[c + d*x]
*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\int \tan^{\frac{p}{2}}(c+dx)\,dx\\
&= \frac{\left(\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\text{Subst}\left(\int \frac{x^{p/2}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}\\
&= \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{2+p}{4},\frac{6+p}{4},-\tan^2(c+dx)\right)\tan(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{2+p}{4},\frac{6+p}{4},-\tan^2(c+dx)\right)\tan(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+p)}$$

```
[In] Integrate[Sqrt[b*Tan[c + d*x]^p],x]
[Out] (2*Hypergeometric2F1[1, (2 + p)/4, (6 + p)/4, -Tan[c + d*x]^2]*Tan[c + d*x]
*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + p))
```

Maple [F]

$$\int \sqrt{b\left(\tan^{p}\left(dx+c\right)\right)}dx$$

```
[In] int((b*tan(d*x+c)^p)^(1/2),x)
[Out] int((b*tan(d*x+c)^p)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{b\tan^{p}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*tan(d*x+c)^p)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{b\tan^{p}(c+dx)}\,dx = \int \sqrt{b\tan^{p}\left(c+dx\right)}\,dx$$

```
[In] integrate((b*tan(d*x+c)**p)**(1/2),x)
[Out] Integral(sqrt(b*tan(c + d*x)**p), x)
```

Maxima [F]

$$\int \sqrt{b\tan^{p}(c+dx)}\,dx = \int \sqrt{b\tan\left(dx+c\right)^{p}}\,dx$$

```
[In] integrate((b*tan(d*x+c)^p)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(d*x + c)^p), x)
```

Giac [F]

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \int \sqrt{b\tan\left(dx+c\right)^p}\,dx$$

```
[In] integrate((b*tan(d*x+c)^p)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*tan(d*x + c)^p), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \int \sqrt{b\tan\left(c+d\,x\right)^p}\,dx$$

```
[In] int((b*tan(c + d*x)^p)^(1/2),x)
[Out] int((b*tan(c + d*x)^p)^(1/2), x)
```

3.49 $\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{2-p}{4},\frac{6-p}{4},-\tan^2(c+dx)\right)\tan(c+dx)}{d(2-p)\sqrt{b\tan^p(c+dx)}}$$

```
[Out] 2*hypergeom([1, 1/2-1/4*p],[3/2-1/4*p],-tan(d*x+c)^2)*tan(d*x+c)/d/(2-p)/(b
*tan(d*x+c)^p)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \frac{2\tan(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{2-p}{4},\frac{6-p}{4},-\tan^2(c+dx)\right)}{d(2-p)\sqrt{b\tan^p(c+dx)}}$$

```
[In] Int[1/Sqrt[b*Tan[c + d*x]^p],x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - p)/4, (6 - p)/4, -Tan[c + d*x]^2]*Tan[c + d*x]
)/(d*(2 - p)*Sqrt[b*Tan[c + d*x]^p])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan^{\frac{p}{2}}(c+dx) \int \tan^{-\frac{p}{2}}(c+dx)\, dx}{\sqrt{b\tan^p(c+dx)}} \\
&= \frac{\tan^{\frac{p}{2}}(c+dx)\text{Subst}\left(\int \frac{x^{-p/2}}{1+x^2}\, dx, x, \tan(c+dx)\right)}{d\sqrt{b\tan^p(c+dx)}} \\
&= \frac{2\,\text{Hypergeometric2F1}\left(1, \frac{2-p}{4}, \frac{6-p}{4}, -\tan^2(c+dx)\right)\tan(c+dx)}{d(2-p)\sqrt{b\tan^p(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\, dx = -\frac{2\,\text{Hypergeometric2F1}\left(1, \frac{2-p}{4}, \frac{6-p}{4}, -\tan^2(c+dx)\right)\tan(c+dx)}{d(-2+p)\sqrt{b\tan^p(c+dx)}}$$

```
[In] Integrate[1/Sqrt[b*Tan[c + d*x]^p],x]
[Out] (-2*Hypergeometric2F1[1, (2 - p)/4, (6 - p)/4, -Tan[c + d*x]^2]*Tan[c + d*x
])/(d*(-2 + p)*Sqrt[b*Tan[c + d*x]^p])
```

Maple [F]

$$\int \frac{1}{\sqrt{b\left(\tan^{p}\left(dx+c\right)\right)}}dx$$

```
[In] int(1/(b*tan(d*x+c)^p)^(1/2),x)
[Out] int(1/(b*tan(d*x+c)^p)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{b\tan^{p}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*tan(d*x+c)^p)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^{p}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan^{p}\left(c+dx\right)}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c)**p)**(1/2),x)
[Out] Integral(1/sqrt(b*tan(c + d*x)**p), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\tan^{p}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(dx+c\right)^{p}}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c)^p)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*tan(d*x + c)^p), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan(dx+c)^p}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c)^p)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*tan(d*x + c)^p), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan(c+d\,x)^p}}\,dx$$

```
[In] int(1/(b*tan(c + d*x)^p)^(1/2),x)
[Out] int(1/(b*tan(c + d*x)^p)^(1/2), x)
```

3.50 $\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3p),\frac{3(2-p)}{4},-\tan^2(c+dx)\right)\tan^{1-p}(c+dx)}{bd(2-3p)\sqrt{b\tan^p(c+dx)}}$$

```
[Out] 2*hypergeom([1, 1/2-3/4*p],[3/2-3/4*p],-tan(d*x+c)^2)*tan(d*x+c)^(1-p)/b/d/
(2-3*p)/(b*tan(d*x+c)^p)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3740, 3557, 371}

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \frac{2\tan^{1-p}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3p),\frac{3(2-p)}{4},-\tan^2(c+dx)\right)}{bd(2-3p)\sqrt{b\tan^p(c+dx)}}$$

```
[In] Int[(b*Tan[c + d*x]^p)^(-3/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - 3*p)/4, (3*(2 - p))/4, -Tan[c + d*x]^2]*Tan[c
+ d*x]^(1 - p))/(b*d*(2 - 3*p)*Sqrt[b*Tan[c + d*x]^p])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan^{\frac{p}{2}}(c+dx)\int \tan^{-\frac{3p}{2}}(c+dx)\,dx}{b\sqrt{b\tan^p(c+dx)}}\\
&= \frac{\tan^{\frac{p}{2}}(c+dx)\text{Subst}\left(\int \frac{x^{-3p/2}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{bd\sqrt{b\tan^p(c+dx)}}\\
&= \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3p),\frac{3(2-p)}{4},-\tan^2(c+dx)\right)\tan^{1-p}(c+dx)}{bd(2-3p)\sqrt{b\tan^p(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3p),-\frac{3}{4}(-2+p),-\tan^2(c+dx)\right)\tan(c+dx)}{d\left(1-\frac{3p}{2}\right)(b\tan^p(c+dx))^{3/2}}$$

```
[In] Integrate[(b*Tan[c + d*x]^p)^(-3/2),x]
```

```
[Out] (Hypergeometric2F1[1, (2 - 3*p)/4, (-3*(-2 + p))/4, -Tan[c + d*x]^2]*Tan[c
+ d*x])/(d*(1 - (3*p)/2)*(b*Tan[c + d*x]^p)^(3/2))
```

Maple [F]

$$\int \frac{1}{\left(b \left(\tan^{p}\left(d x +c \right)\right)\right)^{\frac{3}{2}}}d x$$

```
[In] int(1/(b*tan(d*x+c)^p)^(3/2),x)
```

```
[Out] int(1/(b*tan(d*x+c)^p)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(b \tan^p(c+dx))^{3/2}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*tan(d*x+c)^p)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{(b \tan^p(c+dx))^{3/2}} dx = \int \frac{1}{\left(b \tan^{p}{\left(c + d x \right)}\right)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(1/(b*tan(d*x+c)**p)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**p)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(b \tan^p(c+dx))^{3/2}} dx = \int { \frac{1}{\left(b \tan \left(d x + c\right)^{p}\right)^{\frac{3}{2}}} }\,{d x}$$

```
[In] integrate(1/(b*tan(d*x+c)^p)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\tan{(dx+c)}^p)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c)^p)^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c)^p)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\tan{(c+d\,x)}^p)^{3/2}}\,dx$$

```
[In] int(1/(b*tan(c + d*x)^p)^(3/2),x)
[Out] int(1/(b*tan(c + d*x)^p)^(3/2), x)
```

3.51 $\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-5p),\frac{1}{4}(6-5p),-\tan^2(c+dx)\right)\tan^{1-2p}(c+dx)}{b^2d(2-5p)\sqrt{b\tan^p(c+dx)}}$$

```
[Out] 2*hypergeom([1, 1/2-5/4*p],[3/2-5/4*p],-tan(d*x+c)^2)*tan(d*x+c)^(1-2*p)/b^
2/d/(2-5*p)/(b*tan(d*x+c)^p)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3740, 3557, 371}

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \frac{2\tan^{1-2p}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-5p),\frac{1}{4}(6-5p),-\tan^2(c+dx)\right)}{b^2d(2-5p)\sqrt{b\tan^p(c+dx)}}$$

```
[In] Int[(b*Tan[c + d*x]^p)^(-5/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - 5*p)/4, (6 - 5*p)/4, -Tan[c + d*x]^2]*Tan[c +
d*x]^(1 - 2*p))/(b^2*d*(2 - 5*p)*Sqrt[b*Tan[c + d*x]^p])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\tan^{\frac{p}{2}}(c+dx)\int\tan^{-\frac{5p}{2}}(c+dx)\,dx}{b^2\sqrt{b\tan^p(c+dx)}}\\ &= \frac{\tan^{\frac{p}{2}}(c+dx)\text{Subst}\left(\int\frac{x^{-5p/2}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{b^2d\sqrt{b\tan^p(c+dx)}}\\ &= \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-5p),\frac{1}{4}(6-5p),-\tan^2(c+dx)\right)\tan^{1-2p}(c+dx)}{b^2d(2-5p)\sqrt{b\tan^p(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int\frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-5p),\frac{1}{4}(6-5p),-\tan^2(c+dx)\right)\tan(c+dx)}{d\left(1-\frac{5p}{2}\right)(b\tan^p(c+dx))^{5/2}}$$

```
[In] Integrate[(b*Tan[c + d*x]^p)^(-5/2),x]
```

```
[Out] (Hypergeometric2F1[1, (2 - 5*p)/4, (6 - 5*p)/4, -Tan[c + d*x]^2]*Tan[c + d*
x])/(d*(1 - (5*p)/2)*(b*Tan[c + d*x]^p)^(5/2))
```

Maple [F]

$$\int \frac{1}{\left(b\left(\tan^{p}\left(dx+c\right)\right)\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/(b*tan(d*x+c)^p)^(5/2),x)
```

```
[Out] int(1/(b*tan(d*x+c)^p)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(b\tan^{p}(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*tan(d*x+c)^p)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{(b\tan^{p}(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\tan^{p}\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c)**p)**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**p)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\tan^{p}(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\tan\left(dx+c\right)^{p}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c)^p)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(b\tan^p(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan\left(dx+c\right)^p\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*tan(d*x+c)^p)^(5/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c)^p)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^p(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+d\,x\right)^p\right)^{5/2}}\,dx$$

```
[In] int(1/(b*tan(c + d*x)^p)^(5/2),x)
[Out] int(1/(b*tan(c + d*x)^p)^(5/2), x)
```

3.52 $\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx=-\frac{\cot(c+dx)\log(\cos(c+dx))\,(b\tan^p(c+dx))^{\frac{1}{p}}}{d}$$

```
[Out] -cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^p)^(1/p)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3740, 3556}

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx=-\frac{\cot(c+dx)\log(\cos(c+dx))\,(b\tan^p(c+dx))^{\frac{1}{p}}}{d}$$

```
[In] Int[(b*Tan[c + d*x]^p)^p^(-1),x]
```

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*(b*Tan[c + d*x]^p)^p^(-1))/d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
```

```
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \left(\cot(c+dx)\left(b\tan^p(c+dx)\right)^{\frac{1}{p}}\right)\int\tan(c+dx)\,dx$$
$$= -\frac{\cot(c+dx)\log(\cos(c+dx))\left(b\tan^p(c+dx)\right)^{\frac{1}{p}}}{d}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int\left(b\tan^p(c+dx)\right)^{\frac{1}{p}}\,dx = -\frac{\cot(c+dx)\log(\cos(c+dx))\left(b\tan^p(c+dx)\right)^{\frac{1}{p}}}{d}$$

```
[In] Integrate[(b*Tan[c + d*x]^p)^p^(-1),x]
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*(b*Tan[c + d*x]^p)^p^(-1))/d)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.38 (sec) , antiderivative size = 5979, normalized size of antiderivative = 186.84

method	result	size
risch	Expression too large to display	5979

```
[In] int((b*tan(d*x+c)^p)^(1/p),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\, dx = -\frac{b^{\left(\frac{1}{p}\right)}\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\, d}$$

```
[In] integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="fricas")
[Out] -1/2*b^(1/p)*log(1/(tan(d*x + c)^2 + 1))/d
```

Sympy [F]

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\, dx = \int (b\tan^p(c+dx))^{\frac{1}{p}}\, dx$$

```
[In] integrate((b*tan(d*x+c)**p)**(1/p),x)
[Out] Integral((b*tan(c + d*x)**p)**(1/p), x)
```

Maxima [F]

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\, dx = \int (b\tan(dx+c)^p)^{\left(\frac{1}{p}\right)}\, dx$$

```
[In] integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c)^p)^(1/p), x)
```

Giac [F]

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\, dx = \int (b\tan(dx+c)^p)^{\left(\frac{1}{p}\right)}\, dx$$

```
[In] integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c)^p)^(1/p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx = \int \left(b\tan(c+d\,x)^p\right)^{1/p}\,dx$$

```
[In] int((b*tan(c + d*x)^p)^(1/p),x)
[Out] int((b*tan(c + d*x)^p)^(1/p), x)
```

3.53 $\int (a(b\tan(c+dx))^p)^n\,dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int (a(b\tan(c+dx))^p)^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(c+dx)\right)\tan(c+dx)\,(a(b\tan(c+dx))^p)^n}{d(1+np)}$$

```
[Out] hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(d*x+c)^2)*tan(d*x+c)*(a*(b*ta
n(d*x+c))^p)^n/d/(n*p+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\int (a(b\tan(c+dx))^p)^n\,dx = \frac{\tan(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(c+dx)\right)(a(b\tan(c+dx))^p)^n}{d(np+1)}$$

```
[In] Int[(a*(b*Tan[c + d*x])^p)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(a*(b*Tan[c + d*x])^p)^n)/(d*(1 + n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((b\tan(c+dx))^{-np}\left(a(b\tan(c+dx))^p\right)^n\right)\int(b\tan(c+dx))^{np}\,dx \\
&= \frac{\left(b(b\tan(c+dx))^{-np}\left(a(b\tan(c+dx))^p\right)^n\right)\text{Subst}\left(\int\frac{x^{np}}{b^2+x^2}\,dx,x,b\tan(c+dx)\right)}{d} \\
&= \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(c+dx)\right)\tan(c+dx)\left(a(b\tan(c+dx))^p\right)^n}{d(1+np)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int\left(a(b\tan(c+dx))^p\right)^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(c+dx)\right)\tan(c+dx)\left(a(b\tan(c+dx))^p\right)^n}{d(1+np)}$$

```
[In] Integrate[(a*(b*Tan[c + d*x])^p)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(a*(b*Tan[c + d*x])^p)^n)/(d*(1 + n*p))
```

Maple [F]

$$\int (a(b\tan (dx + c))^p)^n \, dx$$

```
[In] int((a*(b*tan(d*x+c))^p)^n,x)
[Out] int((a*(b*tan(d*x+c))^p)^n,x)
```

Fricas [F]

$$\int (a(b\tan(c+dx))^p)^n \, dx = \int ((b\tan (dx + c))^p \, a)^n \, dx$$

```
[In] integrate((a*(b*tan(d*x+c))^p)^n,x, algorithm="fricas")
[Out] integral(((b*tan(d*x + c))^p*a)^n, x)
```

Sympy [F]

$$\int (a(b\tan(c+dx))^p)^n \, dx = \int (a(b\tan (c + dx))^p)^n \, dx$$

```
[In] integrate((a*(b*tan(d*x+c))**p)**n,x)
[Out] Integral((a*(b*tan(c + d*x))**p)**n, x)
```

Maxima [F]

$$\int (a(b\tan(c+dx))^p)^n \, dx = \int ((b\tan (dx + c))^p \, a)^n \, dx$$

```
[In] integrate((a*(b*tan(d*x+c))^p)^n,x, algorithm="maxima")
[Out] integrate(((b*tan(d*x + c))^p*a)^n, x)
```

Giac [F]

$$\int (a(b\tan(c+dx))^p)^n \, dx = \int ((b\tan(dx+c))^p a)^n \, dx$$

```
[In] integrate((a*(b*tan(d*x+c))^p)^n,x, algorithm="giac")
[Out] integrate(((b*tan(d*x + c))^p*a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a(b\tan(c+dx))^p)^n \, dx = \int (a\,(b\tan(c+d\,x))^p)^n \, dx$$

```
[In] int((a*(b*tan(c + d*x))^p)^n,x)
[Out] int((a*(b*tan(c + d*x))^p)^n, x)
```

3.54 $\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 257

$$\begin{aligned}
\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx &= -\frac{21\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b}+\frac{21\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b}\\
&\quad+\frac{21\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b}\\
&\quad-\frac{21\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b}\\
&\quad-\frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd}-\frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3}
\end{aligned}$$

```
[Out] -21/64*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b*2^(1/2)+21/
64*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b*2^(1/2)+21/128*
ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))*d^(1/2)/b*2^(1/
2)-21/128*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))*d^(1/
2)/b*2^(1/2)-7/16*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d-1/4*cos(b*x+a)^4*(d
*tan(b*x+a))^(7/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2671, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$= -\frac{21\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} + \frac{21\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}b} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} + \frac{21\sqrt{d}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}b} - \frac{21\sqrt{d}\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}b} - \frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd}$$

```
[In] Int[Sin[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-21*Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2
]*b) + (21*Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*
Sqrt[2]*b) + (21*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[
d*Tan[a + b*x]]])/(64*Sqrt[2]*b) - (21*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a
+ b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b) - (7*Cos[a + b*x]^2*
(d*Tan[a + b*x])^(3/2))/(16*b*d) - (Cos[a + b*x]^4*(d*Tan[a + b*x])^(7/2))/
(4*b*d^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
```

```
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{d\text{Subst}\left(\int \frac{x^{9/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} + \frac{(7d)\text{Subst}\left(\int \frac{x^{5/2}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{8b}$$

$$= -\frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} + \frac{(21d)\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{32b}$$

$$= -\frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} + \frac{(21d)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{16b}$$

$$= -\frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} - \frac{(21d)\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{32b} + \frac{(21d)\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{32b}$$

$$= -\frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} + \frac{\left(21\sqrt{d}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{\left(21\sqrt{d}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{(21d)\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64b} + \frac{(21d)\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64b}$$

$$= \frac{21\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} - \frac{21\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} - \frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} + \frac{\left(21\sqrt{d}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{\left(21\sqrt{d}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b}$$

$$= -\frac{21\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} + \frac{21\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} + \frac{21\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} - \frac{21\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} - \frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.47

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{\left(21\arcsin(\cos(a+bx)-\sin(a+bx))\csc(a+bx)\sqrt{\sin(2(a+bx))}+21\csc(a+bx)\log\left(\cos(a+bx)+\right.\right.}{}$$

```
[In] Integrate[Sin[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -1/64*((21*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Csc[a + b*x]*Sqrt[Sin[2*(a +
 b*x)]] + 21*Csc[a + b*x]*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a +
 b*x)]]]*Sqrt[Sin[2*(a + b*x)]] + 18*Sin[2*(a + b*x)] - 2*Sin[4*(a + b*x)])
*Sqrt[d*Tan[a + b*x]])/b
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(197) = 394$.

Time = 13.87 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.41

method	result
default	$\left(16\sqrt{2}\,\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\,(\cos^3(bx+a))\sin(bx+a)+16(\cos^2(bx+a))\sin(bx+a)\sqrt{2}\,\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}-44\cos(bx+a)\sin(bx+\right.$

```
[In] int(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/b*(16*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x
+a)^3*sin(b*x+a)+16*cos(b*x+a)^2*sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)
/(cos(b*x+a)+1)^2)^(1/2)-44*cos(b*x+a)*sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(
b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-44*sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a
)/(cos(b*x+a)+1)^2)^(1/2)+21*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(
b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+c
sc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)
+2)/(-1+cos(b*x+a)))-21*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a
)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*
x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(
-1+cos(b*x+a)))-42*arctan((-sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos
(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+42*arctan((sin(b*x+a)*2^
(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos
(b*x+a))))*(d*tan(b*x+a))^(1/2)*cos(b*x+a)/(cos(b*x+a)+1)/(-cos(b*x+a)*sin(
b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 947, normalized size of antiderivative = 3.68

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/256*(16*(4*cos(b*x + a)^3 - 11*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x
+ a))*sin(b*x + a) + 21*b*(-d^2/b^4)^(1/4)*log(9261/2*d^2*cos(b*x + a)*sin(
b*x + a) + 9261/2*(b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)^2 - b*d*(-d^2/b^4)^(1/
4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 9261/4*(2
*b^2*d*cos(b*x + a)^2 - b^2*d)*sqrt(-d^2/b^4)) - 21*b*(-d^2/b^4)^(1/4)*log(
```

```
9261/2*d^2*cos(b*x + a)*sin(b*x + a) - 9261/2*(b^3*(-d^2/b^4)^(3/4)*cos(b*x
 + a)^2 - b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x +
a)/cos(b*x + a)) - 9261/4*(2*b^2*d*cos(b*x + a)^2 - b^2*d)*sqrt(-d^2/b^4))
+ 21*I*b*(-d^2/b^4)^(1/4)*log(9261/2*d^2*cos(b*x + a)*sin(b*x + a) - 9261/2
*(I*b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)^2 + I*b*d*(-d^2/b^4)^(1/4)*cos(b*x +
a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 9261/4*(2*b^2*d*cos(b*
x + a)^2 - b^2*d)*sqrt(-d^2/b^4)) - 21*I*b*(-d^2/b^4)^(1/4)*log(9261/2*d^2*
cos(b*x + a)*sin(b*x + a) - 9261/2*(-I*b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)^2
- I*b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos
(b*x + a)) + 9261/4*(2*b^2*d*cos(b*x + a)^2 - b^2*d)*sqrt(-d^2/b^4)) + 21*b
*(-d^2/b^4)^(1/4)*log(9261*d^2 + 18522*(b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)*s
in(b*x + a) - b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(
b*x + a))) - 21*b*(-d^2/b^4)^(1/4)*log(9261*d^2 - 18522*(b^3*(-d^2/b^4)^(3/
4)*cos(b*x + a)*sin(b*x + a) - b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*
sin(b*x + a)/cos(b*x + a))) + 21*I*b*(-d^2/b^4)^(1/4)*log(9261*d^2 - 18522*
(I*b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) + I*b*d*(-d^2/b^4)^(1/4)*
cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 21*I*b*(-d^2/b^4)^(1/4
)*log(9261*d^2 - 18522*(-I*b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) -
 I*b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))))
/b
```

Sympy [F]

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(a+bx)}\sin^4(a+bx)\,dx$$

```
[In] integrate(sin(b*x+a)**4*(d*tan(b*x+a))**(1/2),x)
[Out] Integral(sqrt(d*tan(a + b*x))*sin(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.88

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= \frac{21\,d^6\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\right.}{\sqrt{d}}\right.}{128\,bd^5}$$

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/128*(21*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a)
+ sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x
 + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 8*(11*(d*tan(b
*x + a))^(7/2)*d^6 + 7*(d*tan(b*x + a))^(3/2)*d^8)/(d^4*tan(b*x + a)^4 + 2*
d^4*tan(b*x + a)^2 + d^4))/(b*d^5)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.95

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= \frac{\frac{42\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{42\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}-\frac{21\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan\cdots}\right.}{b}}{\cdots}$$

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/128*(42*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2
*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 42*sqrt(2)*abs(d)^(3/2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b -
 21*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*
sqrt(abs(d)) + abs(d))/b + 21*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) - sqr
t(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 8*(11*sqrt(d*tan(b*x +
 a))*d^5*tan(b*x + a)^3 + 7*sqrt(d*tan(b*x + a))*d^5*tan(b*x + a))/((d^2*ta
n(b*x + a)^2 + d^2)^2*b))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sin\left(a+b\,x\right)^4\,\sqrt{d\tan\left(a+b\,x\right)}\,dx$$

```
[In] int(sin(a + b*x)^4*(d*tan(a + b*x))^(1/2),x)
```

```
[Out] int(sin(a + b*x)^4*(d*tan(a + b*x))^(1/2), x)
```

3.55 $\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 227

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{3\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}+\frac{3\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}+\frac{3\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b}-\frac{3\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b}-\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd}$$

```
[Out] -3/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b*2^(1/2)+3/8*a
rctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b*2^(1/2)+3/16*ln(d^(
1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))*d^(1/2)/b*2^(1/2)-3/1
6*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))*d^(1/2)/b*2^(
1/2)-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2671, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$= -\frac{3\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}+\frac{3\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}b}$$
$$+\frac{3\sqrt{d}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b}$$
$$-\frac{3\sqrt{d}\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b}$$
$$-\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd}$$

```
[In] Int[Sin[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-3*Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*
b) + (3*Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt
[2]*b) + (3*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan
[a + b*x]]])/(8*Sqrt[2]*b) - (3*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x]
+ Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) - (Cos[a + b*x]^2*(d*Tan[a +
 b*x])^(3/2))/(2*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{d\text{Subst}\left(\int \frac{x^{5/2}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd} + \frac{(3d)\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{4b}$$
$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd} + \frac{(3d)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2b}$$
$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd} - \frac{(3d)\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4b} + \frac{(3d)\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4b}$$
$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd} + \frac{\left(3\sqrt{d}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{\left(3\sqrt{d}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{(3d)\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8b} + \frac{(3d)\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8b}$$
$$= \frac{3\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} - \frac{3\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd} + \frac{\left(3\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{\left(3\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}$$

$$= -\frac{3\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} + \frac{3\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}$$
$$+ \frac{3\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b}$$
$$- \frac{3\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b}$$
$$- \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.46

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx =$$
$$-\frac{\left(3\arcsin(\cos(a+bx)-\sin(a+bx))\csc(a+bx)+3\csc(a+bx)\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin}\right.\right.}{8b}$$

```
[In] Integrate[Sin[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -1/8*((3*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Csc[a + b*x] + 3*Csc[a + b*x]*
Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]] + 2*Sqrt[Sin[2*(a
 + b*x)]])*Sqrt[Sin[2*(a + b*x)]]*Sqrt[d*Tan[a + b*x]])/b
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(171) = 342$.

Time = 0.92 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.33

method	result
default	$-\frac{\left(4\cos(bx+a)\sin(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}+4\sin(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}-6\arctan\left(\frac{\sin(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\,s}{(\cos(bx+}}}{-1+\cos(b}\right.\right.}{}$

```
[In] int(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/b*(4*cos(b*x+a)*sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a
)+1)^2)^(1/2)+4*sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2
)^(1/2)-6*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)
^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+6*arctan((-sin(b*x+a)*2^(1/2)*(-co
```

```
s(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))-
3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot
(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(
b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))+3*ln(-
(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a
)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a)
)^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a))))*(d*tan(b*x+
a))^(1/2)*cos(b*x+a)/(cos(b*x+a)+1)/(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^
2)^(1/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 934, normalized size of antiderivative = 4.11

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*(16*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)*sin(b*x + a) - 3*b
*(-d^2/b^4)^(1/4)*log(27/2*d^2*cos(b*x + a)*sin(b*x + a) + 27/2*(b^3*(-d^2/
b^4)^(3/4)*cos(b*x + a)^2 - b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)*sin(b*x + a))
*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 27/4*(2*b^2*d*cos(b*x + a)^2 - b^2*d)*
sqrt(-d^2/b^4)) + 3*b*(-d^2/b^4)^(1/4)*log(27/2*d^2*cos(b*x + a)*sin(b*x +
a) - 27/2*(b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)^2 - b*d*(-d^2/b^4)^(1/4)*cos(b
*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 27/4*(2*b^2*d*cos
(b*x + a)^2 - b^2*d)*sqrt(-d^2/b^4)) - 3*I*b*(-d^2/b^4)^(1/4)*log(27/2*d^2*
cos(b*x + a)*sin(b*x + a) - 27/2*(I*b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)^2 + I
*b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*
x + a)) + 27/4*(2*b^2*d*cos(b*x + a)^2 - b^2*d)*sqrt(-d^2/b^4)) + 3*I*b*(-d
^2/b^4)^(1/4)*log(27/2*d^2*cos(b*x + a)*sin(b*x + a) - 27/2*(-I*b^3*(-d^2/b
^4)^(3/4)*cos(b*x + a)^2 - I*b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)*sin(b*x + a)
)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 27/4*(2*b^2*d*cos(b*x + a)^2 - b^2*d)
*sqrt(-d^2/b^4)) - 3*b*(-d^2/b^4)^(1/4)*log(27*d^2 + 54*(b^3*(-d^2/b^4)^(3/
4)*cos(b*x + a)*sin(b*x + a) - b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*
sin(b*x + a)/cos(b*x + a))) + 3*b*(-d^2/b^4)^(1/4)*log(27*d^2 - 54*(b^3*(-d
^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) - b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)
^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 3*I*b*(-d^2/b^4)^(1/4)*log(27*d^2
- 54*(I*b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) + I*b*d*(-d^2/b^4)^(
1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 3*I*b*(-d^2/b^4)^
(1/4)*log(27*d^2 - 54*(-I*b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) -
I*b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))))/
b
```

Sympy [F]

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan{(a+bx)}}\sin^2{(a+bx)}\,dx$$

```
[In] integrate(sin(b*x+a)**2*(d*tan(b*x+a))**(1/2),x)
[Out] Integral(sqrt(d*tan(a + b*x))*sin(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.85

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{\cdots}\right.}{\sqrt{d}}\cdots}{16\,bd^3}$$

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] 1/16*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(b
*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a) +
sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x +
 a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 8*(d*tan(b*x + a
))^(3/2)*d^4/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{\frac{8\sqrt{d\tan(bx+a)}d^3\tan(bx+a)}{\left(d^2\tan(bx+a)^2+d^2\right)b} - \frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b} - \frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b} + \cdots}{16\,d}$$

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*(8*sqrt(d*tan(b*x + a))*d^3*tan(b*x + a)/((d^2*tan(b*x + a)^2 + d^2)*
b) - 6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sq
rt(d*tan(b*x + a)))/sqrt(abs(d)))/b - 6*sqrt(2)*abs(d)^(3/2)*arctan(-1/2*sq
rt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 3*s
qrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(
abs(d)) + abs(d))/b - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) - sqrt(2)*s
qrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sin(a+b\,x)^2\,\sqrt{d\tan(a+b\,x)}\,dx$$

```
[In] int(sin(a + b*x)^2*(d*tan(a + b*x))^(1/2),x)
```

```
[Out] int(sin(a + b*x)^2*(d*tan(a + b*x))^(1/2), x)
```

3.56 $\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx=-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

```
[Out] -2*d/b/(d*tan(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 30}

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx=-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Csc[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{d\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
[Out] (-2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\frac{2d}{b\sqrt{d\tan(bx+a)}}$	17
default	$-\frac{2d}{b\sqrt{d\tan(bx+a)}}$	17

```
[In] int(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*d/b/(d*tan(b*x+a))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(16) = 32.

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)}{b\sin(bx+a)}$$

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)/(b*sin(b*x + a))
```

Sympy [F]

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(a+bx)}\csc^2(a+bx)\,dx$$

```
[In] integrate(csc(b*x+a)**2*(d*tan(b*x+a))**(1/2),x)
[Out] Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\sqrt{d\tan(bx+a)}}{b\tan(bx+a)}$$

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(d*tan(b*x + a))/(b*tan(b*x + a))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d}{\sqrt{d\tan(bx+a)}b}$$

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] -2*d/(sqrt(d*tan(b*x + a))*b)
```

Mupad [B] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.67

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{\sin(2a+2bx)\sqrt{\frac{d\sin(2a+2bx)}{\cos(2a+2bx)+1}}}{b\sin(a+bx)^2}$$

```
[In] int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^2,x)
[Out] -(sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2))/(b*
sin(a + b*x)^2)
```

3.57 $\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx=-\frac{2d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

```
[Out] -2*d/b/(d*tan(b*x+a))^(1/2)-2/5*d^3/b/(d*tan(b*x+a))^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 14}

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx=-\frac{2d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Csc[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
[Out] (-2*d^3)/(5*b*(d*Tan[a + b*x])^(5/2)) - (2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
```

```
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\text{Subst}\left(\int \frac{d^2+x^2}{x^{7/2}}\,dx, x, d\tan(a+bx)\right)}{b} \\
&= \frac{d\text{Subst}\left(\int \left(\frac{d^2}{x^{7/2}}+\frac{1}{x^{3/2}}\right)\,dx, x, d\tan(a+bx)\right)}{b} \\
&= -\frac{2d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d(4+\csc^2(a+bx))}{5b\sqrt{d\tan(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
[Out] (-2*d*(4 + Csc[a + b*x]^2))/(5*b*Sqrt[d*Tan[a + b*x]])
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{2\sqrt{d\tan(bx+a)}\,(4(\cot^3(bx+a))-5\cot(bx+a)(\csc^2(bx+a)))}{5b}$	43

```
[In] int(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5/b*(d*tan(b*x+a))^(1/2)*(4*cot(b*x+a)^3-5*cot(b*x+a)*csc(b*x+a)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(4\cos\left(bx+a\right)^3-5\cos\left(bx+a\right)\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{5\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
[Out] -2/5*(4*cos(b*x + a)^3 - 5*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))/
((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F]

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan\left(a+bx\right)}\csc^4\left(a+bx\right)\,dx$$

```
[In] integrate(csc(b*x+a)**4*(d*tan(b*x+a))**(1/2),x)
[Out] Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(5\,d^2\tan\left(bx+a\right)^2+d^2\right)d}{5\,\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}\,b}$$

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] -2/5*(5*d^2*tan(b*x + a)^2 + d^2)*d/((d*tan(b*x + a))^(5/2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(5\,d^4\tan\left(bx+a\right)^2+d^4\right)}{5\,\sqrt{d\tan\left(bx+a\right)}bd^3\tan\left(bx+a\right)^2}$$

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2/5*(5*d^4*tan(b*x + a)^2 + d^4)/(sqrt(d*tan(b*x + a))*b*d^3*tan(b*x + a)^
2)
```

Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.49

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$= \frac{8\sqrt{-\frac{d\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,2\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,2\mathrm{i}-\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{5\,b\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^3}$$

```
[In] int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^4,x)
```

```
[Out] (8*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*(exp(
a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*2i - exp(a*6i + b*x*6i)*1i - 1i))/(5
*b*(exp(a*2i + b*x*2i) - 1)^3)
```

3.58 $\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d^5}{9b(d\tan(a+bx))^{9/2}} - \frac{4d^3}{5b(d\tan(a+bx))^{5/2}} - \frac{2d}{b\sqrt{d\tan(a+bx)}}$$

```
[Out] -2*d/b/(d*tan(b*x+a))^(1/2)-2/9*d^5/b/(d*tan(b*x+a))^(9/2)-4/5*d^3/b/(d*tan
(b*x+a))^(5/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2671, 276}

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d^5}{9b(d\tan(a+bx))^{9/2}} - \frac{4d^3}{5b(d\tan(a+bx))^{5/2}} - \frac{2d}{b\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Csc[a + b*x]^6*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d^5)/(9*b*(d*Tan[a + b*x])^(9/2)) - (4*d^3)/(5*b*(d*Tan[a + b*x])^(5/2)
) - (2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\text{Subst}\left(\int \frac{(d^2+x^2)^2}{x^{11/2}}\,dx, x, d\tan(a+bx)\right)}{b} \\
&= \frac{d\text{Subst}\left(\int \left(\frac{d^4}{x^{11/2}}+\frac{2d^2}{x^{7/2}}+\frac{1}{x^{3/2}}\right)\,dx, x, d\tan(a+bx)\right)}{b} \\
&= -\frac{2d^5}{9b(d\tan(a+bx))^{9/2}}-\frac{4d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{2d(-21+20\cos(2(a+bx))-4\cos(4(a+bx)))\csc^4(a+bx)}{45b\sqrt{d\tan(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]^6*Sqrt[d*Tan[a + b*x]],x]
[Out] (2*d*(-21 + 20*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4)/(45*b
*Sqrt[d*Tan[a + b*x]])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{2\cot(bx+a)(\csc^4(bx+a))\sqrt{d\tan(bx+a)}\,(32(\cos^4(bx+a))-72(\cos^2(bx+a))+45)}{45b}$	52

```
[In] int(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/b*cot(b*x+a)*csc(b*x+a)^4*(d*tan(b*x+a))^(1/2)*(32*cos(b*x+a)^4-72*co
s(b*x+a)^2+45)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(32\,\cos\left(bx+a\right)^5-72\,\cos\left(bx+a\right)^3+45\,\cos\left(bx+a\right)\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{45\left(b\cos\left(bx+a\right)^4-2\,b\cos\left(bx+a\right)^2+b\right)\sin\left(bx+a\right)}$$

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(32*cos(b*x + a)^5 - 72*cos(b*x + a)^3 + 45*cos(b*x + a))*sqrt(d*sin(
b*x + a)/cos(b*x + a))/((b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)*sin(b*x
 + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**6*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(45\,d^4\tan\left(bx+a\right)^4+18\,d^4\tan\left(bx+a\right)^2+5\,d^4\right)d}{45\left(d\tan\left(bx+a\right)\right)^{\frac{9}{2}}b}$$

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/45*(45*d^4*tan(b*x + a)^4 + 18*d^4*tan(b*x + a)^2 + 5*d^4)*d/((d*tan(b*x
 + a))^(9/2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(45\,d^6\tan\left(bx+a\right)^4+18\,d^6\tan\left(bx+a\right)^2+5\,d^6\right)}{45\,\sqrt{d\tan\left(bx+a\right)}bd^5\tan\left(bx+a\right)^4}$$

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2/45*(45*d^6*tan(b*x + a)^4 + 18*d^6*tan(b*x + a)^2 + 5*d^6)/(sqrt(d*tan(b
*x + a))*b*d^5*tan(b*x + a)^4)
```

Mupad [B] (verification not implemented)

Time = 8.13 (sec) , antiderivative size = 356, normalized size of antiderivative = 5.65

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{45\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)} + \frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{45\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^2} - \frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{15\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3} - \frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{9\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^4} - \frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^5}$$

```
[In] int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^6,x)
```

```
[Out] ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*
x*2i) + 1))^(1/2)*64i)/(45*b*(exp(a*2i + b*x*2i) - 1)^2) - ((exp(a*2i + b*x
*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2
)*64i)/(45*b*(exp(a*2i + b*x*2i) - 1)) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(ex
p(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*32i)/(15*b*(exp(
a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)
*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*64i)/(9*b*(exp(a*2i + b*x*2i) -
1)^4) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a
*2i + b*x*2i) + 1))^(1/2)*32i)/(9*b*(exp(a*2i + b*x*2i) - 1)^5)
```

3.59 $\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}} +\frac{5\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{12b}$$

```
[Out] -5/6*d*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)-1/3*d*sin(b*x+a)^3/b/(d*tan(b*x+a)
)^(1/2)-5/12*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Ellip
ticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2678, 2681, 2653, 2720}

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}-\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}} +\frac{5\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{12b}$$

```
[In] Int[Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-5*d*Sin[a + b*x])/(6*b*Sqrt[d*Tan[a + b*x]]) - (d*Sin[a + b*x]^3)/(3*b*Sq
rt[d*Tan[a + b*x]]) + (5*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin
[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(12*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{5}{6}\int\sin(a+bx)\sqrt{d\tan(a+bx)}\,dx\\
&= -\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{5}{12}\int\csc(a+bx)\sqrt{d\tan(a+bx)}\,dx\\
&= -\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}\\
&\quad+\frac{\left(5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{12\sqrt{\sin(a+bx)}}
\end{aligned}$$

$$= -\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}} - \frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{1}{12}\left(5\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx$$

$$= -\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}} - \frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{5\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{12b}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.79 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.32

$$\int\sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx =$$

$$-\frac{\cos(2(a+bx))\sec(a+bx)\left(-5\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sec^2(a+bx)+(-6\right.}{6b\sqrt{\sec^2(a+bx)}\sqrt{\tan(a+bx)}\left(-1+\tan^2(a\right.}$$

```
[In] Integrate[Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -1/6*(Cos[2*(a + b*x)]*Sec[a + b*x]*(-5*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)
^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sec[a + b*x]^2 + (-6 + Cos[2*(a + b*x)])*Sq
rt[Sec[a + b*x]^2]*Sqrt[Tan[a + b*x]])*Sqrt[d*Tan[a + b*x]])/(b*Sqrt[Sec[a
+ b*x]^2]*Sqrt[Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.25 (sec) , antiderivative size = 1740, normalized size of antiderivative = 16.57

method	result	size
default	Expression too large to display	1740

```
[In] int(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/b*csc(b*x+a)*(-6*I*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b
*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b
*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+6*I*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc
(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+c
sc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)+6*I*(cot(b*x+
a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x
```

+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)+6*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b
*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*
I,1/2*2^(1/2))*cos(b*x+a)-6*I*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+
cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-
cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)+6*(cot(b*x+a)-csc(b*x+a
))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*E
llipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)
-32*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b
*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/
2))*cos(b*x+a)-8*2^(1/2)*cos(b*x+a)^3*sin(b*x+a)+6*(cot(b*x+a)-csc(b*x+a))^
(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*Elli
pticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+6*(cot(b*x+a)
-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a
))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-
32*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*
x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2
))+3*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-2*cot(b*x+a)*2^(1/
2)*(-cot(b*x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))^2)^(1/2)-2*csc(b*x+a)*2^(1/2)*
(-cot(b*x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))^2)^(1/2)-2*cot(b*x+a)+2)*cos(b*x+
a)-3*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(2*cot(b*x+a)*2^(1/2
)*(-cot(b*x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))^2)^(1/2)+2*csc(b*x+a)*2^(1/2)*(
-cot(b*x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))^2)^(1/2)-2*cot(b*x+a)+2)*cos(b*x+a
)+6*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((-sin(b*x+a)*2^(
1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(
b*x+a)))*cos(b*x+a)-6*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arcta
n((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b
*x+a)-1)/(-1+cos(b*x+a)))*cos(b*x+a)+28*sin(b*x+a)*2^(1/2)*cos(b*x+a)+3*(-c
os(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-2*cot(b*x+a)*2^(1/2)*(-cot
(b*x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))^2)^(1/2)-2*csc(b*x+a)*2^(1/2)*(-cot(b*
x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))^2)^(1/2)-2*cot(b*x+a)+2)-3*(-cos(b*x+a)*s
in(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(2*cot(b*x+a)*2^(1/2)*(-cot(b*x+a)*csc(
b*x+a)^2*(-1+cos(b*x+a))^2)^(1/2)+2*csc(b*x+a)*2^(1/2)*(-cot(b*x+a)*csc(b*x
+a)^2*(-1+cos(b*x+a))^2)^(1/2)-2*cot(b*x+a)+2)+6*(-cos(b*x+a)*sin(b*x+a)/(c
os(b*x+a)+1)^2)^(1/2)*arctan((-sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(
cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))-6*(-cos(b*x+a)*sin(b*
x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*
x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a))))*(d*tan(b*x+a))
^(1/2)*2^(1/2)

Fricas [F]

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\sin(bx+a)^3\,dx$$

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**3*(d*tan(b*x+a))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\sin(bx+a)^3\,dx$$

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(b*x + a))*sin(b*x + a)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]e
xt_reduce Error: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sin\left(a+b\,x\right)^3\,\sqrt{d\tan\left(a+b\,x\right)}\,dx$$

```
[In] int(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2),x)
[Out] int(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2), x)
```

3.60 $\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{2b}$$

```
[Out] -d*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)-1/2*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(
1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^
(1/2)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2678, 2681, 2653, 2720}

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{2b} - \frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Sin[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -((d*Sin[a + b*x])/(b*Sqrt[d*Tan[a + b*x]])) + (Csc[a + b*x]*EllipticF[a -
Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(2*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{1}{2}\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx\\
&= -\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{\left(\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{2\sqrt{\sin(a+bx)}}\\
&= -\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{1}{2}\left(\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\
&= -\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{2b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.79 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$=\frac{\cos(a+bx)\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{d\tan(a+bx)}}{b}$$

```
[In] Integrate[Sin[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (Cos[a + b*x]*(-1 + Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[
Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(92) = 184.

Time = 0.74 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.67

method	result
default	$-\frac{\sqrt{d\tan(bx+a)}\left(-\cot(bx+a)\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+a}\right.\right.}{}$

```
[In] int(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(d*tan(b*x+a))^(1/2)*(-cot(b*x+a)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-cs
c(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+c
sc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-csc(b*x+a)*(cot(b*x+a)-csc(b*x+a))
^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*Ell
ipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(b*x+a))*2^(
1/2)
```

Fricas [F]

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx=\int \sqrt{d\tan\left(bx+a\right)}\sin\left(bx+a\right)\,dx$$

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

Sympy [F]

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan{(a+bx)}}\sin{(a+bx)}\,dx$$

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))**(1/2),x)
[Out] Integral(sqrt(d*tan(a + b*x))*sin(a + b*x), x)
```

Maxima [F]

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan{(bx+a)}}\sin{(bx+a)}\,dx$$

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

Giac [F]

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan{(bx+a)}}\sin{(bx+a)}\,dx$$

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sin{(a+b\,x)}\,\sqrt{d\tan{(a+b\,x)}}\,dx$$

```
[In] int(sin(a + b*x)*(d*tan(a + b*x))^(1/2),x)
[Out] int(sin(a + b*x)*(d*tan(a + b*x))^(1/2), x)
```

3.61 $\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] -csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1
/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2681, 2653, 2720}

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{b}$$

```
[In] Int[Csc[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Ta
n[a + b*x]])/b
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
```

```
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{\sqrt{\sin(a+bx)}}\\ &= \left(\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{b}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int\csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\sqrt[4]{-1}\cos(a+bx)\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}}{b\sqrt{\tan(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*(-1)^(1/4)*Cos[a + b*x]*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x
]]], -1]*Sqrt[Sec[a + b*x]^2]*Sqrt[d*Tan[a + b*x]])/(b*Sqrt[Tan[a + b*x]])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.30

method	result
default	$\frac{\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)\sqrt{d\tan(bx+a}}{b}$

[In] int(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/b*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*(d*tan(b*x+a))^(1/2)*(cot(b*x+a)+csc(b*x+a))*2^(1/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{\sqrt{i\,d}F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+\sqrt{-i\,d}F(\arcsin(\cos(bx+a)-i\,\sin(bx+a))\,|-1)}{b}$$

[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")

[Out] -(sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1))/b

Sympy [F]

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(a+bx)}\csc(a+bx)\,dx$$

[In] integrate(csc(b*x+a)*(d*tan(b*x+a))**(1/2),x)

[Out] Integral(sqrt(d*tan(a + b*x))*csc(a + b*x), x)

Maxima [F]

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan\left(bx+a\right)}\csc\left(bx+a\right)\,dx$$

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a), x)
```

Giac [F]

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan\left(bx+a\right)}\csc\left(bx+a\right)\,dx$$

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \frac{\sqrt{d\tan\left(a+b\,x\right)}}{\sin\left(a+b\,x\right)}\,dx$$

```
[In] int((d*tan(a + b*x))^(1/2)/sin(a + b*x),x)
[Out] int((d*tan(a + b*x))^(1/2)/sin(a + b*x), x)
```

3.62 $\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b}$$

```
[Out] -2/3*d*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)-2/3*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^
2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2
*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2679, 2681, 2653, 2720}

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b} - \frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Csc[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d*Csc[a + b*x])/(3*b*Sqrt[d*Tan[a + b*x]]) + (2*Csc[a + b*x]*EllipticF[
a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{2}{3}\int\csc(a+bx)\sqrt{d\tan(a+bx)}\,dx\\
&= -\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{\left(2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{3\sqrt{\sin(a+bx)}}\\
&= -\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\\
&\quad+\frac{1}{3}\left(2\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\
&= -\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.49

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$= \frac{2\cos(2(a+bx))\csc^3(a+bx)(d\tan(a+bx))^{3/2}\left(\sqrt{\sec^2(a+bx)}+2\sqrt[4]{-1}\,\mathrm{EllipticF}\left(i\mathrm{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a\right.\right.\right.}{3bd\sqrt{\sec^2(a+bx)}\,(-1+\tan^2(a+bx))}$$

```
[In] Integrate[Csc[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (2*Cos[2*(a + b*x)]*Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2)*(Sqrt[Sec[a + b*x
]^2] + 2*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]
*Tan[a + b*x]^(3/2)))/(3*b*d*Sqrt[Sec[a + b*x]^2]*(-1 + Tan[a + b*x]^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(92) = 184.

Time = 0.75 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.83

method	result
default	$\frac{\left(-2\sin(bx+a)\cos(bx+a)\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\cot(\right.\right.}{}$

```
[In] int(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b*(-2*sin(b*x+a)*cos(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(cot(b*x+a)
-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)
-cot(b*x+a))^(1/2),1/2*2^(1/2))-2*sin(b*x+a)*(cot(b*x+a)-csc(b*x+a))^(1/2)*
(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF(
(1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(b*x+a))*(d*tan(b*x
+a))^(1/2)/(cos(b*x+a)^2-1)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(\left(\cos(bx+a)^2-1\right)\sqrt{i\,d}F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+\left(\cos(bx+a)^2-1\right)\sqrt{-i\,d}F(\right.}{3\left(b\cos(bx+a)^2-b\right)}$$

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*((cos(b*x + a)^2 - 1)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin
(b*x + a)), -1) + (cos(b*x + a)^2 - 1)*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x
 + a) - I*sin(b*x + a)), -1) - sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x +
a))/(b*cos(b*x + a)^2 - b)
```

Sympy [F]

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(a+bx)}\csc^3(a+bx)\,dx$$

```
[In] integrate(csc(b*x+a)**3*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**3, x)
```

Maxima [F]

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\csc(bx+a)^3\,dx$$

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^3, x)
```

Giac [F]

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\csc(bx+a)^3\,dx$$

[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \frac{\sqrt{d\tan(a+b\,x)}}{\sin(a+b\,x)^3}\,dx$$

[In] int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^3,x)

[Out] int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^3, x)

3.63 $\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}}-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}+\frac{4\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{7b}$$

```
[Out] -4/7*d*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)-2/7*d*csc(b*x+a)^3/b/(d*tan(b*x+a)
)^(1/2)-4/7*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Ellipt
icF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2679, 2681, 2653, 2720}

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}-\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}}+\frac{4\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{7b}$$

```
[In] Int[Csc[a + b*x]^5*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-4*d*Csc[a + b*x])/(7*b*Sqrt[d*Tan[a + b*x]]) - (2*d*Csc[a + b*x]^3)/(7*b*
Sqrt[d*Tan[a + b*x]]) + (4*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[S
in[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(7*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}+\frac{6}{7}\int\csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx\\ &= -\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}}-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}+\frac{4}{7}\int\csc(a+bx)\sqrt{d\tan(a+bx)}\,dx\\ &= -\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}}-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}\\ &\quad+\frac{\left(4\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{7\sqrt{\sin(a+bx)}}\end{aligned}$$

$$= -\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}} - \frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}} + \frac{1}{7}\left(4\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx$$

$$= -\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}} - \frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}} + \frac{4\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{7b}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.18

$$\int\csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d\cos(2(a+bx))\csc^3(a+bx)\left((-2+\cos(2(a+bx)))\sec^2(a+bx)^{3/2}-4\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-\right.\right.}{7b\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}\left(-1+\tan^2(a+bx)\right)}$$

```
[In] Integrate[Csc[a + b*x]^5*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d*Cos[2*(a + b*x)]*Csc[a + b*x]^3*((-2 + Cos[2*(a + b*x)])*(Sec[a + b*x
]^2)^(3/2) - 4*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]
], -1]*Tan[a + b*x]^(7/2)))/(7*b*Sqrt[Sec[a + b*x]^2]*Sqrt[d*Tan[a + b*x]]*
(-1 + Tan[a + b*x]^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(116) = 232.

Time = 0.70 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.29

method	result
default	$\frac{\sqrt{-\frac{d(\csc(bx+a)-\cot(bx+a))}{\left(\csc^2(bx+a)\right)(1-\cos(bx+a))^2-1}}\left(\left(\csc^2(bx+a)\right)(1-\cos(bx+a))^2-1\right)\left(\sin^3(bx+a)\right)\left(\left(\csc^8(bx+a)\right)(1-\cos(bx+a))^8+32\left(\csc^3(bx+a\right.\right.}{56b(1-\cos(bx+a))^3\sqrt{\csc(bx}}$

```
[In] int(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/56/b*(-d/(csc(b*x+a)^2*(1-cos(b*x+a))^2-1)*(csc(b*x+a)-cot(b*x+a)))^(1/2)
*(csc(b*x+a)^2*(1-cos(b*x+a))^2-1)/(1-cos(b*x+a))^3*sin(b*x+a)^3*(csc(b*x+a
)^8*(1-cos(b*x+a))^8+32*csc(b*x+a)^3*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(2-2*c
sc(b*x+a)+2*cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+cs
```

```
c(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*(1-cos(b*x+a))^3+10*csc(b*x+a)^6*(1
-cos(b*x+a))^6-10*csc(b*x+a)^2*(1-cos(b*x+a))^2-1)/(csc(b*x+a)*(1-cos(b*x+a
))*(csc(b*x+a)^2*(1-cos(b*x+a))^2-1))^(1/2)/(csc(b*x+a)^3*(1-cos(b*x+a))^3-
csc(b*x+a)+cot(b*x+a))^(1/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.50

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx =$$
$$-\frac{2\left(2\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\sqrt{i\,d}F(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)+2\left(\cos(bx\right.\right.}{7\left(b\cos\right.}$$

```
[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
[Out] -2/7*(2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(I*d)*elliptic_f(arcsin
(cos(b*x + a) + I*sin(b*x + a)), -1) + 2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2
 + 1)*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - (2
*cos(b*x + a)^3 - 3*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos
(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [F]

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(a+bx)}\csc^5(a+bx)\,dx$$

```
[In] integrate(csc(b*x+a)**5*(d*tan(b*x+a))**(1/2),x)
[Out] Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**5, x)
```

Maxima [F]

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\csc(bx+a)^5\,dx$$

```
[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^5, x)
```

Giac [F]

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\csc(bx+a)^5\,dx$$

```
[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \frac{\sqrt{d\tan(a+b\,x)}}{\sin(a+b\,x)^5}\,dx$$

```
[In] int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^5,x)
[Out] int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^5, x)
```

3.64 $\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 277

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{45d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{45d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} + \frac{45d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} - \frac{45d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3}$$

```
[Out] 45/64*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-45/6
4*d^(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)+45/128*d
^(3/2)*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2
)-45/128*d^(3/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a)
)/b*2^(1/2)+45/16*d*(d*tan(b*x+a))^(1/2)/b-9/16*cos(b*x+a)^2*(d*tan(b*x+a))
^(5/2)/b/d-1/4*cos(b*x+a)^4*(d*tan(b*x+a))^(9/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2671, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{45d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{45d^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}b} + \frac{45d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}b} - \frac{45d^{3/2}\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}b} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3} + \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd}$$

```
[In] Int[Sin[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (45*d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]
*b) - (45*d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*S
qrt[2]*b) + (45*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d
*Tan[a + b*x]]])/(64*Sqrt[2]*b) - (45*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a +
 b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b) + (45*d*Sqrt[d*Tan[a
+ b*x]])/(16*b) - (9*Cos[a + b*x]^2*(d*Tan[a + b*x])^(5/2))/(16*b*d) - (Cos
[a + b*x]^4*(d*Tan[a + b*x])^(9/2))/(4*b*d^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{d\text{Subst}\left(\int \frac{x^{11/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3} + \frac{(9d)\text{Subst}\left(\int \frac{x^{7/2}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{8b}$$

$$= -\frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3} + \frac{(45d)\text{Subst}\left(\int \frac{x^{3/2}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{32b}$$

$$= \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3} - \frac{(45d^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\tan(a+bx)\right)}{32b}$$

$$= \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3} - \frac{(45d^3)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{16b}$$

$$= \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3} - \frac{(45d^2)\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{32b} - \frac{(45d^2)\,\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{32b}$$

$$= \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3} + \frac{(45d^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{(45d^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} - \frac{(45d^2)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64b} - \frac{(45d^2)\,\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64b}$$

$$= \frac{45d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} - \frac{45d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3} - \frac{(45d^{3/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} + \frac{(45d^{3/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b}$$

$$= \frac{45d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{45d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} + \frac{45d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} - \frac{45d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.44

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{d\csc(a+bx)\left(-143\sin(a+bx)-45\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}+45\log\left(\cos(a+\right.\right.}{}$$

```
[In] Integrate[Sin[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/64*(d*Csc[a + b*x]*(-143*Sin[a + b*x] - 45*ArcSin[Cos[a + b*x] - Sin[a +
 b*x]]*Sqrt[Sin[2*(a + b*x)]] + 45*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[S
in[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x)]] - 14*Sin[3*(a + b*x)] + Sin[5*(a +
 b*x)])*Sqrt[d*Tan[a + b*x]])/b
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. 2(213) = 426.

Time = 3.46 (sec) , antiderivative size = 890, normalized size of antiderivative = 3.21

method	result	size
default	Expression too large to display	890

```
[In] int(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/b*(d*tan(b*x+a))^(1/2)*(-16*2^(1/2)*cos(b*x+a)^4+68*cos(b*x+a)^2*2^(1
/2)+45*cot(b*x+a)*ln(-2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(
1/2)*csc(b*x+a)-2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*c
ot(b*x+a)-2*cot(b*x+a)+2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-4
5*cot(b*x+a)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(2*(-cos(b*x
+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*csc(b*x+a)+2*(-cos(b*x+a)*si
n(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*cot(b*x+a)-2*cot(b*x+a)+2)-90*cot(
b*x+a)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b*x+a)*2
^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+co
s(b*x+a)))+90*cot(b*x+a)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ar
ctan((-sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+c
os(b*x+a)-1)/(-1+cos(b*x+a)))+45*csc(b*x+a)*ln(-2*(-cos(b*x+a)*sin(b*x+a)/(
cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*csc(b*x+a)-2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*
x+a)+1)^2)^(1/2)*2^(1/2)*cot(b*x+a)-2*cot(b*x+a)+2)*(-cos(b*x+a)*sin(b*x+a)
/(cos(b*x+a)+1)^2)^(1/2)-45*csc(b*x+a)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+
1)^2)^(1/2)*ln(2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*cs
c(b*x+a)+2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*cot(b*x+
a)-2*cot(b*x+a)+2)+128*2^(1/2)-90*csc(b*x+a)*(-cos(b*x+a)*sin(b*x+a)/(cos(b
```

```
*x+a)+1)^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b
*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+90*csc(b*x+a)*(-cos(b*x+a)
*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((-sin(b*x+a)*2^(1/2)*(-cos(b*x+a
)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a))))*d*2^(1
/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 954, normalized size of antiderivative = 3.44

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/256*(45*I*(-d^6/b^4)^(1/4)*b*log(182250*d^5*cos(b*x + a)^2 + 182250*sqrt
(-d^6/b^4)*b^2*d^2*cos(b*x + a)*sin(b*x + a) - 91125*d^5 - 182250*(I*(-d^6/
b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) - I*(-d^6/b^4)^(3/4)*b^3*cos(b*x
 + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 45*I*(-d^6/b^4)^(1/4)*b*log(1
82250*d^5*cos(b*x + a)^2 + 182250*sqrt(-d^6/b^4)*b^2*d^2*cos(b*x + a)*sin(b
*x + a) - 91125*d^5 - 182250*(-I*(-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*
x + a) + I*(-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x
 + a))) - 45*(-d^6/b^4)^(1/4)*b*log(182250*d^5*cos(b*x + a)^2 - 182250*sqrt
(-d^6/b^4)*b^2*d^2*cos(b*x + a)*sin(b*x + a) - 91125*d^5 + 182250*((-d^6/b^
4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) + (-d^6/b^4)^(3/4)*b^3*cos(b*x + a
)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 45*(-d^6/b^4)^(1/4)*b*log(182250*
d^5*cos(b*x + a)^2 - 182250*sqrt(-d^6/b^4)*b^2*d^2*cos(b*x + a)*sin(b*x + a
) - 91125*d^5 - 182250*((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) +
(-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 4
5*(-d^6/b^4)^(1/4)*b*log(-91125*d^5 + 182250*((-d^6/b^4)^(1/4)*b*d^3*cos(b*
x + a)*sin(b*x + a) - (-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x +
 a)/cos(b*x + a))) + 45*(-d^6/b^4)^(1/4)*b*log(-91125*d^5 - 182250*((-d^6/b
^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) - (-d^6/b^4)^(3/4)*b^3*cos(b*x +
a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 45*I*(-d^6/b^4)^(1/4)*b*log(-911
25*d^5 - 182250*(I*(-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) + I*(-d
^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 45*I
*(-d^6/b^4)^(1/4)*b*log(-91125*d^5 - 182250*(-I*(-d^6/b^4)^(1/4)*b*d^3*cos(
b*x + a)*sin(b*x + a) - I*(-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b
*x + a)/cos(b*x + a))) + 16*(4*d*cos(b*x + a)^4 - 17*d*cos(b*x + a)^2 - 32*
d)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/b
```

Sympy [F(-1)]

Timed out.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**4*(d*tan(b*x+a))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.85

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{90\sqrt{2}d^{\frac{13}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+90\sqrt{2}d^{\frac{13}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+45\sqrt{2}d^{\frac{13}{2}}\log}{}$$

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] -1/128*(90*sqrt(2)*d^(13/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*
tan(b*x + a)))/sqrt(d)) + 90*sqrt(2)*d^(13/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*
sqrt(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 45*sqrt(2)*d^(13/2)*log(d*tan(
b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 45*sqrt(2)*d^(13/2)*
log(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 256*sqrt(d
*tan(b*x + a))*d^6 - 8*(17*(d*tan(b*x + a))^(5/2)*d^8 + 13*sqrt(d*tan(b*x +
 a))*d^10)/(d^4*tan(b*x + a)^4 + 2*d^4*tan(b*x + a)^2 + d^4))/(b*d^5)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.91

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{1}{128}d\left(\frac{90\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{90\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}\right.$$

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*d*(90*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
+ 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 90*sqrt(2)*sqrt(abs(d))*arctan(
-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/
b + 45*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a
))*sqrt(abs(d)) + abs(d))/b - 45*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) -
sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 256*sqrt(d*tan(b*x
+ a))/b - 8*(17*sqrt(d*tan(b*x + a))*d^4*tan(b*x + a)^2 + 13*sqrt(d*tan(b*x
 + a))*d^4)/((d^2*tan(b*x + a)^2 + d^2)^2*b))
```

Mupad [F(-1)]

Timed out.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx=\int \sin\left(a+b\,x\right)^4\,{\left(d\tan(a+b\,x)\right)}^{3/2}\,dx$$

```
[In] int(sin(a + b*x)^4*(d*tan(a + b*x))^(3/2),x)
[Out] int(sin(a + b*x)^4*(d*tan(a + b*x))^(3/2), x)
```

3.65 $\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 247

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{5d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{5d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} + \frac{5d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} - \frac{5d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd}$$

```
[Out] 5/8*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-5/8*d^(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)+5/16*d^(3/2)*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)-5/16*d^(3/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)+5/2*d*(d*tan(b*x+a))^(1/2)/b-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(5/2)/b/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2671, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx &= \frac{5d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} \\ &- \frac{5d^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}b} \\ &+ \frac{5d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b} \\ &- \frac{5d^{3/2}\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b} \\ &+ \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd}\end{aligned}$$

```
[In] Int[Sin[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (5*d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b
) - (5*d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[
2]*b) + (5*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[
a + b*x]]])/(8*Sqrt[2]*b) - (5*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] +
 Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) + (5*d*Sqrt[d*Tan[a + b*x]])/
(2*b) - (Cos[a + b*x]^2*(d*Tan[a + b*x])^(5/2))/(2*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{d\text{Subst}\left(\int \frac{x^{7/2}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{b}\\
&= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd} + \frac{(5d)\text{Subst}\left(\int \frac{x^{3/2}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{4b}\\
&= \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd}\\
&\quad - \frac{(5d^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\tan(a+bx)\right)}{4b}\\
&= \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd}\\
&\quad - \frac{(5d^3)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2b}\\
&= \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd}\\
&\quad - \frac{(5d^2)\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4b}\\
&\quad - \frac{(5d^2)\,\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4b}\\
&= \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd}\\
&\quad + \frac{(5d^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b}\\
&\quad + \frac{(5d^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b}\\
&\quad - \frac{(5d^2)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8b}\\
&\quad - \frac{(5d^2)\,\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8b}
\end{aligned}
$$

$$= \frac{5d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} - \frac{5d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd} - \frac{(5d^{3/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} + \frac{(5d^{3/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}$$

$$= \frac{5d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{5d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} + \frac{5d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} - \frac{5d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.46

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d\csc(a+bx)\left(17\sin(a+bx)+5\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-5\log\left(\right.\right.}{}$$

```
[In] Integrate[Sin[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (d*Csc[a + b*x]*(17*Sin[a + b*x] + 5*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sq
rt[Sin[2*(a + b*x)]] - 5*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a +
b*x)]]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)])*Sqrt[d*Tan[a + b*x]])/(8
*b)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. 2(187) = 374.

Time = 2.33 (sec) , antiderivative size = 1083, normalized size of antiderivative = 4.38

method	result	size
default	Expression too large to display	1083

```
[In] int(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/b*sin(b*x+a)*(4*cos(b*x+a)^2*sin(b*x+a)*2^(1/2)+5*(-cos(b*x+a)*sin(b*
x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(
b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+c
sc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)
+2)/(-1+cos(b*x+a)))*cos(b*x+a)-5*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)
^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+
3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)
-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))*c
os(b*x+a)+10*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((-sin(b
*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)
/(-1+cos(b*x+a)))*cos(b*x+a)-10*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(
1/2)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(
1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*cos(b*x+a)+16*sin(b*x+a)*2^(1/2)+5*(-co
s(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*co
t(b*x+a)-2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)
*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*
x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))-5*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1
)^2)^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a
)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*
x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)
))+10*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((-sin(b*x+a)*2
^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+co
s(b*x+a)))-10*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b
*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)
/(-1+cos(b*x+a))))*(d*tan(b*x+a))^(1/2)*d/(-1+cos(b*x+a))/(cos(b*x+a)+1)*2^
(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 942, normalized size of antiderivative = 3.81

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/32*(5*I*(-d^6/b^4)^(1/4)*b*log(250*d^5*cos(b*x + a)^2 + 250*sqrt(-d^6/b^
4)*b^2*d^2*cos(b*x + a)*sin(b*x + a) - 125*d^5 - 250*(I*(-d^6/b^4)^(1/4)*b*
d^3*cos(b*x + a)*sin(b*x + a) - I*(-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt
(d*sin(b*x + a)/cos(b*x + a))) - 5*I*(-d^6/b^4)^(1/4)*b*log(250*d^5*cos(b*x
 + a)^2 + 250*sqrt(-d^6/b^4)*b^2*d^2*cos(b*x + a)*sin(b*x + a) - 125*d^5 -
250*(-I*(-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) + I*(-d^6/b^4)^(3/
4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 5*(-d^6/b^4)^(1
/4)*b*log(250*d^5*cos(b*x + a)^2 - 250*sqrt(-d^6/b^4)*b^2*d^2*cos(b*x + a)*
sin(b*x + a) - 125*d^5 + 250*((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x +
 a) + (-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)
)) + 5*(-d^6/b^4)^(1/4)*b*log(250*d^5*cos(b*x + a)^2 - 250*sqrt(-d^6/b^4)*b
^2*d^2*cos(b*x + a)*sin(b*x + a) - 125*d^5 - 250*((-d^6/b^4)^(1/4)*b*d^3*co
s(b*x + a)*sin(b*x + a) + (-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b
*x + a)/cos(b*x + a))) - 5*(-d^6/b^4)^(1/4)*b*log(-125*d^5 + 250*((-d^6/b^4
)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) - (-d^6/b^4)^(3/4)*b^3*cos(b*x + a)
^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 5*(-d^6/b^4)^(1/4)*b*log(-125*d^5
- 250*((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) - (-d^6/b^4)^(3/4)*
b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 5*I*(-d^6/b^4)^(1/
4)*b*log(-125*d^5 - 250*(I*(-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a)
 + I*(-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))
) - 5*I*(-d^6/b^4)^(1/4)*b*log(-125*d^5 - 250*(-I*(-d^6/b^4)^(1/4)*b*d^3*co
s(b*x + a)*sin(b*x + a) - I*(-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin
(b*x + a)/cos(b*x + a))) - 16*(d*cos(b*x + a)^2 + 4*d)*sqrt(d*sin(b*x + a)/
cos(b*x + a)))/b
```

Sympy [F]

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(a+bx)})^{\frac{3}{2}}\sin^2{(a+bx)}\,dx$$

```
[In] integrate(sin(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(3/2)*sin(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.83

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{10\sqrt{2}d^{\frac{9}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+10\sqrt{2}d^{\frac{9}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+5\sqrt{2}d^{\frac{9}{2}}\log\left(d\,\mathrm{t}\right.}{}$$

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(10*sqrt(2)*d^(9/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*ta
n(b*x + a)))/sqrt(d)) + 10*sqrt(2)*d^(9/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 5*sqrt(2)*d^(9/2)*log(d*tan(b*x +
 a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 5*sqrt(2)*d^(9/2)*log(d*t
an(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan(b*x
+ a))*d^6/(d^2*tan(b*x + a)^2 + d^2) - 32*sqrt(d*tan(b*x + a))*d^4)/(b*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.91

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{1}{16}d\left(\frac{10\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{10\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\right.$$

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*d*(10*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) +
 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 10*sqrt(2)*sqrt(abs(d))*arctan(-
1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b
 + 5*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))
*sqrt(abs(d)) + abs(d))/b - 5*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) - sqr
t(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 8*sqrt(d*tan(b*x + a))
*d^2/((d^2*tan(b*x + a)^2 + d^2)*b) - 32*sqrt(d*tan(b*x + a))/b)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \sin(a+b\,x)^2\,(d\tan(a+b\,x))^{3/2}\,dx$$

```
[In] int(sin(a + b*x)^2*(d*tan(a + b*x))^(3/2),x)
[Out] int(sin(a + b*x)^2*(d*tan(a + b*x))^(3/2), x)
```

3.66 $\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] 2*d*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 30}

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

```
[In] Int[Csc[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, d\tan(a+bx)\right)}{b}\\
&= \frac{2d\sqrt{d\tan(a+bx)}}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$
\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\sqrt{d\tan(a+bx)}}{b}
$$

```
[In] Integrate[Csc[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
[Out] (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{2d\sqrt{d\tan(bx+a)}}{b}$	17
default	$\frac{2d\sqrt{d\tan(bx+a)}}{b}$	17

```
[In] int(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*d*(d*tan(b*x+a))^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$
\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,d\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{b}
$$

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] 2*d*sqrt(d*sin(b*x + a)/cos(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,(d\tan{(bx+a)})^{\frac{3}{2}}}{b\tan{(bx+a)}}$$

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] 2*(d*tan(b*x + a))^(3/2)/(b*tan(b*x + a))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,\sqrt{d\tan{(bx+a)}}d}{b}$$

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] 2*sqrt(d*tan(b*x + a))*d/b
```

Mupad [B] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{b}$$

```
[In] int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^2,x)
[Out] (2*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/b
```

3.67 $\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d^3}{3b(d\tan(a+bx))^{3/2}} + \frac{2d\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] 2*d*(d*tan(b*x+a))^(1/2)/b-2/3*d^3/b/(d*tan(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2671, 14}

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{2d^3}{3b(d\tan(a+bx))^{3/2}}$$

```
[In] Int[Csc[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^3)/(3*b*(d*Tan[a + b*x])^(3/2)) + (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
```

```
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{d\text{Subst}\left(\int \frac{d^2+x^2}{x^{5/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= \frac{d\text{Subst}\left(\int \left(\frac{d^2}{x^{5/2}}+\frac{1}{\sqrt{x}}\right)\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{2d^3}{3b(d\tan(a+bx))^{3/2}}+\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d(-4+\csc^2(a+bx))\sqrt{d\tan(a+bx)}}{3b}$$

```
[In] Integrate[Csc[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
[Out] (-2*d*(-4 + Csc[a + b*x]^2)*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{2\sqrt{d\tan(bx+a)}\,d\left(4\left(\cot^2(bx+a)\right)-3\left(\csc^2(bx+a)\right)\right)}{3b}$	38

```
[In] int(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3/b*(d*tan(b*x+a))^(1/2)*d*(4*cot(b*x+a)^2-3*csc(b*x+a)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(4\,d\cos\left(bx+a\right)^2-3\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{3\left(b\cos\left(bx+a\right)^2-b\right)}$$

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] 2/3*(4*d*cos(b*x + a)^2 - 3*d)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(b*x
 + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**4*(d*tan(b*x+a))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\,d^3\left(\frac{1}{(d\tan(bx+a))^{\frac{3}{2}}}-\frac{3\,\sqrt{d\tan(bx+a)}}{d^2}\right)}{3\,b}$$

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] -2/3*d^3*(1/(d*tan(b*x + a))^(3/2) - 3*sqrt(d*tan(b*x + a))/d^2)/b
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2}{3}\,d\left(\frac{3\,\sqrt{d\tan\left(bx+a\right)}}{b} - \frac{d}{\sqrt{d\tan\left(bx+a\right)}b\tan\left(bx+a\right)}\right)$$

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] 2/3*d*(3*sqrt(d*tan(b*x + a))/b - d/(sqrt(d*tan(b*x + a))*b*tan(b*x + a)))
```

Mupad [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.44

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{8\,d\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}\,\left(11\,\cos\left(2\,a+2\,b\,x\right)-5\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)-7\right)}{3\,b\,\left(15\,\cos\left(2\,a+2\,b\,x\right)-6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)-10\right)}$$

```
[In] int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^4,x)
[Out] (8*d*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2)*(11*cos(2*a + 2*b*
x) - 5*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) - 7))/(3*b*(15*cos(2*a + 2*b*x)
- 6*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) - 10))
```

3.68 $\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d^5}{7b(d\tan(a+bx))^{7/2}} - \frac{4d^3}{3b(d\tan(a+bx))^{3/2}} + \frac{2d\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] 2*d*(d*tan(b*x+a))^(1/2)/b-2/7*d^5/b/(d*tan(b*x+a))^(7/2)-4/3*d^3/b/(d*tan(
b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 276}

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d^5}{7b(d\tan(a+bx))^{7/2}} - \frac{4d^3}{3b(d\tan(a+bx))^{3/2}} + \frac{2d\sqrt{d\tan(a+bx)}}{b}$$

```
[In] Int[Csc[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^5)/(7*b*(d*Tan[a + b*x])^(7/2)) - (4*d^3)/(3*b*(d*Tan[a + b*x])^(3/2)
) + (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d\text{Subst}\left(\int \frac{(d^2+x^2)^2}{x^{9/2}}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= \frac{d\text{Subst}\left(\int \left(\frac{d^4}{x^{9/2}}+\frac{2d^2}{x^{5/2}}+\frac{1}{\sqrt{x}}\right)dx, x, d\tan(a+bx)\right)}{b}\\ &= -\frac{2d^5}{7b(d\tan(a+bx))^{7/2}}-\frac{4d^3}{3b(d\tan(a+bx))^{3/2}}+\frac{2d\sqrt{d\tan(a+bx)}}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d(-32+8\csc^2(a+bx)+3\csc^4(a+bx))\sqrt{d\tan(a+bx)}}{21b}$$

```
[In] Integrate[Csc[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
[Out] (-2*d*(-32 + 8*Csc[a + b*x]^2 + 3*Csc[a + b*x]^4)*Sqrt[d*Tan[a + b*x]])/(21
*b)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{2(\csc^4(bx+a))\sqrt{d\tan(bx+a)}\,d(32(\cos^4(bx+a))-56(\cos^2(bx+a))+21)}{21b}$	47

```
[In] int(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/b*csc(b*x+a)^4*(d*tan(b*x+a))^(1/2)*d*(32*cos(b*x+a)^4-56*cos(b*x+a)^2
+21)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(32\,d\cos\left(bx+a\right)^4-56\,d\cos\left(bx+a\right)^2+21\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{21\left(b\cos\left(bx+a\right)^4-2\,b\cos\left(bx+a\right)^2+b\right)}$$

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(32*d*cos(b*x + a)^4 - 56*d*cos(b*x + a)^2 + 21*d)*sqrt(d*sin(b*x + a)
/cos(b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**6*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,d^5\left(\frac{21\,\sqrt{d\tan(bx+a)}}{d^4}-\frac{14\,d^2\tan(bx+a)^2+3\,d^2}{(d\tan(bx+a))^{\frac{7}{2}}\,d^2}\right)}{21\,b}$$

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21*d^5*(21*sqrt(d*tan(b*x + a))/d^4 - (14*d^2*tan(b*x + a)^2 + 3*d^2)/((d
*tan(b*x + a))^(7/2)*d^2))/b
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2}{21}\,d\left(\frac{21\sqrt{d\tan(bx+a)}}{b} - \frac{14\,d^4\tan(bx+a)^2+3\,d^4}{\sqrt{d\tan(bx+a)}bd^3\tan(bx+a)^3}\right)$$

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/21*d*(21*sqrt(d*tan(b*x + a))/b - (14*d^4*tan(b*x + a)^2 + 3*d^4)/(sqrt(d
*tan(b*x + a))*b*d^3*tan(b*x + a)^3))
```

Mupad [B] (verification not implemented)

Time = 7.08 (sec) , antiderivative size = 292, normalized size of antiderivative = 4.63

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{\left(\frac{20\,d}{21\,b}-\frac{64\,d\,e^{a\,2i+b\,x\,2i}}{21\,b}\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{e^{a\,2i+b\,x\,2i}-1} + \frac{20\,d\,\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{21\,b\,\left(e^{a\,2i+b\,x\,2i}-1\right)^2} - \frac{24\,d\,\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{7\,b\,\left(e^{a\,2i+b\,x\,2i}-1\right)^3} - \frac{16\,d\,\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{7\,b\,\left(e^{a\,2i+b\,x\,2i}-1\right)^4}$$

```
[In] int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^6,x)
```

```
[Out] (20*d*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i
 + b*x*2i) + 1))^(1/2))/(21*b*(exp(a*2i + b*x*2i) - 1)^2) - (((20*d)/(21*b)
 - (64*d*exp(a*2i + b*x*2i))/(21*b))*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(ex
p(a*2i + b*x*2i) + 1))^(1/2))/(exp(a*2i + b*x*2i) - 1) - (24*d*(exp(a*2i +
b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(
1/2))/(7*b*(exp(a*2i + b*x*2i) - 1)^3) - (16*d*(exp(a*2i + b*x*2i) + 1)*(-(
d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(7*b*(exp(
a*2i + b*x*2i) - 1)^4)
```

3.69 $\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} - \frac{7d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] 7/2*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*P
i+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2*d*
sin(b*x+a)^3*(d*tan(b*x+a))^(1/2)/b+7/3*d^3*sin(b*x+a)^3/b/(d*tan(b*x+a))^(
3/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2674, 2678, 2681, 2652, 2719}

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} - \frac{7d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[In] Int[Sin[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (7*d^3*Sin[a + b*x]^3)/(3*b*(d*Tan[a + b*x])^(3/2)) - (7*d^2*EllipticE[a -
Pi/4 + b*x, 2]*Sin[a + b*x])/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x
]]) + (2*d*Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b} - (7d^2)\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx \\
&= \frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} + \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{1}{2}(7d^2)\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx \\
&= \frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} + \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b} \\
&\quad - \frac{\left(7d^2\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}
\end{aligned}$$

$$= \frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} + \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{(7d^2\sin(a+bx))\int\sqrt{\sin(2a+2bx)}\,dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$= \frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} - \frac{7d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\left(-28\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sec(a+bx)+2\cos(a+bx)(13+\cos(2(a+\dots\right.}{12b\sqrt{\sec^2(a+bx)}}$$

```
[In] Integrate[Sin[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] ((-28*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x] + 2*Co
s[a + b*x]*(13 + Cos[2*(a + b*x)])*Sqrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(
3/2))/(12*b*Sqrt[Sec[a + b*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(123) = 246.

Time = 0.82 (sec) , antiderivative size = 409, normalized size of antiderivative = 3.72

method	result
default	$-\frac{\sin(bx+a)\left(42\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\dots\right.\right.}{}$

```
[In] int(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/b*sin(b*x+a)*(42*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b
*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+
a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-21*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*
x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b
*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-2*2^(1/2)*cos(b*x+a)^4+42*(
1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)
-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-2
1*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x
```

```
+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2)
)+11*cos(b*x+a)^2*2^(1/2)-21*2^(1/2)*cos(b*x+a)+12*2^(1/2))*(d*tan(b*x+a))^
(1/2)*d/(cos(b*x+a)^2-1)*2^(1/2)
```

Fricas [F]

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sin(bx+a)^3\,dx$$

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] integral(-(d*cos(b*x + a)^2 - d)*sqrt(d*tan(b*x + a))*sin(b*x + a)*tan(b*x
+ a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sin(bx+a)^3\,dx$$

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^(3/2)*sin(b*x + a)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]e
xt_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0]ext_r
educe
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \sin\left(a+b\,x\right)^3\,\left(d\tan(a+b\,x)\right)^{3/2}\,dx$$

```
[In] int(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2),x)
[Out] int(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2), x)
```

3.70 $\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{3d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] 3*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2*d*si
n(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2674, 2681, 2652, 2719}

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{3d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Sin[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-3*d^2*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]
]*Sqrt[d*Tan[a + b*x]]) + (2*d*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \left(3d^2\right)\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx \\
&= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(3d^2\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(3d^2\sin(a+bx)\right)\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} \\
&= -\frac{3d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\cos(a+bx)\left(-1+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)(d\tan(a+bx))^{3/2}}{b}$$

```
[In] Integrate[Sin[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
[Out] (-2*Cos[a + b*x]*(-1 + Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sq
rt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(3/2))/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(95) = 190.

Time = 0.77 (sec) , antiderivative size = 395, normalized size of antiderivative = 5.20

method	result
default	$-\frac{\sin(bx+a)\left(6\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right.\right.}{}$

```
[In] int(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2/b*sin(b*x+a)*(6*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x
+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a)
)^(1/2),1/2*2^(1/2))*cos(b*x+a)-3*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a
)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+
a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+6*(1+csc(b*x+a)-cot(b*x+a))^(1
/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Elliptic
E((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-3*(cot(b*x+a)-csc(b*x+a))^(1
/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*Ellipt
icF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2*2^(1/2)-3*2^(
1/2)*cos(b*x+a)+2*2^(1/2))*(d*tan(b*x+a))^(1/2)*d/(cos(b*x+a)^2-1)*2^(1/2)
```

Fricas [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sin(bx+a)\,dx$$

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(b*x + a))*d*sin(b*x + a)*tan(b*x + a), x)
```

Sympy [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(a+bx))^{\frac{3}{2}}\sin(a+bx)\,dx$$

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))**(3/2),x)
[Out] Integral((d*tan(a + b*x))**(3/2)*sin(a + b*x), x)
```

Maxima [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sin(bx+a)\,dx$$

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^(3/2)*sin(b*x + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]e
xt_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0]ext_r
educe
```

Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \sin\left(a+b\,x\right)\,\left(d\tan(a+b\,x)\right)^{3/2}\,dx$$

```
[In] int(sin(a + b*x)*(d*tan(a + b*x))^(3/2),x)
[Out] int(sin(a + b*x)*(d*tan(a + b*x))^(3/2), x)
```

3.71 $\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] 2*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2*d*si
n(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2673, 2681, 2652, 2719}

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{2d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Csc[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^2*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]
]*Sqrt[d*Tan[a + b*x]]) + (2*d*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2673

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Dist[b^2*((m + 2)/(a^2*(n - 1))), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \left(2d^2\right)\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx \\
&= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(2d^2\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(2d^2\sin(a+bx)\right)\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} \\
&= -\frac{2d^2E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$
$$-\frac{2\cos(a+bx)\left(-3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)(d\tan(a+bx))^{3/2}}{3b}$$

```
[In] Integrate[Csc[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
[Out] (-2*Cos[a + b*x]*(-3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*
Sqrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(95) = 190.

Time = 0.62 (sec) , antiderivative size = 381, normalized size of antiderivative = 5.01

method	result
default	$-\frac{\sin(bx+a)\left(2\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\right.\right.}{}$

```
[In] int(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/b*sin(b*x+a)*(2*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a
))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^
(1/2),1/2*2^(1/2))*cos(b*x+a)-(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+
cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-c
ot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+2*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*
(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1
+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-(cot(b*x+a)-csc(b*x+a))^(1/2)*(-
csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1
+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-2^(1/2)*cos(b*x+a)+2^(1/2))*(d*t
an(b*x+a))^(1/2)*d/(cos(b*x+a)^2-1)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.79

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{-i\sqrt{id}dE(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)+i\sqrt{-id}dE(\arcsin(\cos(bx+a)-i\sin$$

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(I*d)*d*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + I*s
qrt(-I*d)*d*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) + I*sqrt(
I*d)*d*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) - I*sqrt(-I*d)
*d*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) + 2*d*sqrt(d*sin(b
*x + a)/cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(a+bx))^{\frac{3}{2}}\csc(a+bx)\,dx$$

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(3/2)*csc(a + b*x), x)
```

Maxima [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\csc(bx+a)\,dx$$

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a), x)
```

Giac [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\csc(bx+a)\,dx$$

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \frac{(d\tan(a+b\,x))^{3/2}}{\sin(a+b\,x)}\,dx$$

```
[In] int((d*tan(a + b*x))^(3/2)/sin(a + b*x),x)
[Out] int((d*tan(a + b*x))^(3/2)/sin(a + b*x), x)
```

3.72 $\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{4d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] -4*d^2*cos(b*x+a)/b/(d*tan(b*x+a))^(1/2)+4*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/
sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b
*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2*d*csc(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2673, 2681, 2650, 2652, 2719}

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{4d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[In] Int[Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-4*d^2*Cos[a + b*x])/(b*Sqrt[d*Tan[a + b*x]]) - (4*d^2*EllipticE[a - Pi/4
+ b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]]) + (
2*d*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2673

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Dist[b^2*((m + 2)/(a^2*(n - 1))), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b} + (2d^2)\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b} + \frac{\left(2d^2\sqrt{\sin(a+bx)}\right)\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$
\begin{aligned}
&= -\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b} \\
&\quad - \frac{\left(4d^2\sqrt{\sin(a+bx)}\right)\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= -\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{(4d^2\sin(a+bx))\int\sqrt{\sin(2a+2bx)}\,dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} \\
&= -\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{4d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.64 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.70

$$
\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx =
$$

$$
-\frac{2\cos(a+bx)\left(-6+3\csc^2(a+bx)+4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)(d\tan}{3b}
$$

```
[In] Integrate[Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]*(-6 + 3*Csc[a + b*x]^2 + 4*Hypergeometric2F1[3/4, 3/2, 7/4
, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(119) = 238.

Time = 0.67 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.64

method	result
default	$-\frac{\csc(bx+a)\sqrt{d\tan(bx+a)}\,d\left(-4\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+}\right.\right.}{}$

```
[In] int(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*csc(b*x+a)*(d*tan(b*x+a))^(1/2)*d*(-4*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*
(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1
+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+2*(cot(b*x+a)-csc(b*x
+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)
*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-4*(1+csc
(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(
```

```
b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*(cot
(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-co
t(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*2^
(1/2)*cos(b*x+a)-2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.73

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{2\left(i\sqrt{i}\,ddE(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\,|-1)\sin\left(bx+a\right)-i\sqrt{-i}\,ddE(\arcsin\left(\cos\left(bx+a\right)-i\right.\right.}{}$$

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(I*sqrt(I*d)*d*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin
(b*x + a) - I*sqrt(-I*d)*d*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a))
, -1)*sin(b*x + a) - I*sqrt(I*d)*d*elliptic_f(arcsin(cos(b*x + a) + I*sin(b
*x + a)), -1)*sin(b*x + a) + I*sqrt(-I*d)*d*elliptic_f(arcsin(cos(b*x + a)
- I*sin(b*x + a)), -1)*sin(b*x + a) + (2*d*cos(b*x + a)^2 - d)*sqrt(d*sin(b
*x + a)/cos(b*x + a)))/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}\csc\left(bx+a\right)^3\,dx$$

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a)^3, x)
```

Giac [F]

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\csc(bx+a)^3\,dx$$

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \frac{(d\tan(a+b\,x))^{3/2}}{\sin(a+b\,x)^3}\,dx$$

```
[In] int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^3,x)
[Out] int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^3, x)
```

3.73 $\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 277

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{77d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{77d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{77d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{77d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{77d(d\tan(a+bx))^{3/2}}{48b} - \frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3}$$

```
[Out] 77/64*d^(5/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-77/6
4*d^(5/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-77/128*d
^(5/2)*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2
)+77/128*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a)
)/b*2^(1/2)+77/48*d*(d*tan(b*x+a))^(3/2)/b-11/16*cos(b*x+a)^2*(d*tan(b*x+a)
)^(7/2)/b/d-1/4*cos(b*x+a)^4*(d*tan(b*x+a))^(11/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2671, 294, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = &\frac{77d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b}\\ &-\frac{77d^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}b}\\ &-\frac{77d^{5/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}b}\\ &+\frac{77d^{5/2}\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}b}\\ &-\frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3}+\frac{77d(d\tan(a+bx))^{3/2}}{48b}\\ &-\frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd}\end{aligned}$$

```
[In] Int[Sin[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (77*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]
*b) - (77*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*S
qrt[2]*b) - (77*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d
*Tan[a + b*x]]])/(64*Sqrt[2]*b) + (77*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a +
 b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b) + (77*d*(d*Tan[a + b*
x])^(3/2))/(48*b) - (11*Cos[a + b*x]^2*(d*Tan[a + b*x])^(7/2))/(16*b*d) - (
Cos[a + b*x]^4*(d*Tan[a + b*x])^(11/2))/(4*b*d^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\text{Subst}\left(\int \frac{x^{13/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}\\
&= -\frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3}+\frac{(11d)\text{Subst}\left(\int \frac{x^{9/2}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{8b}\\
&= -\frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd}-\frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3}\\
&\quad+\frac{(77d)\text{Subst}\left(\int \frac{x^{5/2}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{32b}\\
&= \frac{77d(d\tan(a+bx))^{3/2}}{48b}-\frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd}\\
&\quad-\frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3}-\frac{(77d^3)\,\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{32b}\\
&= \frac{77d(d\tan(a+bx))^{3/2}}{48b}-\frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd}\\
&\quad-\frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3}-\frac{(77d^3)\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{16b}\\
&= \frac{77d(d\tan(a+bx))^{3/2}}{48b}-\frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd}\\
&\quad-\frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3}+\frac{(77d^3)\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{32b}\\
&\quad-\frac{(77d^3)\,\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{32b}
\end{aligned}$$

$$= \frac{77d(d\tan(a+bx))^{3/2}}{48b} - \frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3} - \frac{(77d^{5/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} - \frac{(77d^{5/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} - \frac{(77d^3)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64b} - \frac{(77d^3)\,\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64b}$$

$$= -\frac{77d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{77d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{77d(d\tan(a+bx))^{3/2}}{48b} - \frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3} - \frac{(77d^{5/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} + \frac{(77d^{5/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b}$$

$$= \frac{77d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{77d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{77d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{77d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{77d(d\tan(a+bx))^{3/2}}{48b} - \frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.51

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{d\Big(128+204\cos^2(a+bx)+231\arcsin(\cos(a+bx)-\sin(a+bx))\cot(a+bx)\csc(a+bx)\sqrt{\sin}}{}$$

```
[In] Integrate[Sin[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (d*(128 + 204*Cos[a + b*x]^2 + 231*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Cot[
a + b*x]*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]] + 231*Cot[a + b*x]*Csc[a + b*x
]*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a +
 b*x)]] - 6*Cot[a + b*x]*Sin[4*(a + b*x)])*(d*Tan[a + b*x])^(3/2))/(192*b)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(213) = 426.

Time = 48.45 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.90

method	result
default	$-\frac{\tan(bx+a)\sqrt{d\tan(bx+a)}\left(48(\cos^5(bx+a))\sqrt{2}-48\sqrt{2}\,(\cos^4(bx+a))-228(\cos^3(bx+a))\sqrt{2}+228(\cos^2(bx+a))\sqrt{2}-462\sqrt{-\frac{\cos(bx+a)}{(\cos(bx}}\right.}{}$

```
[In] int(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/384/b*tan(b*x+a)*(d*tan(b*x+a))^(1/2)*(48*cos(b*x+a)^5*2^(1/2)-48*2^(1/2
)*cos(b*x+a)^4-228*cos(b*x+a)^3*2^(1/2)+228*cos(b*x+a)^2*2^(1/2)-462*(-cos(
b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((-sin(b*x+a)*2^(1/2)*(-cos
(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*c
os(b*x+a)+462*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b
*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)
/(-1+cos(b*x+a)))*cos(b*x+a)-231*cos(b*x+a)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*
x+a)+1)^2)^(1/2)*ln(2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/
2)*csc(b*x+a)+2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*cot
(b*x+a)-2*cot(b*x+a)+2)+231*cos(b*x+a)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+
1)^2)^(1/2)*ln(-2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*c
sc(b*x+a)-2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*cot(b*x
+a)-2*cot(b*x+a)+2)-128*2^(1/2)*cos(b*x+a)+128*2^(1/2))/(-1+cos(b*x+a))*d^2
*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 1048, normalized size of antiderivative = 3.78

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/768*(231*(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(-456533/2*d^8*cos(b*x + a)
*sin(b*x + a) + 456533/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-d^10/b^
4) + 456533/2*((-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x + a) - (-d^10/b
^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 231*(-d^
10/b^4)^(1/4)*b*cos(b*x + a)*log(-456533/2*d^8*cos(b*x + a)*sin(b*x + a) +
456533/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-d^10/b^4) - 456533/2*((
-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x + a) - (-d^10/b^4)^(3/4)*b^3*co
s(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 231*I*(-d^10/b^4)^(1/4)*
b*cos(b*x + a)*log(-456533/2*d^8*cos(b*x + a)*sin(b*x + a) - 456533/4*(2*b^
2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-d^10/b^4) - 456533/2*(I*(-d^10/b^4)^(
1/4)*b*d^5*cos(b*x + a)*sin(b*x + a) + I*(-d^10/b^4)^(3/4)*b^3*cos(b*x + a)
^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 231*I*(-d^10/b^4)^(1/4)*b*cos(b*x
+ a)*log(-456533/2*d^8*cos(b*x + a)*sin(b*x + a) - 456533/4*(2*b^2*d^3*cos(
b*x + a)^2 - b^2*d^3)*sqrt(-d^10/b^4) - 456533/2*(-I*(-d^10/b^4)^(1/4)*b*d^
5*cos(b*x + a)*sin(b*x + a) - I*(-d^10/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(
d*sin(b*x + a)/cos(b*x + a))) - 231*(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(45
6533*d^8 + 913066*((-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)^2 - (-d^10/b^4)^(3/
4)*b^3*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 231*
(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(456533*d^8 - 913066*((-d^10/b^4)^(1/4)
*b*d^5*cos(b*x + a)^2 - (-d^10/b^4)^(3/4)*b^3*cos(b*x + a)*sin(b*x + a))*sq
rt(d*sin(b*x + a)/cos(b*x + a))) + 231*I*(-d^10/b^4)^(1/4)*b*cos(b*x + a)*l
og(456533*d^8 - 913066*(I*(-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)^2 + I*(-d^10
/b^4)^(3/4)*b^3*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)
)) - 231*I*(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(456533*d^8 - 913066*(-I*(-d
^10/b^4)^(1/4)*b*d^5*cos(b*x + a)^2 - I*(-d^10/b^4)^(3/4)*b^3*cos(b*x + a)*
sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 16*(12*d^2*cos(b*x + a)^
4 - 57*d^2*cos(b*x + a)^2 - 32*d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))*sin(b
*x + a))/(b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**4*(d*tan(b*x+a))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.87

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{231\,d^8\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\right.}{\sqrt{d}}\right.}{3}$$

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
[Out] -1/384*(231*d^8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*t
an(b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a
) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b
*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 256*(d*tan(b
*x + a))^(3/2)*d^6 - 24*(19*(d*tan(b*x + a))^(7/2)*d^8 + 15*(d*tan(b*x + a)
)^(3/2)*d^10)/(d^4*tan(b*x + a)^4 + 2*d^4*tan(b*x + a)^2 + d^4))/(b*d^5)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{1}{384}\,d^2\left(\frac{462\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd}+\frac{462\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd}\right.$$

[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="giac")

[Out] -1/384*d^2*(462*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) + 462*sqrt(2)*abs(d)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) - 231*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) + 231*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) - 256*sqrt(d*tan(b*x + a))*tan(b*x + a)/b - 24*(19*sqrt(d*tan(b*x + a))*d^4*tan(b*x + a)^3 + 15*sqrt(d*tan(b*x + a))*d^4*tan(b*x + a))/((d^2*tan(b*x + a)^2 + d^2)^2*b))

Mupad [F(-1)]

Timed out.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \sin\left(a+b\,x\right)^4\left(d\tan(a+b\,x)\right)^{5/2}\,dx$$

[In] int(sin(a + b*x)^4*(d*tan(a + b*x))^(5/2),x)

[Out] int(sin(a + b*x)^4*(d*tan(a + b*x))^(5/2), x)

3.74 $\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 247

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{7d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{7d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{7d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{7d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd}$$

```
[Out] 7/8*d^(5/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-7/8*d^
(5/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-7/16*d^(5/2)
*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)+7/16
*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1
/2)+7/6*d*(d*tan(b*x+a))^(3/2)/b-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(7/2)/b/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2671, 294, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{7d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{7d^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}b} - \frac{7d^{5/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b} + \frac{7d^{5/2}\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b} + \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd}$$

```
[In] Int[Sin[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (7*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b
) - (7*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[
2]*b) - (7*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[
a + b*x]]])/(8*Sqrt[2]*b) + (7*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] +
 Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) + (7*d*(d*Tan[a + b*x])^(3/2)
)/(6*b) - (Cos[a + b*x]^2*(d*Tan[a + b*x])^(7/2))/(2*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{d\text{Subst}\left(\int \frac{x^{9/2}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} + \frac{(7d)\text{Subst}\left(\int \frac{x^{5/2}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{4b}$$

$$= \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} - \frac{(7d^3)\,\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{4b}$$

$$= \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} - \frac{(7d^3)\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2b}$$

$$= \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} + \frac{(7d^3)\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4b} - \frac{(7d^3)\,\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4b}$$

$$= \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} - \frac{(7d^{5/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} - \frac{(7d^{5/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} - \frac{(7d^3)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8b} - \frac{(7d^3)\,\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8b}$$

$$= -\frac{7d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{7d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} - \frac{(7d^{5/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} + \frac{(7d^{5/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}$$

$$= \frac{7d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{7d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{7d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{7d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.51

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{d\Big(16+12\cos^2(a+bx)+21\arcsin(\cos(a+bx)-\sin(a+bx))\cot(a+bx)\csc(a+bx)\sqrt{\sin(2(a+bx))}\cdots}{\cdots}$$

```
[In] Integrate[Sin[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (d*(16 + 12*Cos[a + b*x]^2 + 21*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Cot[a +
 b*x]*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]] + 21*Cot[a + b*x]*Csc[a + b*x]*Lo
g[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x
)]])*(d*Tan[a + b*x])^(3/2))/(24*b)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(187) = 374$.

Time = 3.59 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.02

method	result
default	$\frac{\tan(bx+a)\sqrt{d\tan(bx+a)}\left(12\left(\cos^3(bx+a)\right)\sqrt{2}+21\cos(bx+a)\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\ \ln\left(2\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\ \sqrt{2}\csc(bx+a)+2\right.\right.}{}$

[In] int(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/48/b*tan(b*x+a)*(d*tan(b*x+a))^(1/2)*(12*cos(b*x+a)^3*2^(1/2)+21*cos(b*x+a)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*csc(b*x+a)+2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*cot(b*x+a)-2*cot(b*x+a)+2)-21*cos(b*x+a)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*csc(b*x+a)-2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*cot(b*x+a)-2*cot(b*x+a)+2)-12*cos(b*x+a)^2*2^(1/2)+42*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((-sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*cos(b*x+a)-42*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*cos(b*x+a)+16*2^(1/2)*cos(b*x+a)-16*2^(1/2))/(-1+cos(b*x+a))*d^2*2^(1/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 1035, normalized size of antiderivative = 4.19

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Too large to display}$$

[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")

[Out] -1/96*(21*(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(-343/2*d^8*cos(b*x + a)*sin(b*x + a) + 343/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-d^10/b^4) + 343/2*((-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x + a) - (-d^10/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 21*(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(-343/2*d^8*cos(b*x + a)*sin(b*x + a) + 343/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-d^10/b^4) - 343/2*((-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x + a) - (-d^10/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 21*I*(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(-34

```
3/2*d^8*cos(b*x + a)*sin(b*x + a) - 343/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d
^3)*sqrt(-d^10/b^4) - 343/2*(I*(-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x
 + a) + I*(-d^10/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x
 + a))) + 21*I*(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(-343/2*d^8*cos(b*x + a)
*sin(b*x + a) - 343/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-d^10/b^4)
- 343/2*(-I*(-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x + a) - I*(-d^10/b^
4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 21*(-d^10
/b^4)^(1/4)*b*cos(b*x + a)*log(343*d^8 + 686*((-d^10/b^4)^(1/4)*b*d^5*cos(b
*x + a)^2 - (-d^10/b^4)^(3/4)*b^3*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x
 + a)/cos(b*x + a))) + 21*(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(343*d^8 - 68
6*((-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)^2 - (-d^10/b^4)^(3/4)*b^3*cos(b*x +
 a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 21*I*(-d^10/b^4)^(1/
4)*b*cos(b*x + a)*log(343*d^8 - 686*(I*(-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)
^2 + I*(-d^10/b^4)^(3/4)*b^3*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)
/cos(b*x + a))) - 21*I*(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(343*d^8 - 686*(
-I*(-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)^2 - I*(-d^10/b^4)^(3/4)*b^3*cos(b*x
 + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 16*(3*d^2*cos(b*x
+ a)^2 + 4*d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))*sin(b*x + a))/(b*cos(b*x
+ a))
```

Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**2*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.85

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{21\,d^6\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}\right.}{\sqrt{d}}\right.}{48\,bd^3}$$

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/48*(21*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a)
+ sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x
 + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 24*(d*tan(b*x
+ a))^(3/2)*d^6/(d^2*tan(b*x + a)^2 + d^2) - 32*(d*tan(b*x + a))^(3/2)*d^4)
/(b*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.02

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{1}{48}\left(\frac{24\sqrt{d\tan(bx+a)}d^2\tan(bx+a)}{\left(d^2\tan(bx+a)^2+d^2\right)b} - \frac{42\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd} - \frac{42\ldots}{}\right.$$

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/48*(24*sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)/((d^2*tan(b*x + a)^2 + d^2)*
b) - 42*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*s
qrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) - 42*sqrt(2)*abs(d)^(3/2)*arctan(-
1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(
b*d) + 21*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x
+ a))*sqrt(abs(d)) + abs(d))/(b*d) - 21*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x
+ a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) + 32*sqrt(
d*tan(b*x + a))*tan(b*x + a)/b)*d^2
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \sin(a+b\,x)^2\,(d\tan(a+b\,x))^{5/2}\,dx$$

```
[In] int(sin(a + b*x)^2*(d*tan(a + b*x))^(5/2),x)
```

```
[Out] int(sin(a + b*x)^2*(d*tan(a + b*x))^(5/2), x)
```

3.75 $\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx=\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] 2/3*d*(d*tan(b*x+a))^(3/2)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 30}

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx=\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

```
[In] Int[Csc[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{d\text{Subst}\left(\int \sqrt{x}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= \frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

```
[In] Integrate[Csc[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
[Out] (2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2d(d\tan(bx+a))^{\frac{3}{2}}}{3b}$	17
default	$\frac{2d(d\tan(bx+a))^{\frac{3}{2}}}{3b}$	17

```
[In] int(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/3*d*(d*tan(b*x+a))^(3/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2\,d^2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\,\sin(bx+a)}{3\,b\cos(bx+a)}$$

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
[Out] 2/3*d^2*sqrt(d*sin(b*x + a)/cos(b*x + a))*sin(b*x + a)/(b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**2*(d*tan(b*x+a))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2\,(d\tan(bx+a))^{\frac{5}{2}}}{3\,b\tan(bx+a)}$$

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
[Out] 2/3*(d*tan(b*x + a))^(5/2)/(b*tan(b*x + a))
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2\,\sqrt{d\tan(bx+a)}d^2\tan(bx+a)}{3\,b}$$

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] 2/3*sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 3.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.80

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2\,d^2\,\sin(2\,a+2\,b\,x)\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}}{3\,b\,(\cos(2\,a+2\,b\,x)+1)}$$

```
[In] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^2,x)
[Out] (2*d^2*sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2)
)/(3*b*(cos(2*a + 2*b*x) + 1))
```

3.76 $\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2d^3}{b\sqrt{d\tan(a+bx)}}+\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] -2*d^3/b/(d*tan(b*x+a))^(1/2)+2/3*d*(d*tan(b*x+a))^(3/2)/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 14}

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d(d\tan(a+bx))^{3/2}}{3b}-\frac{2d^3}{b\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Csc[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d^3)/(b*Sqrt[d*Tan[a + b*x]]) + (2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
```

```
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\text{Subst}\left(\int \frac{d^2+x^2}{x^{3/2}}\,dx, x, d\tan(a+bx)\right)}{b} \\
&= \frac{d\text{Subst}\left(\int \left(\frac{d^2}{x^{3/2}}+\sqrt{x}\right)\,dx, x, d\tan(a+bx)\right)}{b} \\
&= -\frac{2d^3}{b\sqrt{d\tan(a+bx)}}+\frac{2d(d\tan(a+bx))^{3/2}}{3b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2d(-1+3\cot^2(a+bx))\,(d\tan(a+bx))^{3/2}}{3b}$$

```
[In] Integrate[Csc[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
[Out] (-2*d*(-1 + 3*Cot[a + b*x]^2)*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{2\sqrt{d\tan(bx+a)}\,d^2(4\cot(bx+a)-\sec(bx+a)\csc(bx+a))}{3b}$	42

```
[In] int(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3/b*(d*tan(b*x+a))^(1/2)*d^2*(4*cot(b*x+a)-sec(b*x+a)*csc(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2\left(4\,d^2\cos\left(bx+a\right)^2-d^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{3\,b\cos\left(bx+a\right)\sin\left(bx+a\right)}$$

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
[Out] -2/3*(4*d^2*cos(b*x + a)^2 - d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(
b*x + a)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**4*(d*tan(b*x+a))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2\,d^3\left(\frac{3}{\sqrt{d\tan(bx+a)}}-\frac{(d\tan(bx+a))^{\frac{3}{2}}}{d^2}\right)}{3\,b}$$

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
[Out] -2/3*d^3*(3/sqrt(d*tan(b*x + a)) - (d*tan(b*x + a))^(3/2)/d^2)/b
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2}{3}\,d^2\left(\frac{\sqrt{d\tan\left(bx+a\right)}\tan\left(bx+a\right)}{b} - \frac{3\,d}{\sqrt{d\tan\left(bx+a\right)}b}\right)$$

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] 2/3*d^2*(sqrt(d*tan(b*x + a))*tan(b*x + a)/b - 3*d/(sqrt(d*tan(b*x + a))*b)
)
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{4\,d^2\left(\sin\left(2\,a+2\,b\,x\right)+\sin\left(4\,a+4\,b\,x\right)\right)\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}}{3\,b\sin\left(2\,a+2\,b\,x\right)^2}$$

```
[In] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^4,x)
[Out] -(4*d^2*(sin(2*a + 2*b*x) + sin(4*a + 4*b*x))*((d*sin(2*a + 2*b*x))/(cos(2*
a + 2*b*x) + 1))^(1/2))/(3*b*sin(2*a + 2*b*x)^2)
```

3.77 $\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2d^5}{5b(d\tan(a+bx))^{5/2}} - \frac{4d^3}{b\sqrt{d\tan(a+bx)}} + \frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] -4*d^3/b/(d*tan(b*x+a))^(1/2)-2/5*d^5/b/(d*tan(b*x+a))^(5/2)+2/3*d*(d*tan(b
*x+a))^(3/2)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 276}

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2d^5}{5b(d\tan(a+bx))^{5/2}} - \frac{4d^3}{b\sqrt{d\tan(a+bx)}} + \frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

```
[In] Int[Csc[a + b*x]^6*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d^5)/(5*b*(d*Tan[a + b*x])^(5/2)) - (4*d^3)/(b*Sqrt[d*Tan[a + b*x]]) +
(2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d\text{Subst}\left(\int \frac{(d^2+x^2)^2}{x^{7/2}}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= \frac{d\text{Subst}\left(\int \left(\frac{d^4}{x^{7/2}}+\frac{2d^2}{x^{3/2}}+\sqrt{x}\right)\,dx, x, d\tan(a+bx)\right)}{b}\\ &= -\frac{2d^5}{5b(d\tan(a+bx))^{5/2}}-\frac{4d^3}{b\sqrt{d\tan(a+bx)}}+\frac{2d(d\tan(a+bx))^{3/2}}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2d\left(-5+3\cot^2(a+bx)\left(9+\csc^2(a+bx)\right)\right)(d\tan(a+bx))^{3/2}}{15b}$$

```
[In] Integrate[Csc[a + b*x]^6*(d*Tan[a + b*x])^(5/2),x]
[Out] (-2*d*(-5 + 3*Cot[a + b*x]^2*(9 + Csc[a + b*x]^2))*(d*Tan[a + b*x])^(3/2))/
(15*b)
```

Maple [A] (verified)

Time = 73.87 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{2\sec(bx+a)\left(\csc^3(bx+a)\right)\sqrt{d\tan(bx+a)}\,d^2\left(32\left(\cos^4(bx+a)\right)-40\left(\cos^2(bx+a)\right)+5\right)}{15b}$	55

```
[In] int(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/b*sec(b*x+a)*csc(b*x+a)^3*(d*tan(b*x+a))^(1/2)*d^2*(32*cos(b*x+a)^4-40
*cos(b*x+a)^2+5)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2\left(32\,d^2\cos\left(bx+a\right)^4-40\,d^2\cos\left(bx+a\right)^2+5\,d^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{15\left(b\cos\left(bx+a\right)^3-b\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}$$

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(32*d^2*cos(b*x + a)^4 - 40*d^2*cos(b*x + a)^2 + 5*d^2)*sqrt(d*sin(b*
x + a)/cos(b*x + a))/((b*cos(b*x + a)^3 - b*cos(b*x + a))*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**6*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2\,d^5\left(\frac{5\,(d\tan(bx+a))^{\frac{3}{2}}}{d^4}-\frac{3\,\left(10\,d^2\tan(bx+a)^2+d^2\right)}{(d\tan(bx+a))^{\frac{5}{2}}\,d^2}\right)}{15\,b}$$

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*d^5*(5*(d*tan(b*x + a))^(3/2)/d^4 - 3*(10*d^2*tan(b*x + a)^2 + d^2)/((
d*tan(b*x + a))^(5/2)*d^2))/b
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2}{15}\,d^2\left(\frac{5\,\sqrt{d\tan\left(bx+a\right)}\tan\left(bx+a\right)}{b} - \frac{3\,\left(10\,d^3\tan\left(bx+a\right)^2+d^3\right)}{\sqrt{d\tan\left(bx+a\right)}bd^2\tan\left(bx+a\right)^2}\right)$$

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*d^2*(5*sqrt(d*tan(b*x + a))*tan(b*x + a)/b - 3*(10*d^3*tan(b*x + a)^2
+ d^3)/(sqrt(d*tan(b*x + a))*b*d^2*tan(b*x + a)^2))
```

Mupad [B] (verification not implemented)

Time = 6.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.13

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{32\,d^2\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,2\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,3\mathrm{i}+\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,2\mathrm{i}-\mathrm{e}^{a\,8\mathrm{i}+b\,x\,8\mathrm{i}}\,2\mathrm{i}-2\mathrm{i}\right)}{15\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^3\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}$$

```
[In] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^6,x)
```

```
[Out] (32*d^2*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*
(exp(a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*x*6i)*2i - ex
p(a*8i + b*x*8i)*2i - 2i))/(15*b*(exp(a*2i + b*x*2i) - 1)^3*(exp(a*2i + b*x
*2i) + 1))
```

3.78 $\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx=\frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}}+\frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}}-\frac{5d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{4b}+\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] 5/2*d^3*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)+d^3*sin(b*x+a)^3/b/(d*tan(b*x+a))
^(1/2)+5/4*d^2*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Ell
ipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)
/b+2/3*d*sin(b*x+a)^3*(d*tan(b*x+a))^(3/2)/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2674, 2678, 2681, 2653, 2720}

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx=\frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}}-\frac{5d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{4b}+\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[In] Int[Sin[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (5*d^3*Sin[a + b*x])/(2*b*Sqrt[d*Tan[a + b*x]]) + (d^3*Sin[a + b*x]^3)/(b*S
qrt[d*Tan[a + b*x]]) - (5*d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqr
t[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(4*b) + (2*d*Sin[a + b*x]^3*(d*Ta
n[a + b*x])^(3/2))/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \left(3d^2\right)\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= \frac{d^3 \sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{2}(5d^2)\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= \frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}} + \frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{4}(5d^2)\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= \frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}} + \frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{\left(5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{4\sqrt{\sin(a+bx)}}$$

$$= \frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}} + \frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{4}\left(5d^2\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx$$

$$= \frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}} + \frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{5d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{4b} + \frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$

$$-\frac{\csc(a+bx)\sqrt{\sec^2(a+bx)}\left(120\sqrt[4]{-1}\cos(2(a+bx))\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)+(22\right.}{48b\tan^{\frac{3}{2}}(a+bx)\,(}$$

```
[In] Integrate[Sin[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -1/48*(Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]*(120*(-1)^(1/4)*Cos[2*(a + b*x)]*E
llipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1] + (22 + 77*Cos[2*(a
+ b*x)] + 22*Cos[4*(a + b*x)] - Cos[6*(a + b*x)])*Sqrt[Sec[a + b*x]^2]*Sqrt
[Tan[a + b*x]])*(d*Tan[a + b*x])^(5/2))/(b*Tan[a + b*x]^(3/2)*(-1 + Tan[a +
 b*x]^2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.73 (sec) , antiderivative size = 1840, normalized size of antiderivative = 13.43

method	result	size
default	Expression too large to display	1840

```
[In] int(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/b*tan(b*x+a)*(-6*I*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(
b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*
x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)^2+6*I*EllipticPi((1+csc(b*x+a
)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*
(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*cos(b*x+a)^2
-6*I*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(
b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*cos(b*x+a)+6*I*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1
/2+1/2*I,1/2*2^(1/2))*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*
x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*cos(b*x+a)-6*EllipticPi((1+csc(b*
x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(1+csc(b*x+a)-cot(b*x+a))^(1/
2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*cos(b*x+a
)^2+72*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(co
t(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^
(1/2))*cos(b*x+a)^2-6*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*
x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+
a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)^2+8*2^(1/2)*cos(b*x+a)^4*sin(b*
x+a)-6*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+cs
c(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1
/2*I,1/2*2^(1/2))*cos(b*x+a)+72*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+
1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)
-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-6*(cot(b*x+a)-csc(b*x+a))^(1/2)*
(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi
((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)-3*(-cos(
b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-2*cot(b*x+a)*2^(1/2)*(-cot(b*
x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))^2)^(1/2)-2*csc(b*x+a)*2^(1/2)*(-cot(b*x+a
)*csc(b*x+a)^2*(-1+cos(b*x+a))^2)^(1/2)-2*cot(b*x+a)+2)*cos(b*x+a)^2+3*(-co
s(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(2*cot(b*x+a)*2^(1/2)*(-cot(b
*x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))^2)^(1/2)+2*csc(b*x+a)*2^(1/2)*(-cot(b*x+
a)*csc(b*x+a)^2*(-1+cos(b*x+a))^2)^(1/2)-2*cot(b*x+a)+2)*cos(b*x+a)^2-6*(-c
os(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((-sin(b*x+a)*2^(1/2)*(-
cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a))
)*cos(b*x+a)^2+6*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((si
n(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)
-1)/(-1+cos(b*x+a)))*cos(b*x+a)^2-52*cos(b*x+a)^2*sin(b*x+a)*2^(1/2)-3*(-co
```

```
s(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-2*cot(b*x+a)*2^(1/2)*(-cot(
b*x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))^2)^(1/2)-2*csc(b*x+a)*2^(1/2)*(-cot(b*x
+a)*csc(b*x+a)^2*(-1+cos(b*x+a))^2)^(1/2)-2*cot(b*x+a)+2)*cos(b*x+a)+3*(-co
s(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(2*cot(b*x+a)*2^(1/2)*(-cot(b
*x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))^2)^(1/2)+2*csc(b*x+a)*2^(1/2)*(-cot(b*x+
a)*csc(b*x+a)^2*(-1+cos(b*x+a))^2)^(1/2)-2*cot(b*x+a)+2)*cos(b*x+a)-6*(-cos
(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((-sin(b*x+a)*2^(1/2)*(-co
s(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*
cos(b*x+a)+6*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b*
x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/
(-1+cos(b*x+a)))*cos(b*x+a)-16*sin(b*x+a)*2^(1/2))*(d*tan(b*x+a))^(1/2)*d^2
/(cos(b*x+a)^2-1)*2^(1/2)
```

Fricas [F]

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\sin(bx+a)^3\,dx$$

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
[Out] integral(-(d^2*cos(b*x + a)^2 - d^2)*sqrt(d*tan(b*x + a))*sin(b*x + a)*tan(
b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**3*(d*tan(b*x+a))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\sin(bx+a)^3\,dx$$

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^(5/2)*sin(b*x + a)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]ext_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0]ext_reduce

Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \sin\left(a+b\,x\right)^3\,\left(d\tan(a+b\,x)\right)^{5/2}\,dx$$

[In] int(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2),x)

[Out] int(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2), x)

3.79 $\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}} - \frac{5d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{6b} + \frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] 5/3*d^3*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)+5/6*d^2*csc(b*x+a)*(sin(a+1/4*Pi+
b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*
b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2/3*d*sin(b*x+a)*(d*tan(b*x+a))^(3/2)
/b
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2674, 2678, 2681, 2653, 2720}

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}} - \frac{5d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{6b} + \frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[In] Int[Sin[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (5*d^3*Sin[a + b*x])/(3*b*Sqrt[d*Tan[a + b*x]]) - (5*d^2*Csc[a + b*x]*Ellip
ticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(6*b)
+ (2*d*Sin[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{3}\left(5d^2\right)\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{6}(5d^2)\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{\left(5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{6\sqrt{\sin(a+bx)}}$$

$$= \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{6}\left(5d^2\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx$$

$$= \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}} - \frac{5d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{6b} + \frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.23

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{\cos(2(a+bx))\csc(a+bx)\sqrt{\sec^2(a+bx)}\left(10\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)+(7+\right.}{6b\tan^{\frac{3}{2}}(a+bx)\left(-1+\tan^2(a+bx)\right)}$$

```
[In] Integrate[Sin[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -1/6*(Cos[2*(a + b*x)]*Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]*(10*(-1)^(1/4)*Ell
ipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1] + (7 + 3*Cos[2*(a + b*
x)])*Sqrt[Sec[a + b*x]^2]*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(5/2))/(b*Ta
n[a + b*x]^(3/2)*(-1 + Tan[a + b*x]^2))
```

Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.16

method	result
default	$-\frac{\sqrt{d\tan(bx+a)}\,\sin(bx+a)\left(-5\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+a}\right.\right.}{}$

```
[In] int(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b*(d*tan(b*x+a))^(1/2)*sin(b*x+a)*(-5*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-
csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1
+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+3*sin(b*x+a)*2^(1/2)*
cos(b*x+a)-5*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)
*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),
1/2*2^(1/2))+2*tan(b*x+a)*2^(1/2))*d^2/(cos(b*x+a)^2-1)*2^(1/2)
```

Fricas [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\sin(bx+a)\;dx$$

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d^2*sin(b*x + a)*tan(b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\sin(bx+a)\,dx$$

[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")

[Out] integrate((d*tan(b*x + a))^(5/2)*sin(b*x + a), x)

Giac [F(-2)]

Exception generated.

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]ext_reduce Error: Bad Argument TypeDone

Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \sin(a+b\,x)\,(d\tan(a+b\,x))^{5/2}\,dx$$

[In] int(sin(a + b*x)*(d*tan(a + b*x))^(5/2),x)

[Out] int(sin(a + b*x)*(d*tan(a + b*x))^(5/2), x)

3.80 $\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b} + \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] 1/3*d^2*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(
cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2/3*
d*csc(b*x+a)*(d*tan(b*x+a))^(3/2)/b
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2674, 2681, 2653, 2720}

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b}$$

```
[In] Int[Csc[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -1/3*(d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*
Sqrt[d*Tan[a + b*x]])/b + (2*d*Csc[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{3}d^2\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx \\
&= \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{\left(d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{3\sqrt{\sin(a+bx)}} \\
&= \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} \\
&\quad - \frac{1}{3}\left(d^2\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx \\
&= -\frac{d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b} \\
&\quad + \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.51 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d^2\cos(a+bx)\left(\sec^2(a+bx) - \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)}{3b}$$

[In] Integrate[Csc[a + b*x]*(d*Tan[a + b*x])^(5/2),x]

[Out] (2*d^2*Cos[a + b*x]*(Sec[a + b*x]^2 - Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/(3*b)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(95) = 190.

Time = 1.63 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.69

method	result
default	$-\frac{\sqrt{d\tan(bx+a)}\,\sin(bx+a)\left(-\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)}\right.\right.}{}$

[In] int(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/3/b*(d*tan(b*x+a))^(1/2)*sin(b*x+a)*(-(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+tan(b*x+a)*2^(1/2))*d^2/(cos(b*x+a)^2-1)*2^(1/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{\sqrt{i}\,dd^2\cos(bx+a)\,F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1) + \sqrt{-i}\,dd^2\cos(bx+a)\,F(\text{arcs}}{3\,b\cos(bx+a)}$$

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(I*d)*d^2*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x
+ a)), -1) + sqrt(-I*d)*d^2*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) - I
*sin(b*x + a)), -1) + 2*d^2*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x +
 a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)\,dx$$

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a), x)
```

Giac [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)\,dx$$

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \frac{(d\tan(a+b\,x))^{5/2}}{\sin\left(a+b\,x\right)}\,dx$$

```
[In] int((d*tan(a + b*x))^(5/2)/sin(a + b*x),x)
[Out] int((d*tan(a + b*x))^(5/2)/sin(a + b*x), x)
```

3.81 $\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b} + \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] -2/3*d^2*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF
(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2/3
*d*csc(b*x+a)*(d*tan(b*x+a))^(3/2)/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2673, 2681, 2653, 2720}

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b} + \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[In] Int[Csc[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqr
t[d*Tan[a + b*x]])/(3*b) + (2*d*Csc[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2673

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Dist[b^2*((m + 2)/(a^2*(n - 1))), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{1}{3}\left(2d^2\right)\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx \\
&= \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} \\
&\quad + \frac{\left(2d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{3\sqrt{\sin(a+bx)}} \\
&= \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} \\
&\quad + \frac{1}{3}\left(2d^2\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx
\end{aligned}$$

$$= \frac{2d^2 \csc(a+bx) \operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right) \sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b} + \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.61 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d^2\cos(a+bx)\left(\sec^2(a+bx)+2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx}\right.}{3b}$$

```
[In] Integrate[Csc[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
[Out] (2*d^2*Cos[a + b*x]*(Sec[a + b*x]^2 + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -T
an[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(95) = 190.

Time = 2.50 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.69

method	result
default	$-\frac{\sqrt{d\tan(bx+a)}\,\sin(bx+a)\left(2\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+a}\right.\right.}{}$

```
[In] int(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/3/b*(d*tan(b*x+a))^(1/2)*sin(b*x+a)*(2*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-c
sc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+
csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+2*(cot(b*x+a)-csc(b*x+
a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*
EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+tan(b*x+a)*2^(1/2))*
d^2/(cos(b*x+a)^2-1)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$
$$-\frac{2\left(\sqrt{i\,d}d^2\cos(bx+a)\,F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+\sqrt{-i\,d}d^2\cos(bx+a)\,F(\arcsin(\cos(b\right.}{3\,b\cos(bx+a)}$$

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(I*d)*d^2*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x
 + a)), -1) + sqrt(-I*d)*d^2*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) -
I*sin(b*x + a)), -1) - d^2*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x +
a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**3*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^3\,dx$$

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^3, x)
```

Giac [F]

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^3\,dx$$

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \frac{(d\tan(a+b\,x))^{5/2}}{\sin(a+b\,x)^3}\,dx$$

```
[In] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^3,x)
[Out] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^3, x)
```

3.82 $\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{4d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b} + \frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] -4/3*d^3*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)-4/3*d^2*csc(b*x+a)*(sin(a+1/4*Pi
+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2
*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2/3*d*csc(b*x+a)^3*(d*tan(b*x+a))^(3
/2)/b
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2673, 2679, 2681, 2653, 2720}

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{4d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b} + \frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[In] Int[Csc[a + b*x]^5*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-4*d^3*Csc[a + b*x])/(3*b*Sqrt[d*Tan[a + b*x]]) + (4*d^2*Csc[a + b*x]*Elli
pticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b)
 + (2*d*Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2673

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Dist[b^2*((m + 2)/(a^2*(n - 1))), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \left(2d^2\right)\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$
$$+ \frac{1}{3}(4d^2)\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$
$$+ \frac{\left(4d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{3\sqrt{\sin(a+bx)}}$$

$$= -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$
$$+ \frac{1}{3}\left(4d^2\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx$$

$$= -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$
$$+ \frac{4d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b}$$
$$+ \frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$
$$-\frac{2d\csc^3(a+bx)\left(\cos(2(a+bx))\sqrt{\sec^2(a+bx)} + 2\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sin(}{3b\sqrt{\sec^2(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]^5*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d*Csc[a + b*x]^3*(Cos[2*(a + b*x)]*Sqrt[Sec[a + b*x]^2] + 2*(-1)^(1/4)*
EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sin[2*(a + b*x)]*Sq
rt[Tan[a + b*x]])*(d*Tan[a + b*x])^(3/2))/(3*b*Sqrt[Sec[a + b*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(121) = 242$.

Time = 20.39 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.35

method	result
default	$-\frac{\sqrt{d\tan(bx+a)}\,d^2\left(-4\cos(bx+a)\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,(\sin^3(bx+a)\right.}{}$

[In] int(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/3/b*(d*tan(b*x+a))^(1/2)*d^2/(-1+cos(b*x+a))^2/(cos(b*x+a)+1)^2*(-4*cos(b*x+a)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*sin(b*x+a)^3*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-4*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*sin(b*x+a)^3*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*cos(b*x+a)*2^(1/2)*sin(b*x+a)^2-sin(b*x+a)*tan(b*x+a)*2^(1/2))*2^(1/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.47

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$

$$-\frac{2\left(2\left(d^2\cos\left(bx+a\right)^3-d^2\cos\left(bx+a\right)\right)\sqrt{i\,d}F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)+2\left(d^2\cos\left(bx\right.\right.\right.}{3\left(b\cos\left(bx\right.\right.}$$

[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")

[Out] -2/3*(2*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + 2*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - (2*d^2*cos(b*x + a)^2 - d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x + a)^3 - b*cos(b*x + a))

Sympy [F(-1)]

Timed out.

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**5*(d*tan(b*x+a))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^5\,dx$$

```
[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^5, x)
```

Giac [F]

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^5\,dx$$

```
[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \frac{(d\tan(a+b\,x))^{5/2}}{\sin(a+b\,x)^5}\,dx$$

```
[In] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^5,x)
[Out] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^5, x)
```

3.83 $\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}}-\frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}} + \frac{40d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] -40/21*d^3*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)-20/21*d^3*csc(b*x+a)^3/b/(d*ta
n(b*x+a))^(1/2)-40/21*d^2*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*
Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*
x+a))^(1/2)/b+2/3*d*csc(b*x+a)^5*(d*tan(b*x+a))^(3/2)/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2673, 2679, 2681, 2653, 2720}

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}}-\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}} + \frac{40d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[In] Int[Csc[a + b*x]^7*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-40*d^3*Csc[a + b*x])/(21*b*Sqrt[d*Tan[a + b*x]]) - (20*d^3*Csc[a + b*x]^3
)/(21*b*Sqrt[d*Tan[a + b*x]]) + (40*d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b
*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(21*b) + (2*d*Csc[a + b
*x]^5*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2673

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Dist[b^2*((m + 2)/(a^2*(n - 1))), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{1}{3}\left(10d^2\right)\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= -\frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}} + \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{1}{7}(20d^2)\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= -\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}} - \frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}} + \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{1}{21}(40d^2)\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= -\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}} - \frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}} + \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{\left(40d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{21\sqrt{\sin(a+bx)}}$$

$$= -\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}} - \frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}} + \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{1}{21}\left(40d^2\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx$$

$$= -\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}} - \frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}} + \frac{40d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.93

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{d^2\csc(a+bx)\left((1+10\cos(2(a+bx))-5\cos(4(a+bx)))\csc^3(a+bx)\sec(a+bx)\sqrt{\sec^2(a+bx)}+80\sqrt[4]{-\right.}{21b\sqrt{\sec^2(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]^7*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -1/21*(d^2*Csc[a + b*x]*((1 + 10*Cos[2*(a + b*x)] - 5*Cos[4*(a + b*x)])*Csc
[a + b*x]^3*Sec[a + b*x]*Sqrt[Sec[a + b*x]^2] + 80*(-1)^(1/4)*EllipticF[I*A
rcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sqrt[Tan[a + b*x]])*Sqrt[d*Tan[a
 + b*x]])/(b*Sqrt[Sec[a + b*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. 2(147) = 294.

Time = 151.87 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.26

method	result
default	$\frac{\sin(bx+a)\tan(bx+a)\left(40\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)}\right.\right.}{}$

```
[In] int(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/b*sin(b*x+a)*tan(b*x+a)*(40*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+
a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a
)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^4*sin(b*x+a)+40*(1+csc(b*x+a)-c
ot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(
1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^3*si
n(b*x+a)-40*sin(b*x+a)*cos(b*x+a)^2*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(cot(b*
x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*EllipticF((1+csc(b*
x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-40*sin(b*x+a)*cos(b*x+a)*(1+csc(b*x+a)-
cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^
(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-20*2^(1/2)*cos
(b*x+a)^4+30*cos(b*x+a)^2*2^(1/2)-7*2^(1/2))*(d*tan(b*x+a))^(1/2)*d^2/(-1+c
os(b*x+a))^3/(cos(b*x+a)+1)^3*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.49

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$
$$-\frac{2\left(20\left(d^2\cos(bx+a)^5-2\,d^2\cos(bx+a)^3+d^2\cos(bx+a)\right)\sqrt{i\,d}F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\mid\right.}{}$$

```
[In] integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/21*(20*(d^2*cos(b*x + a)^5 - 2*d^2*cos(b*x + a)^3 + d^2*cos(b*x + a))*sq
rt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + 20*(d^2*cos
(b*x + a)^5 - 2*d^2*cos(b*x + a)^3 + d^2*cos(b*x + a))*sqrt(-I*d)*elliptic_
f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - (20*d^2*cos(b*x + a)^4 - 30*
d^2*cos(b*x + a)^2 + 7*d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x +
 a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**7*(d*tan(b*x+a))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^7\,dx$$

```
[In] integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^7, x)
```

Giac [F]

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^7\,dx$$

```
[In] integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \frac{(d\tan(a+b\,x))^{5/2}}{\sin(a+b\,x)^7}\,dx$$

```
[In] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^7,x)
[Out] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^7, x)
```

3.84 $\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 257

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b\sqrt{d}}+\frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b\sqrt{d}} - \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b\sqrt{d}} + \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b\sqrt{d}} - \frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3}$$

```
[Out] -5/64*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)/d^(1/2)+5/64
*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)/d^(1/2)-5/128*ln(
d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)/d^(1/2)+
5/128*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)
/d^(1/2)-5/16*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d-1/4*cos(b*x+a)^4*(d*tan
(b*x+a))^(5/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2671, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b\sqrt{d}}+\frac{5\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}b\sqrt{d}} -\frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3} -\frac{5\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}b\sqrt{d}} +\frac{5\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}b\sqrt{d}} -\frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd}$$

```
[In] Int[Sin[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-5*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*Sqrt[
d]) + (5*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*
Sqrt[d]) - (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b
*x]]])/(64*Sqrt[2]*b*Sqrt[d]) + (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqr
t[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b*Sqrt[d]) - (5*Cos[a + b*x]^2*Sqrt
[d*Tan[a + b*x]])/(16*b*d) - (Cos[a + b*x]^4*(d*Tan[a + b*x])^(5/2))/(4*b*d
^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{d\text{Subst}\left(\int \frac{x^{7/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}\\
&= -\frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3} + \frac{(5d)\text{Subst}\left(\int \frac{x^{3/2}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{8b}\\
&= -\frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3}\\
&\quad + \frac{(5d)\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\tan(a+bx)\right)}{32b}\\
&= -\frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3}\\
&\quad + \frac{(5d)\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{16b}\\
&= -\frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3}\\
&\quad + \frac{5\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{32b}\\
&\quad + \frac{5\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{32b}\\
&= -\frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3}\\
&\quad + \frac{5\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64b}\\
&\quad + \frac{5\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64b}\\
&\quad - \frac{5\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b\sqrt{d}}\\
&\quad - \frac{5\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b\sqrt{d}}
\end{aligned}
$$

$$= -\frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b\sqrt{d}} + \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b\sqrt{d}} - \frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3} + \frac{5\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b\sqrt{d}} - \frac{5\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b\sqrt{d}}$$

$$= -\frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b\sqrt{d}} + \frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b\sqrt{d}} - \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b\sqrt{d}} + \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b\sqrt{d}} - \frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.47

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sec(a+bx)\left(-7\sin(a+bx)-5\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}+5\log\left(\cos(a+bx)+\right.\right.}{64b\sqrt{d\tan(a+bx)}}$$

```
[In] Integrate[Sin[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (Sec[a + b*x]*(-7*Sin[a + b*x] - 5*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt
[Sin[2*(a + b*x)]] + 5*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*
x)]]]*Sqrt[Sin[2*(a + b*x)]] - 6*Sin[3*(a + b*x)] + Sin[5*(a + b*x)]))/(64*
b*Sqrt[d*Tan[a + b*x]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(197) = 394$.

Time = 9.16 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.35

method	result
default	$\frac{\sin(bx+a)\left(16\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\,\sqrt{2}\,(\cos^4(bx+a))+16\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\,\sqrt{2}\,(\cos^3(bx+a))-36\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\,\sqrt{2}\right.}{}$

```
[In] int(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/b*sin(b*x+a)*(16*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1
/2)*cos(b*x+a)^4+16*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)
*cos(b*x+a)^3-36*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*co
s(b*x+a)^2-36*cos(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^
(1/2)-5*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3
+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a
)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))+
5*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot
(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(
b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))-10*arc
tan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos
(b*x+a)-1)/(-1+cos(b*x+a)))+10*arctan((-sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin
(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a))))/(cos(b*x+a)
+1)/(d*tan(b*x+a))^(1/2)/(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^
(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 892, normalized size of antiderivative = 3.47

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/256*(5*b*d*(-1/(b^4*d^2))^(1/4)*log(2*b^2*d*sqrt(-1/(b^4*d^2))*cos(b*x +
a)*sin(b*x + a) - 2*cos(b*x + a)^2 + 2*(b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x
+ a)^2 + b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x +
 a)/cos(b*x + a)) + 1) - 5*b*d*(-1/(b^4*d^2))^(1/4)*log(2*b^2*d*sqrt(-1/(b^
4*d^2))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 - 2*(b^3*d*(-1/(b^4*d^
```

```
2))^(3/4)*cos(b*x + a)^2 + b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a)
)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) + 5*I*b*d*(-1/(b^4*d^2))^(1/4)*log
(-2*b^2*d*sqrt(-1/(b^4*d^2))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 -
 2*(I*b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 - I*b*(-1/(b^4*d^2))^(1/4)*
cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) - 5*I*b*d
*(-1/(b^4*d^2))^(1/4)*log(-2*b^2*d*sqrt(-1/(b^4*d^2))*cos(b*x + a)*sin(b*x
+ a) - 2*cos(b*x + a)^2 - 2*(-I*b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 +
 I*b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/co
s(b*x + a)) + 1) + 5*b*d*(-1/(b^4*d^2))^(1/4)*log(2*(b^3*d*(-1/(b^4*d^2))^(
3/4)*cos(b*x + a)^2 - b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqr
t(d*sin(b*x + a)/cos(b*x + a)) - 1) - 5*b*d*(-1/(b^4*d^2))^(1/4)*log(-2*(b^
3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 - b*(-1/(b^4*d^2))^(1/4)*cos(b*x +
a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1) + 5*I*b*d*(-1/(b^4*
d^2))^(1/4)*log(-2*(I*b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 + I*b*(-1/(
b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)
) - 1) - 5*I*b*d*(-1/(b^4*d^2))^(1/4)*log(-2*(-I*b^3*d*(-1/(b^4*d^2))^(3/4)
*cos(b*x + a)^2 - I*b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(
d*sin(b*x + a)/cos(b*x + a)) - 1) + 16*(4*cos(b*x + a)^4 - 9*cos(b*x + a)^2
)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*d)
```

Sympy [F]

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin^4\left(a+bx\right)}{\sqrt{d\tan\left(a+bx\right)}}\,dx$$

```
[In] integrate(sin(b*x+a)**4/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sin(a + b*x)**4/sqrt(d*tan(a + b*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.86

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{10\sqrt{2}d^{\frac{9}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right) + 10\sqrt{2}d^{\frac{9}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right) + 5\sqrt{2}d^{\frac{9}{2}}\log\left(d\,t\right.}{}$$

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/128*(10*sqrt(2)*d^(9/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*ta
n(b*x + a)))/sqrt(d)) + 10*sqrt(2)*d^(9/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 5*sqrt(2)*d^(9/2)*log(d*tan(b*x +
 a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 5*sqrt(2)*d^(9/2)*log(d*t
an(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*(9*(d*tan(b*x +
 a))^(5/2)*d^6 + 5*sqrt(d*tan(b*x + a))*d^8)/(d^4*tan(b*x + a)^4 + 2*d^4*ta
n(b*x + a)^2 + d^4))/(b*d^5)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.96

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{5\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd} + \frac{5\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd} + \frac{5\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{128\,bd} - \frac{5\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{128\,bd} - \frac{9\sqrt{d\tan(bx+a)}d^3\tan(bx+a)^2+5\sqrt{d\tan(bx+a)}d^3}{16\left(d^2\tan(bx+a)^2+d^2\right)^2 b}$$

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt
(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) + 5/64*sqrt(2)*sqrt(abs(d))*arctan(-1
/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b
*d) + 5/128*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*
x + a))*sqrt(abs(d)) + abs(d))/(b*d) - 5/128*sqrt(2)*sqrt(abs(d))*log(d*tan
(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) - 1/1
6*(9*sqrt(d*tan(b*x + a))*d^3*tan(b*x + a)^2 + 5*sqrt(d*tan(b*x + a))*d^3)/
((d^2*tan(b*x + a)^2 + d^2)^2*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(a+b\,x)^4}{\sqrt{d\tan(a+b\,x)}}\,dx$$

```
[In] int(sin(a + b*x)^4/(d*tan(a + b*x))^(1/2),x)
[Out] int(sin(a + b*x)^4/(d*tan(a + b*x))^(1/2), x)
```

3.85 $\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 227

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b\sqrt{d}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b\sqrt{d}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b\sqrt{d}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b\sqrt{d}} - \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd}$$

```
[Out] -1/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)/d^(1/2)+1/8*a
rctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)/d^(1/2)-1/16*ln(d^(
1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)/d^(1/2)+1/1
6*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)/d^(
1/2)-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2671, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b\sqrt{d}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}b\sqrt{d}} - \frac{\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b\sqrt{d}} + \frac{\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b\sqrt{d}} - \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd}$$

```
[In] Int[Sin[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -1/4*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*b*Sqrt[d])
 + ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(4*Sqrt[2]*b*Sqrt[d])
 - Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]/(8*Sq
rt[2]*b*Sqrt[d]) + Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqrt[d*Tan[
a + b*x]]]/(8*Sqrt[2]*b*Sqrt[d]) - (Cos[a + b*x]^2*Sqrt[d*Tan[a + b*x]])/(2
*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{d\text{Subst}\left(\int \frac{x^{3/2}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd} + \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\tan(a+bx)\right)}{4b}$$

$$= -\frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd} + \frac{d\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2b}$$

$$= -\frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4b}$$

$$= -\frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8b} + \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8b} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b\sqrt{d}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b\sqrt{d}}$$

$$= -\frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b\sqrt{d}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b\sqrt{d}} - \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b\sqrt{d}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b\sqrt{d}}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b\sqrt{d}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b\sqrt{d}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b\sqrt{d}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b\sqrt{d}} - \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.48

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{\sec(a+bx)\left(\sin(a+bx)+\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-\log\left(\cos(a+bx)+\sin\right.\right.}{8b\sqrt{d\tan(a+bx)}}$$

```
[In] Integrate[Sin[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -1/8*(Sec[a + b*x]*(Sin[a + b*x] + ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt
[Sin[2*(a + b*x)]] - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)
]]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)]))/(b*Sqrt[d*Tan[a + b*x]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(171) = 342$.

Time = 1.73 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.31

method	result
default	$-\frac{\sin(bx+a)\left(4\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\sqrt{2}\left(\cos^2(bx+a)\right)+4\cos(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}+2\arctan\left(\frac{\sin(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+}{(\cos(}}}{-1+c}\right.\right.}{}$

```
[In] int(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/b*sin(b*x+a)*(4*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/
2)*cos(b*x+a)^2+4*cos(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)
^2)^(1/2)+2*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+
1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+2*arctan((sin(b*x+a)*2^(1/2)*(-c
```

```
os(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))
-ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(
b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b
*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))+ln(-(co
t(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2
*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(
1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a))))/(cos(b*x+a)+1)
/(d*tan(b*x+a))^(1/2)/(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/
2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 877, normalized size of antiderivative = 3.86

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(b*d*(-1/(b^4*d^2))^(1/4)*log(2*b^2*d*sqrt(-1/(b^4*d^2))*cos(b*x + a)*
sin(b*x + a) - 2*cos(b*x + a)^2 + 2*(b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x + a
)^2 + b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)
/cos(b*x + a)) + 1) - b*d*(-1/(b^4*d^2))^(1/4)*log(2*b^2*d*sqrt(-1/(b^4*d^2
))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 - 2*(b^3*d*(-1/(b^4*d^2))^(
3/4)*cos(b*x + a)^2 + b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqr
t(d*sin(b*x + a)/cos(b*x + a)) + 1) + I*b*d*(-1/(b^4*d^2))^(1/4)*log(-2*b^2
*d*sqrt(-1/(b^4*d^2))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 - 2*(I*b
^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 - I*b*(-1/(b^4*d^2))^(1/4)*cos(b*x
 + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) - I*b*d*(-1/(b^4
*d^2))^(1/4)*log(-2*b^2*d*sqrt(-1/(b^4*d^2))*cos(b*x + a)*sin(b*x + a) - 2*
cos(b*x + a)^2 - 2*(-I*b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 + I*b*(-1/
(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a
)) + 1) + b*d*(-1/(b^4*d^2))^(1/4)*log(2*(b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*
x + a)^2 - b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x
 + a)/cos(b*x + a)) - 1) - b*d*(-1/(b^4*d^2))^(1/4)*log(-2*(b^3*d*(-1/(b^4*
d^2))^(3/4)*cos(b*x + a)^2 - b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x +
a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1) + I*b*d*(-1/(b^4*d^2))^(1/4)*log
(-2*(I*b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 + I*b*(-1/(b^4*d^2))^(1/4)
*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1) - I*b*d*
(-1/(b^4*d^2))^(1/4)*log(-2*(-I*b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 -
 I*b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/co
s(b*x + a)) - 1) - 16*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2)/(b*
d)
```

Sympy [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

```
[In] integrate(sin(b*x+a)**2/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/sqrt(d*tan(a + b*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.83

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+\sqrt{2}d^{\frac{5}{2}}\log\left(d\tan\right.}{}$$

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
b*x + a)))/sqrt(d)) + 2*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + sqrt(2)*d^(5/2)*log(d*tan(b*x + a) +
 sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - sqrt(2)*d^(5/2)*log(d*tan(b*x
+ a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan(b*x + a))*d
^4/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.96

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd} + \frac{\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd} + \frac{\sqrt{2}\sqrt{|d|}\log\left(d\tan\left(bx+a\right)+\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{16\,bd} - \frac{\sqrt{2}\sqrt{|d|}\log\left(d\tan\left(bx+a\right)-\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{16\,bd} - \frac{\sqrt{d\tan\left(bx+a\right)}d}{2\left(d^2\tan\left(bx+a\right)^2+d^2\right)b}$$

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(
d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) + 1/8*sqrt(2)*sqrt(abs(d))*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d
) + 1/16*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x +
 a))*sqrt(abs(d)) + abs(d))/(b*d) - 1/16*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x
 + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) - 1/2*sqr
t(d*tan(b*x + a))*d/((d^2*tan(b*x + a)^2 + d^2)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

```
[In] int(sin(a + b*x)^2/(d*tan(a + b*x))^(1/2),x)
```

```
[Out] int(sin(a + b*x)^2/(d*tan(a + b*x))^(1/2), x)
```

3.86 $\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

```
[Out] -2/3*d/b/(d*tan(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 30}

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

```
[In] Int[Csc[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
[Out] (-2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
```

```
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{d\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, d\tan(a+bx)\right)}{b} \\ &= -\frac{2d}{3b(d\tan(a+bx))^{3/2}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

```
[In] Integrate[Csc[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
[Out] (-2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2d}{3b(d\tan(bx+a))^{\frac{3}{2}}}$	17
default	$-\frac{2d}{3b(d\tan(bx+a))^{\frac{3}{2}}}$	17

```
[In] int(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*d/b/(d*tan(b*x+a))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(16) = 32$.

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos{(bx+a)}^2}{3\left(bd\cos{(bx+a)}^2 - bd\right)}$$

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2/(b*d*cos(b*x + a)^2 -
b*d)
```

Sympy [F]

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc^2{(a+bx)}}{\sqrt{d\tan{(a+bx)}}}\,dx$$

```
[In] integrate(csc(b*x+a)**2/(d*tan(b*x+a))**(1/2),x)
[Out] Integral(csc(a + b*x)**2/sqrt(d*tan(a + b*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2}{3\sqrt{d\tan{(bx+a)}}b\tan{(bx+a)}}$$

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] -2/3/(sqrt(d*tan(b*x + a))*b*tan(b*x + a))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2}{3\sqrt{d\tan(bx+a)}b\tan(bx+a)}$$

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] -2/3/(sqrt(d*tan(b*x + a))*b*tan(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 3.43 (sec) , antiderivative size = 102, normalized size of antiderivative = 5.10

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\sqrt{\frac{d\sin(2a+2bx)}{\cos(2a+2bx)+1}}\,(\cos(2a+2bx)+2\cos(4a+4bx)-\cos(6a+6bx)-2)}{3bd\,(15\cos(2a+2bx)-6\cos(4a+4bx)+\cos(6a+6bx)-10)}$$

```
[In] int(1/(sin(a + b*x)^2*(d*tan(a + b*x))^(1/2)),x)
[Out] -(2*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2)*(cos(2*a + 2*b*x) +
 2*cos(4*a + 4*b*x) - cos(6*a + 6*b*x) - 2))/(3*b*d*(15*cos(2*a + 2*b*x) -
6*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) - 10))
```

3.87 $\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d^3}{7b(d\tan(a+bx))^{7/2}} - \frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

```
[Out] -2/7*d^3/b/(d*tan(b*x+a))^(7/2)-2/3*d/b/(d*tan(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 14}

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d^3}{7b(d\tan(a+bx))^{7/2}} - \frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

```
[In] Int[Csc[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
[Out] (-2*d^3)/(7*b*(d*Tan[a + b*x])^(7/2)) - (2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
```

```
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\text{Subst}\left(\int \frac{d^2+x^2}{x^{9/2}}\,dx, x, d\tan(a+bx)\right)}{b}\\
&= \frac{d\text{Subst}\left(\int \left(\frac{d^2}{x^{9/2}}+\frac{1}{x^{5/2}}\right)\,dx, x, d\tan(a+bx)\right)}{b}\\
&= -\frac{2d^3}{7b(d\tan(a+bx))^{7/2}}-\frac{2d}{3b(d\tan(a+bx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2d(-5+2\cos(2(a+bx)))\csc^2(a+bx)}{21b(d\tan(a+bx))^{3/2}}$$

```
[In] Integrate[Csc[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
[Out] (2*d*(-5 + 2*Cos[2*(a + b*x)])*Csc[a + b*x]^2)/(21*b*(d*Tan[a + b*x])^(3/2)
)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\frac{8\left(\cot^3(bx+a)\right)}{21}-\frac{2\cot(bx+a)\left(\csc^2(bx+a)\right)}{3}}{\sqrt{d\tan(bx+a)}\,b}$	43

```
[In] int(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/21/b/(d*tan(b*x+a))^(1/2)*(4*cot(b*x+a)^3-7*cot(b*x+a)*csc(b*x+a)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2\left(4\cos\left(bx+a\right)^4 - 7\cos\left(bx+a\right)^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{21\left(bd\cos\left(bx+a\right)^4 - 2\,bd\cos\left(bx+a\right)^2 + bd\right)}$$

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2/21*(4*cos(b*x + a)^4 - 7*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)
)/(b*d*cos(b*x + a)^4 - 2*b*d*cos(b*x + a)^2 + b*d)
```

Sympy [F]

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc^4\left(a+bx\right)}{\sqrt{d\tan\left(a+bx\right)}}\,dx$$

```
[In] integrate(csc(b*x+a)**4/(d*tan(b*x+a))**(1/2),x)
[Out] Integral(csc(a + b*x)**4/sqrt(d*tan(a + b*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\left(7\,d^2\tan\left(bx+a\right)^2 + 3\,d^2\right)d}{21\,\left(d\tan\left(bx+a\right)\right)^{\frac{7}{2}}\,b}$$

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] -2/21*(7*d^2*tan(b*x + a)^2 + 3*d^2)*d/((d*tan(b*x + a))^(7/2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\left(7\,d^3\tan\left(bx+a\right)^2+3\,d^3\right)}{21\,\sqrt{d\tan\left(bx+a\right)}bd^3\tan\left(bx+a\right)^3}$$

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2/21*(7*d^3*tan(b*x + a)^2 + 3*d^3)/(sqrt(d*tan(b*x + a))*b*d^3*tan(b*x +
a)^3)
```

Mupad [B] (verification not implemented)

Time = 7.66 (sec) , antiderivative size = 530, normalized size of antiderivative = 12.33

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{344\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{105\,b\,d\,\left(e^{a\,2i+b\,x\,2i}-1\right)} + \frac{40\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{21\,b\,d\left(e^{a\,2i+b\,x\,2i}-1\right)^2} + \frac{24\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{35\,b\,d\left(e^{a\,2i+b\,x\,2i}-1\right)^3} - \frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,304i}{105\,b\,d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)} + \frac{16\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{7\,b\,d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^2} + \frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,144i}{35\,b\,d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^3} - \frac{16\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{7\,b\,d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^4}$$

```
[In] int(1/(sin(a + b*x)^4*(d*tan(a + b*x))^(1/2)),x)
```

```
[Out] (344*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i
+ b*x*2i) + 1))^(1/2))/(105*b*d*(exp(a*2i + b*x*2i) - 1)) + (40*(exp(a*2i +
 b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^
(1/2))/(21*b*d*(exp(a*2i + b*x*2i) - 1)^2) + (24*(exp(a*2i + b*x*2i) + 1)*(
```

```
-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(35*b*d*
(exp(a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*
x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*304i)/(105*b*d*(exp(a*2i +
b*x*2i)*1i - 1i)) + (16*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1
i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(7*b*d*(exp(a*2i + b*x*2i)*1i - 1
i)^2) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a
*2i + b*x*2i) + 1))^(1/2)*144i)/(35*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^3) - (
16*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i +
b*x*2i) + 1))^(1/2))/(7*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^4)
```

3.88 $\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d^5}{11b(d\tan(a+bx))^{11/2}} - \frac{4d^3}{7b(d\tan(a+bx))^{7/2}} - \frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

```
[Out] -2/11*d^5/b/(d*tan(b*x+a))^(11/2)-4/7*d^3/b/(d*tan(b*x+a))^(7/2)-2/3*d/b/(d
*tan(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 276}

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d^5}{11b(d\tan(a+bx))^{11/2}} - \frac{4d^3}{7b(d\tan(a+bx))^{7/2}} - \frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

```
[In] Int[Csc[a + b*x]^6/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d^5)/(11*b*(d*Tan[a + b*x])^(11/2)) - (4*d^3)/(7*b*(d*Tan[a + b*x])^(7/
2)) - (2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\text{Subst}\left(\int \frac{(d^2+x^2)^2}{x^{13/2}}\,dx, x, d\tan(a+bx)\right)}{b} \\
&= \frac{d\text{Subst}\left(\int \left(\frac{d^4}{x^{13/2}}+\frac{2d^2}{x^{9/2}}+\frac{1}{x^{5/2}}\right)\,dx, x, d\tan(a+bx)\right)}{b} \\
&= -\frac{2d^5}{11b(d\tan(a+bx))^{11/2}}-\frac{4d^3}{7b(d\tan(a+bx))^{7/2}}-\frac{2d}{3b(d\tan(a+bx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2d(-45+28\cos(2(a+bx))-4\cos(4(a+bx)))\csc^4(a+bx)}{231b(d\tan(a+bx))^{3/2}}$$

```
[In] Integrate[Csc[a + b*x]^6/Sqrt[d*Tan[a + b*x]],x]
[Out] (2*d*(-45 + 28*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4)/(231*
b*(d*Tan[a + b*x])^(3/2))
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{2\cot(bx+a)(\csc^4(bx+a))(32(\cos^4(bx+a))-88(\cos^2(bx+a))+77)}{231b\sqrt{d\tan(bx+a)}}$	52

```
[In] int(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/231/b*cot(b*x+a)*csc(b*x+a)^4*(32*cos(b*x+a)^4-88*cos(b*x+a)^2+77)/(d*ta
n(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2\left(32\cos(bx+a)^6 - 88\cos(bx+a)^4 + 77\cos(bx+a)^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{231\left(bd\cos(bx+a)^6 - 3\,bd\cos(bx+a)^4 + 3\,bd\cos(bx+a)^2 - bd\right)}$$

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(32*cos(b*x + a)^6 - 88*cos(b*x + a)^4 + 77*cos(b*x + a)^2)*sqrt(d*si
n(b*x + a)/cos(b*x + a))/(b*d*cos(b*x + a)^6 - 3*b*d*cos(b*x + a)^4 + 3*b*d
*cos(b*x + a)^2 - b*d)
```

Sympy [F]

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

```
[In] integrate(csc(b*x+a)**6/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)**6/sqrt(d*tan(a + b*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\left(77\,d^4\tan(bx+a)^4 + 66\,d^4\tan(bx+a)^2 + 21\,d^4\right)d}{231\left(d\tan(bx+a)\right)^{\frac{11}{2}}b}$$

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/231*(77*d^4*tan(b*x + a)^4 + 66*d^4*tan(b*x + a)^2 + 21*d^4)*d/((d*tan(b
*x + a))^(11/2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\left(77\,d^5\tan\left(bx+a\right)^4+66\,d^5\tan\left(bx+a\right)^2+21\,d^5\right)}{231\,\sqrt{d\tan\left(bx+a\right)}bd^5\tan\left(bx+a\right)^5}$$

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] -2/231*(77*d^5*tan(b*x + a)^4 + 66*d^5*tan(b*x + a)^2 + 21*d^5)/(sqrt(d*tan
(b*x + a))*b*d^5*tan(b*x + a)^5)
```

Mupad [B] (verification not implemented)

Time = 12.24 (sec) , antiderivative size = 831, normalized size of antiderivative = 12.78

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \text{Too large to display}$$

```
[In] int(1/(sin(a + b*x)^6*(d*tan(a + b*x))^(1/2)),x)
[Out] ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*
x*2i) + 1))^(1/2)*44864i)/(10395*b*d*(exp(a*2i + b*x*2i)*1i - 1i)) - (128*(
exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*
2i) + 1))^(1/2))/(35*b*d*(exp(a*2i + b*x*2i) - 1)^2) - (7136*(exp(a*2i + b*
x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/
2))/(1155*b*d*(exp(a*2i + b*x*2i) - 1)^3) - (1216*(exp(a*2i + b*x*2i) + 1)*
(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(231*b*
d*(exp(a*2i + b*x*2i) - 1)^4) - (160*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2
i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(99*b*d*(exp(a*2i +
b*x*2i) - 1)^5) - (41984*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*
1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(10395*b*d*(exp(a*2i + b*x*2i) -
 1)) - (3904*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(e
xp(a*2i + b*x*2i) + 1))^(1/2))/(1155*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^2) -
((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*
x*2i) + 1))^(1/2)*1088i)/(165*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^3) + (320*(e
xp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2
i) + 1))^(1/2))/(21*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^4) + ((exp(a*2i + b*x*
2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)
*1600i)/(99*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^5) - (64*(exp(a*2i + b*x*2i) +
 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(11
*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^6)
```

3.89 $\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}} - \frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}} + \frac{7E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{20b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] -7/20*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-7/30*d
*sin(b*x+a)^3/b/(d*tan(b*x+a))^(3/2)-1/5*d*sin(b*x+a)^5/b/(d*tan(b*x+a))^(3
/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2678, 2681, 2652, 2719}

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}} - \frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}} + \frac{7\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{20b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Sin[a + b*x]^5/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-7*d*Sin[a + b*x]^3)/(30*b*(d*Tan[a + b*x])^(3/2)) - (d*Sin[a + b*x]^5)/(5
*b*(d*Tan[a + b*x])^(3/2)) + (7*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/
(20*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{7}{10}\int\frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\
&= -\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{7}{20}\int\frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\
&= -\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\\
&\quad+\frac{\left(7\sqrt{\sin(a+bx)}\right)\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{20\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{(7\sin(a+bx))\int\sqrt{\sin(2a+2bx)}\,dx}{20\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{7E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{20b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.94 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$
$$= \frac{\sin(a+bx)\left(-20\sin(2(a+bx))+3\sin(4(a+bx))+28\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\text{se}}\right.}{120b\sqrt{d\tan(a+bx)}}$$

```
[In] Integrate[Sin[a + b*x]^5/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (Sin[a + b*x]*(-20*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + 28*Hypergeometri
c2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*Tan[a + b*x]))/(1
20*b*Sqrt[d*Tan[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(118) = 236.

Time = 1.12 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.64

method	result
default	$-\frac{\left(12(\cos^5(bx+a))\sqrt{2}-38(\cos^3(bx+a))\sqrt{2}+42\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\right.}{}$

```
[In] int(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120/b/(d*tan(b*x+a))^(1/2)*(12*cos(b*x+a)^5*2^(1/2)-38*cos(b*x+a)^3*2^(1
/2)+42*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(co
t(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^
(1/2))-21*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1
+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2
*2^(1/2))+42*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(
b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x
+a))^(1/2),1/2*2^(1/2))-21*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc
(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(
b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+47*2^(1/2)*cos(b*x+a)-21*2^(1/2))*2^(
1/2)
```

Fricas [F]

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)^5}{\sqrt{d\tan(bx+a)}}\,dx$$

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*tan(b*x + a))*sin(b
*x + a)/(d*tan(b*x + a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**5/(d*tan(b*x+a))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)^5}{\sqrt{d\tan(bx+a)}}\,dx$$

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^5/sqrt(d*tan(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)^5}{\sqrt{d\tan(bx+a)}}\,dx$$

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^5/sqrt(d*tan(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)^5}{\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

```
[In] int(sin(a + b*x)^5/(d*tan(a + b*x))^(1/2),x)
[Out] int(sin(a + b*x)^5/(d*tan(a + b*x))^(1/2), x)
```

3.90 $\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} + \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] -1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-1/3*d*s
in(b*x+a)^3/b/(d*tan(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2678, 2681, 2652, 2719}

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} - \frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

```
[In] Int[Sin[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -1/3*(d*Sin[a + b*x]^3)/(b*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 +
b*x, 2]*Sin[a + b*x])/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}+\frac{1}{2}\int\frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\
&= -\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}+\frac{\sqrt{\sin(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}+\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}\,dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}+\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.77 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.24

$$\int\frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$=\frac{\sqrt{d\tan(a+bx)}\left(-\sqrt{\sec^2(a+bx)}(\sin(a+bx)+\sin(3(a+bx)))+4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a\right.\right.}{12bd\sqrt{\sec^2(a+bx)}}$$

```
[In] Integrate[Sin[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (Sqrt[d*Tan[a + b*x]]*(-(Sqrt[Sec[a + b*x]^2]*(Sin[a + b*x] + Sin[3*(a + b*
x)])) + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Ta
n[a + b*x]))/(12*b*d*Sqrt[Sec[a + b*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(94) = 188$.

Time = 0.99 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.77

method	result
default	$\frac{\left(2\left(\cos^3(bx+a)\right)\sqrt{2}-6\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(b\right.\right.}{}$

```
[In] int(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/b/(d*tan(b*x+a))^(1/2)*(2*cos(b*x+a)^3*2^(1/2)-6*(1+csc(b*x+a)-cot(b*x
+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*E
llipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+3*(cot(b*x+a)-csc(b*x
+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)
*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-6*sec(b*x+a)*(1+csc
(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(
b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+3*sec(
b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(co
t(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^
(1/2))-5*2^(1/2)*cos(b*x+a)+3*2^(1/2))*2^(1/2)
```

Fricas [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a)/(d*tan(b*x
 + a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**3/(d*tan(b*x+a))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)^3}{\sqrt{d\tan(bx+a)}}\,dx$$

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)^3}{\sqrt{d\tan(bx+a)}}\,dx$$

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(a+b\,x)^3}{\sqrt{d\tan(a+b\,x)}}\,dx$$

```
[In] int(sin(a + b*x)^3/(d*tan(a + b*x))^(1/2),x)
[Out] int(sin(a + b*x)^3/(d*tan(a + b*x))^(1/2), x)
```

3.91 $\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] -(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),
2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2681, 2652, 2719}

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Sin[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[
d*Tan[a + b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\sin(a+bx)} \int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= \frac{\sin(a+bx) \int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} \\
&= \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$
\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\sin(a+bx)\sqrt{d\tan(a+bx)}}{3bd}
$$

```
[In] Integrate[Sin[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*S
in[a + b*x]*Sqrt[d*Tan[a + b*x]])/(3*b*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(69) = 138.

Time = 0.83 (sec) , antiderivative size = 363, normalized size of antiderivative = 7.72

method	result
default	$-\frac{\left(2\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)-\sqrt{\mathrm{co}}\right.}{}$

[In] int(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2/b/(d*tan(b*x+a))^(1/2)*(2*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(b*x+a)-2^(1/2))*2^(1/2)

Fricas [F]

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)}{\sqrt{d\tan(bx+a)}}\,dx$$

[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d*tan(b*x + a))*sin(b*x + a)/(d*tan(b*x + a)), x)

Sympy [F]

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

[In] integrate(sin(b*x+a)/(d*tan(b*x+a))**(1/2),x)

[Out] Integral(sin(a + b*x)/sqrt(d*tan(a + b*x)), x)

Maxima [F]

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(bx+a\right)}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(bx+a\right)}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)}{\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

```
[In] int(sin(a + b*x)/(d*tan(a + b*x))^(1/2),x)
[Out] int(sin(a + b*x)/(d*tan(a + b*x))^(1/2), x)
```

3.92 $\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}} \, dx = -\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] -2*cos(b*x+a)/b/(d*tan(b*x+a))^(1/2)+2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/
4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^
(1/2)/(d*tan(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2681, 2650, 2652, 2719}

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}} \, dx = -\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Csc[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*Cos[a + b*x])/(b*Sqrt[d*Tan[a + b*x]]) - (2*EllipticE[a - Pi/4 + b*x, 2
]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
```

```
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{\left(2\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{(2\sin(a+bx))\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\cos(a+bx)\left(3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\tan^2(a+bx)\right)}{3b\sqrt{d\tan(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*Cos[a + b*x]*(3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*S
qrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(3*b*Sqrt[d*Tan[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(91) = 182$.

Time = 0.88 (sec) , antiderivative size = 349, normalized size of antiderivative = 4.85

method	result
default	$-\frac{\left(-2\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)+\sqrt{}\right.}{}$

```
[In] int(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(d*tan(b*x+a))^(1/2)*(-2*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+
1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-c
ot(b*x+a))^(1/2),1/2*2^(1/2))+(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+
cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-c
ot(b*x+a))^(1/2),1/2*2^(1/2))-2*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*
(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1
+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+sec(b*x+a)*(1+csc(b*x+a)-cot(b*x
+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*E
llipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.35

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)^2 + i\sqrt{i\,d}E(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)\sin(bx+a) - i\sqrt{-i\,d}}{}$$

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2 + I*sqrt(I*d)*elliptic
_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) - I*sqrt(-I*d)*e
lliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) - I*sqrt(
I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + I
*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x +
 a))/(b*d*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc\left(a+bx\right)}{\sqrt{d\tan\left(a+bx\right)}}\,dx$$

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))**(1/2),x)
[Out] Integral(csc(a + b*x)/sqrt(d*tan(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc\left(bx+a\right)}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

Giac [F]

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc\left(bx+a\right)}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)\,\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

```
[In] int(1/(sin(a + b*x)*(d*tan(a + b*x))^(1/2)),x)
[Out] int(1/(sin(a + b*x)*(d*tan(a + b*x))^(1/2)), x)
```

3.93 $\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}} - \frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}} - \frac{4E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] -4/5*cos(b*x+a)/b/(d*tan(b*x+a))^(1/2)+4/5*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(
a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2
*a)^(1/2)/(d*tan(b*x+a))^(1/2)-2/5*d*csc(b*x+a)/b/(d*tan(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2679, 2681, 2650, 2652, 2719}

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}} - \frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}} - \frac{4\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Csc[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d*Csc[a + b*x])/(5*b*(d*Tan[a + b*x])^(3/2)) - (4*Cos[a + b*x])/(5*b*Sq
rt[d*Tan[a + b*x]]) - (4*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(5*b*Sq
rt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}} + \frac{2}{5}\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}} + \frac{\left(2\sqrt{\sin(a+bx)}\right)\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$= -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}} - \frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}} - \frac{\left(4\sqrt{\sin(a+bx)}\right)\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$= -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}} - \frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}} - \frac{(4\sin(a+bx))\int\sqrt{\sin(2a+2bx)}\,dx}{5\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$= -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}} - \frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}} - \frac{4E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.70 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$
$$= \frac{6(-2+\cos(2(a+bx)))\cot(a+bx)\csc(a+bx)\sqrt{\sec^2(a+bx)} - 8\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+\right.}{15b\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (6*(-2 + Cos[2*(a + b*x)])*Cot[a + b*x]*Csc[a + b*x]*Sqrt[Sec[a + b*x]^2] -
 8*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Tan[a + b
*x]^2)/(15*b*Sqrt[Sec[a + b*x]^2]*Sqrt[d*Tan[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(113) = 226.

Time = 1.03 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.63

method	result
default	$\frac{\left(4\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)-2\sqrt{\cot(}\right.}{}$

```
[In] int(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/b/(d*tan(b*x+a))^(1/2)*(4*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+
1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-c
ot(b*x+a))^(1/2),1/2*2^(1/2))-2*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+
1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)
```

```
-cot(b*x+a))^(1/2),1/2*2^(1/2))+4*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2
)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE(
(1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-2*sec(b*x+a)*(1+csc(b*x+a)-cot
(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/
2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-2*2^(1/2)-cot(b*x
+a)*csc(b*x+a)*2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.32

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\left(\left(i\cos(bx+a)^2-i\right)\sqrt{i\,d}E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a)+\left(-i\cos(bx+a)\right.\right.}{}$$

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*((I*cos(b*x + a)^2 - I)*sqrt(I*d)*elliptic_e(arcsin(cos(b*x + a) + I*s
in(b*x + a)), -1)*sin(b*x + a) + (-I*cos(b*x + a)^2 + I)*sqrt(-I*d)*ellipti
c_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (-I*cos(b*x +
 a)^2 + I)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*
sin(b*x + a) + (I*cos(b*x + a)^2 - I)*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x
+ a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (2*cos(b*x + a)^4 - 3*cos(b*x +
a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/((b*d*cos(b*x + a)^2 - b*d)*sin(b*
x + a))
```

Sympy [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc^3\left(a+bx\right)}{\sqrt{d\tan\left(a+bx\right)}}\,dx$$

```
[In] integrate(csc(b*x+a)**3/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)**3/sqrt(d*tan(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

Giac [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^3\,\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

```
[In] int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2)),x)
[Out] int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2)), x)
```

3.94 $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 257

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{3/2}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{3/2}} + \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{3/2}} - \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{3/2}} + \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3}$$

```
[Out] -3/64*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)+3/64
*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)+3/128*ln(
d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1/2)-
3/128*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)
*2^(1/2)+3/16*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d^3-1/4*cos(b*x+a)^4*(d*t
an(b*x+a))^(3/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2671, 294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{3/2}} + \frac{3\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}bd^{3/2}} + \frac{3\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}bd^{3/2}} - \frac{3\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}bd^{3/2}} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3} + \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3}$$

```
[In] Int[Sin[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*d^(3/
2)) + (3*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*
d^(3/2)) + (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b
*x]]])/(64*Sqrt[2]*b*d^(3/2)) - (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqr
t[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b*d^(3/2)) + (3*Cos[a + b*x]^2*(d*T
an[a + b*x])^(3/2))/(16*b*d^3) - (Cos[a + b*x]^4*(d*Tan[a + b*x])^(3/2))/(4
*b*d^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\text{Subst}\left(\int \frac{x^{5/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}\\
&= -\frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3} + \frac{(3d)\text{Subst}\left(\int \frac{\sqrt{x}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{8b}\\
&= \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3}\\
&\quad + \frac{3\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{32bd}\\
&= \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3}\\
&\quad + \frac{3\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{16bd}\\
&= \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3}\\
&\quad - \frac{3\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{32bd}\\
&\quad + \frac{3\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{32bd}\\
&= \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3}\\
&\quad + \frac{3\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{3/2}}\\
&\quad + \frac{3\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{3/2}}\\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64bd}\\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64bd}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{3/2}} \\
&\quad - \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{3/2}} \\
&\quad + \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{3/2}} \\
&\quad - \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{3/2}} \\
&= -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{3/2}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{3/2}} \\
&\quad + \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{3/2}} \\
&\quad - \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{3/2}} \\
&\quad + \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.48

$$
\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\csc(a+bx)\left(\cos(a+bx)-2\cos(3(a+bx))+\cos(5(a+bx))-3\arcsin(\cos(a+b\right.}{}
$$

```
[In] Integrate[Sin[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Csc[a + b*x]*(Cos[a + b*x] - 2*Cos[3*(a + b*x)] + Cos[5*(a + b*x)] - 3*Arc
Sin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt[Sin[2*(a + b*x)]] - 3*Log[Cos[a + b*x
] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x)]])*Sqrt[d*T
an[a + b*x]])/(64*b*d^2)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(197) = 394$.

Time = 10.06 (sec) , antiderivative size = 621, normalized size of antiderivative = 2.42

method	result
default	$\frac{\csc(bx+a)(-1+\cos(bx+a))\left(16\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\,(\cos^3(bx+a))\sin(bx+a)+16(\cos^2(bx+a))\sin(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(}{(\cos(bx+a)}}\right.}{}$

```
[In] int(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/b*csc(b*x+a)*(-1+cos(b*x+a))*(16*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos
(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^3*sin(b*x+a)+16*cos(b*x+a)^2*sin(b*x+a)*2^(1
/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-12*cos(b*x+a)*sin(b*x+a
)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-12*sin(b*x+a)*2^(
1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-6*arctan((sin(b*x+a)*2
^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+co
s(b*x+a)))-6*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)
+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))-3*ln(-(cot(b*x+a)*cos(b*x+a)-2*
cot(b*x+a)-2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+
a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(
b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))+3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*
x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc
(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)
+csc(b*x+a)+2)/(-1+cos(b*x+a))))/(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^
(1/2)/(d*tan(b*x+a))^(1/2)/d*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 986, normalized size of antiderivative = 3.84

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/256*(3*b*d^2*(-1/(b^4*d^6))^(1/4)*log(1/2*cos(b*x + a)*sin(b*x + a) + 1/2
*(b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 - b*d*(-1/(b^4*d^6))^(1/4)*co
s(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1/4*(2*b^2*d^3
*cos(b*x + a)^2 - b^2*d^3)*sqrt(-1/(b^4*d^6))) - 3*b*d^2*(-1/(b^4*d^6))^(1/
4)*log(1/2*cos(b*x + a)*sin(b*x + a) - 1/2*(b^3*d^4*(-1/(b^4*d^6))^(3/4)*co
```

```
s(b*x + a)^2 - b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*s
in(b*x + a)/cos(b*x + a)) - 1/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-
1/(b^4*d^6))) - 3*I*b*d^2*(-1/(b^4*d^6))^(1/4)*log(1/2*cos(b*x + a)*sin(b*x
 + a) + 1/2*(I*b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 + I*b*d*(-1/(b^4
*d^6))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) +
 1/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-1/(b^4*d^6))) + 3*I*b*d^2*(
-1/(b^4*d^6))^(1/4)*log(1/2*cos(b*x + a)*sin(b*x + a) + 1/2*(-I*b^3*d^4*(-1
/(b^4*d^6))^(3/4)*cos(b*x + a)^2 - I*b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)*
sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1/4*(2*b^2*d^3*cos(b*x +
a)^2 - b^2*d^3)*sqrt(-1/(b^4*d^6))) + 3*b*d^2*(-1/(b^4*d^6))^(1/4)*log(2*(b
^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) - b*d*(-1/(b^4*d^6))^
(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) - 3*b*d^2*(-1/
(b^4*d^6))^(1/4)*log(-2*(b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x
+ a) - b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x
 + a)) + 1) + 3*I*b*d^2*(-1/(b^4*d^6))^(1/4)*log(-2*(I*b^3*d^4*(-1/(b^4*d^6
))^(3/4)*cos(b*x + a)*sin(b*x + a) + I*b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a
)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) - 3*I*b*d^2*(-1/(b^4*d^6))^(1/4
)*log(-2*(-I*b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) - I*b*d
*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1
) - 16*(4*cos(b*x + a)^3 - 3*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)
)*sin(b*x + a))/(b*d^2)
```

Sympy [F]

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin^4{(a+bx)}}{(d\tan{(a+bx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)**4/(d*tan(b*x+a))**(3/2),x)
[Out] Integral(sin(a + b*x)**4/(d*tan(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.88

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(d\right.}{}\right.}{}$$

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/128*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a) +
 sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x
+ a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) + 8*(3*(d*tan(b*x
 + a))^(7/2)*d^4 - (d*tan(b*x + a))^(3/2)*d^6)/(d^4*tan(b*x + a)^4 + 2*d^4*
tan(b*x + a)^2 + d^4))/(b*d^5)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}+\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}-\frac{3\sqrt{2}}{}}{}$$

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*(6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*
sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^2) + 6*sqrt(2)*abs(d)^(3/2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))
/(b*d^2) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b
*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2) + 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(
b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2) + 8*
(3*sqrt(d*tan(b*x + a))*d^3*tan(b*x + a)^3 - sqrt(d*tan(b*x + a))*d^3*tan(b
*x + a))/((d^2*tan(b*x + a)^2 + d^2)^2*b))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{{\sin\left(a+b\,x\right)}^4}{{\left(d\,\tan\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int(sin(a + b*x)^4/(d*tan(a + b*x))^(3/2),x)
```

```
[Out] int(sin(a + b*x)^4/(d*tan(a + b*x))^(3/2), x)
```

3.95 $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 227

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} + \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3}$$

```
[Out] -1/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)+1/8*a
rctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)+1/16*ln(d^(
1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1/2)-1/1
6*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(
1/2)+1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used

= {2671, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}bd^{3/2}} + \frac{\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}bd^{3/2}} - \frac{\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}bd^{3/2}} + \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3}$$

```
[In] Int[Sin[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/4*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*b*d^(3/2))
 + ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(4*Sqrt[2]*b*d^(3/2))
 + Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]/(8*Sq
rt[2]*b*d^(3/2)) - Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqrt[d*Tan[
a + b*x]]]/(8*Sqrt[2]*b*d^(3/2)) + (Cos[a + b*x]^2*(d*Tan[a + b*x])^(3/2))/
(2*b*d^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{d\text{Subst}\left(\int \frac{\sqrt{x}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{4bd}$$

$$= \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3} + \frac{\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2bd}$$

$$=\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3}-\frac{\text{Subst}\left(\int\frac{d-x^2}{d^2+x^4}\,dx,x,\sqrt{d\tan(a+bx)}\right)}{4bd}$$
$$+\frac{\text{Subst}\left(\int\frac{d+x^2}{d^2+x^4}\,dx,x,\sqrt{d\tan(a+bx)}\right)}{4bd}$$

$$=\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3}+\frac{\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx,x,\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}}$$
$$+\frac{\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx,x,\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}}$$
$$+\frac{\text{Subst}\left(\int\frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx,x,\sqrt{d\tan(a+bx)}\right)}{8bd}$$
$$+\frac{\text{Subst}\left(\int\frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx,x,\sqrt{d\tan(a+bx)}\right)}{8bd}$$

$$=\frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}}$$
$$-\frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}}$$
$$+\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3}+\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}}$$
$$-\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}}$$

$$=-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}}$$
$$+\frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}}$$
$$-\frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}}$$
$$+\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.46

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\left(\arcsin(\cos(a+bx)-\sin(a+bx))\csc(a+bx)+\csc(a+bx)\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+\right.\right.}{8bd^2}$$

```
[In] Integrate[Sin[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/8*((ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Csc[a + b*x] + Csc[a + b*x]*Log[
Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]] - 2*Sqrt[Sin[2*(a + b
*x)]])*Sqrt[Sin[2*(a + b*x)]]*Sqrt[d*Tan[a + b*x]])/(b*d^2)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. 2(171) = 342.

Time = 13.24 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.33

method	result
default	$-\frac{\csc(bx+a)(-1+\cos(bx+a))\left(4\cos(bx+a)\sin(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}+4\sin(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}+\ln\left(\frac{2\sin(bx\right.\right.}{}$

```
[In] int(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/b*csc(b*x+a)*(-1+cos(b*x+a))*(4*cos(b*x+a)*sin(b*x+a)*2^(1/2)*(-cos(b
*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+4*sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*
sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+ln((2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*
x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x
+a))^(1/2)-cot(b*x+a)*cos(b*x+a)+2*cot(b*x+a)+2*cos(b*x+a)+sin(b*x+a)-csc(b
*x+a)-2)/(-1+cos(b*x+a)))-ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x
+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(
b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)
/(-1+cos(b*x+a)))+2*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos
(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))+2*arctan((sin(b*x+a)*2^(
1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(
b*x+a))))/(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*tan(b*x+a))^(1
/2)/d*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 971, normalized size of antiderivative = 4.28

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(b*d^2*(-1/(b^4*d^6))^(1/4)*log(1/2*cos(b*x + a)*sin(b*x + a) + 1/2*(b
^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 - b*d*(-1/(b^4*d^6))^(1/4)*cos(b
*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1/4*(2*b^2*d^3*co
s(b*x + a)^2 - b^2*d^3)*sqrt(-1/(b^4*d^6))) - b*d^2*(-1/(b^4*d^6))^(1/4)*lo
g(1/2*cos(b*x + a)*sin(b*x + a) - 1/2*(b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x
 + a)^2 - b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*
x + a)/cos(b*x + a)) - 1/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-1/(b^
4*d^6))) - I*b*d^2*(-1/(b^4*d^6))^(1/4)*log(1/2*cos(b*x + a)*sin(b*x + a) +
 1/2*(I*b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 + I*b*d*(-1/(b^4*d^6))^
(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1/4*(2
*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-1/(b^4*d^6))) + I*b*d^2*(-1/(b^4*d
^6))^(1/4)*log(1/2*cos(b*x + a)*sin(b*x + a) + 1/2*(-I*b^3*d^4*(-1/(b^4*d^6
))^(3/4)*cos(b*x + a)^2 - I*b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)*sin(b*x +
 a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1/4*(2*b^2*d^3*cos(b*x + a)^2 - b^
2*d^3)*sqrt(-1/(b^4*d^6))) + b*d^2*(-1/(b^4*d^6))^(1/4)*log(2*(b^3*d^4*(-1/
(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) - b*d*(-1/(b^4*d^6))^(1/4)*cos(b
*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) - b*d^2*(-1/(b^4*d^6))^(1
/4)*log(-2*(b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) - b*d*(-
1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) +
 I*b*d^2*(-1/(b^4*d^6))^(1/4)*log(-2*(I*b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*
x + a)*sin(b*x + a) + I*b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin
(b*x + a)/cos(b*x + a)) + 1) - I*b*d^2*(-1/(b^4*d^6))^(1/4)*log(-2*(-I*b^3*
d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) - I*b*d*(-1/(b^4*d^6))^(
1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) + 16*sqrt(d*sin
(b*x + a)/cos(b*x + a))*cos(b*x + a)*sin(b*x + a))/(b*d^2)
```

Sympy [F]

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.85

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan\cdots\right.}{\cdots}\right.}{16\cdots}$$

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(b*x
 + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a) + sq
rt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x + a
) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) + 8*(d*tan(b*x + a))
^(3/2)*d^2/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\frac{8\sqrt{d\tan(bx+a)}\,d\tan(bx+a)}{\left(d^2\tan(bx+a)^2+d^2\right)b}+\frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}+\frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\cdots\right.}{\cdots}\right.}{bd^2}\cdots}{\cdots}$$

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(8*sqrt(d*tan(b*x + a))*d*tan(b*x + a)/((d^2*tan(b*x + a)^2 + d^2)*b)
+ 2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(
d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^2) + 2*sqrt(2)*abs(d)^(3/2)*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d
^2) - sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a)
)*sqrt(abs(d)) + abs(d))/(b*d^2) + sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a)
- sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin(a+b\,x)^2}{(d\tan(a+b\,x))^{3/2}}\,dx$$

```
[In] int(sin(a + b*x)^2/(d*tan(a + b*x))^(3/2),x)
[Out] int(sin(a + b*x)^2/(d*tan(a + b*x))^(3/2), x)
```

3.96 $\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

```
[Out] -2/5*d/b/(d*tan(b*x+a))^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 30}

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

```
[In] Int[Csc[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
[Out] (-2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
```

```
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d\text{Subst}\left(\int \frac{1}{x^{7/2}}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= -\frac{2d}{5b(d\tan(a+bx))^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

```
[In] Integrate[Csc[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
[Out] (-2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2d}{5b(d\tan(bx+a))^{\frac{5}{2}}}$	17
default	$-\frac{2d}{5b(d\tan(bx+a))^{\frac{5}{2}}}$	17

```
[In] int(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/5*d/b/(d*tan(b*x+a))^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)^3}{5\left(bd^2\cos(bx+a)^2 - bd^2\right)\sin(bx+a)}$$

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^3/((b*d^2*cos(b*x + a)^2
 - b*d^2)*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
[Out] Integral(csc(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{5\,(d\tan(bx+a))^{\frac{3}{2}}\,b\tan(bx+a)}$$

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] -2/5/((d*tan(b*x + a))^(3/2)*b*tan(b*x + a))
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{5\,\sqrt{d\tan(bx+a)}bd\tan(bx+a)^2}$$

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] -2/5/(sqrt(d*tan(b*x + a))*b*d*tan(b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 7.07 (sec) , antiderivative size = 381, normalized size of antiderivative = 19.05

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,14\mathrm{i}}{5\,b\,d^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)} - \frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,8\mathrm{i}}{15\,b\,d^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^2} - \frac{16\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{5\,b\,d^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)} - \frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{15\,b\,d^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)^2} + \frac{8\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{5\,b\,d^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)^3}$$

```
[In] int(1/(sin(a + b*x)^2*(d*tan(a + b*x))^(3/2)),x)
```

```
[Out] (8*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i +
b*x*2i) + 1))^(1/2))/(5*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)^3) - ((exp(a*2i
+ b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))
^(1/2)*8i)/(15*b*d^2*(exp(a*2i + b*x*2i) - 1)^2) - (16*(exp(a*2i + b*x*2i)
+ 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(5
*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(
a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*32i)/(15*b*d^2*(ex
p(a*2i + b*x*2i)*1i - 1i)^2) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b
*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*14i)/(5*b*d^2*(exp(a*2i +
b*x*2i) - 1))
```

3.97 $\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2d^3}{9b(d\tan(a+bx))^{9/2}}-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

```
[Out] -2/9*d^3/b/(d*tan(b*x+a))^(9/2)-2/5*d/b/(d*tan(b*x+a))^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 14}

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2d^3}{9b(d\tan(a+bx))^{9/2}}-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

```
[In] Int[Csc[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^3)/(9*b*(d*Tan[a + b*x])^(9/2)) - (2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
```

```
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{d\text{Subst}\left(\int \frac{d^2+x^2}{x^{11/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= \frac{d\text{Subst}\left(\int \left(\frac{d^2}{x^{11/2}}+\frac{1}{x^{7/2}}\right)\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{2d^3}{9b(d\tan(a+bx))^{9/2}} - \frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2(4+\csc^2(a+bx)-5\csc^4(a+bx))}{45bd\sqrt{d\tan(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
[Out] (2*(4 + Csc[a + b*x]^2 - 5*Csc[a + b*x]^4))/(45*b*d*Sqrt[d*Tan[a + b*x]])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\frac{8\left(\cot^4(bx+a)\right)}{45}-\frac{2\left(\cot^2(bx+a)\right)\left(\csc^2(bx+a)\right)}{5}}{bd\sqrt{d\tan(bx+a)}}$	48

```
[In] int(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/45/b/(d*tan(b*x+a))^(1/2)/d*(4*cot(b*x+a)^4-9*cot(b*x+a)^2*csc(b*x+a)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(35) = 70.

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.95

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\left(4\cos\left(bx+a\right)^5 - 9\cos\left(bx+a\right)^3\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{45\left(bd^2\cos\left(bx+a\right)^4 - 2\,bd^2\cos\left(bx+a\right)^2 + bd^2\right)\sin\left(bx+a\right)}$$

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] 2/45*(4*cos(b*x + a)^5 - 9*cos(b*x + a)^3)*sqrt(d*sin(b*x + a)/cos(b*x + a)
)/((b*d^2*cos(b*x + a)^4 - 2*b*d^2*cos(b*x + a)^2 + b*d^2)*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc^4\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)**4/(d*tan(b*x+a))**(3/2),x)
[Out] Integral(csc(a + b*x)**4/(d*tan(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(9\,d^2\tan\left(bx+a\right)^2 + 5\,d^2\right)d}{45\,\left(d\tan\left(bx+a\right)\right)^{\frac{9}{2}}\,b}$$

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] -2/45*(9*d^2*tan(b*x + a)^2 + 5*d^2)*d/((d*tan(b*x + a))^(9/2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(9\,d^4\tan\left(bx+a\right)^2+5\,d^4\right)}{45\,\sqrt{d\tan\left(bx+a\right)}bd^5\tan\left(bx+a\right)^4}$$

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] -2/45*(9*d^4*tan(b*x + a)^2 + 5*d^4)/(sqrt(d*tan(b*x + a))*b*d^5*tan(b*x +
a)^4)
```

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 684, normalized size of antiderivative = 15.91

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] int(1/(sin(a + b*x)^4*(d*tan(a + b*x))^(3/2)),x)
[Out] ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*
x*2i) + 1))^(1/2)*6088i)/(945*b*d^2*(exp(a*2i + b*x*2i) - 1)) + ((exp(a*2i
+ b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))
^(1/2)*4024i)/(945*b*d^2*(exp(a*2i + b*x*2i) - 1)^2) + ((exp(a*2i + b*x*2i)
 + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*20
0i)/(63*b*d^2*(exp(a*2i + b*x*2i) - 1)^3) + ((exp(a*2i + b*x*2i) + 1)*(-(d*
(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*64i)/(63*b*d^
2*(exp(a*2i + b*x*2i) - 1)^4) + (1184*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*
2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(189*b*d^2*(exp(a*2
i + b*x*2i)*1i - 1i)) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*
1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*4192i)/(945*b*d^2*(exp(a*2i + b*x
*2i)*1i - 1i)^2) - (2176*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*
1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(315*b*d^2*(exp(a*2i + b*x*2i)*1
i - 1i)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2)*512i)/(63*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)
^4) + (32*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(
a*2i + b*x*2i) + 1))^(1/2))/(9*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)^5)
```

3.98 $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2d^5}{13b(d\tan(a+bx))^{13/2}} - \frac{4d^3}{9b(d\tan(a+bx))^{9/2}} - \frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

```
[Out] -2/13*d^5/b/(d*tan(b*x+a))^(13/2)-4/9*d^3/b/(d*tan(b*x+a))^(9/2)-2/5*d/b/(d
*tan(b*x+a))^(5/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 276}

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2d^5}{13b(d\tan(a+bx))^{13/2}} - \frac{4d^3}{9b(d\tan(a+bx))^{9/2}} - \frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

```
[In] Int[Csc[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^5)/(13*b*(d*Tan[a + b*x])^(13/2)) - (4*d^3)/(9*b*(d*Tan[a + b*x])^(9/
2)) - (2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d\text{Subst}\left(\int \frac{(d^2+x^2)^2}{x^{15/2}}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= \frac{d\text{Subst}\left(\int \left(\frac{d^4}{x^{15/2}}+\frac{2d^2}{x^{11/2}}+\frac{1}{x^{7/2}}\right)\,dx, x, d\tan(a+bx)\right)}{b}\\ &= -\frac{2d^5}{13b(d\tan(a+bx))^{13/2}}-\frac{4d^3}{9b(d\tan(a+bx))^{9/2}}-\frac{2d}{5b(d\tan(a+bx))^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{64+16\csc^2(a+bx)+10\csc^4(a+bx)-90\csc^6(a+bx)}{585bd\sqrt{d\tan(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
[Out] (64 + 16*Csc[a + b*x]^2 + 10*Csc[a + b*x]^4 - 90*Csc[a + b*x]^6)/(585*b*d*S
qrt[d*Tan[a + b*x]])
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{2(\cot^2(bx+a))(\csc^4(bx+a))(32(\cos^4(bx+a))-104(\cos^2(bx+a))+117)}{585b\sqrt{d\tan(bx+a)}\,d}$	57

```
[In] int(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/585/b*cot(b*x+a)^2*csc(b*x+a)^4*(32*cos(b*x+a)^4-104*cos(b*x+a)^2+117)/(
d*tan(b*x+a))^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(53) = 106.

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.68

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\left(32\cos(bx+a)^7 - 104\cos(bx+a)^5 + 117\cos(bx+a)^3\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{585\left(bd^2\cos(bx+a)^6 - 3\,bd^2\cos(bx+a)^4 + 3\,bd^2\cos(bx+a)^2 - bd^2\right)\sin(bx+a)}$$

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/585*(32*cos(b*x + a)^7 - 104*cos(b*x + a)^5 + 117*cos(b*x + a)^3)*sqrt(d*
sin(b*x + a)/cos(b*x + a))/((b*d^2*cos(b*x + a)^6 - 3*b*d^2*cos(b*x + a)^4
+ 3*b*d^2*cos(b*x + a)^2 - b*d^2)*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)**6/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)**6/(d*tan(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(117\,d^4\tan(bx+a)^4 + 130\,d^4\tan(bx+a)^2 + 45\,d^4\right)d}{585\,\left(d\tan(bx+a)\right)^{\frac{13}{2}}\,b}$$

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/585*(117*d^4*tan(b*x + a)^4 + 130*d^4*tan(b*x + a)^2 + 45*d^4)*d/((d*tan
(b*x + a))^(13/2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(117\,d^6\tan\left(bx+a\right)^4+130\,d^6\tan\left(bx+a\right)^2+45\,d^6\right)}{585\,\sqrt{d\tan\left(bx+a\right)}bd^7\tan\left(bx+a\right)^6}$$

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] -2/585*(117*d^6*tan(b*x + a)^4 + 130*d^6*tan(b*x + a)^2 + 45*d^6)/(sqrt(d*t
an(b*x + a))*b*d^7*tan(b*x + a)^6)
```

Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 987, normalized size of antiderivative = 15.18

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] int(1/(sin(a + b*x)^6*(d*tan(a + b*x))^(3/2)),x)
[Out] (128*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i
+ b*x*2i) + 1))^(1/2))/(11*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)^3) - ((exp(a*
2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) +
1))^(1/2)*294464i)/(45045*b*d^2*(exp(a*2i + b*x*2i) - 1)^2) - ((exp(a*2i +
b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(
1/2)*24608i)/(2145*b*d^2*(exp(a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b*x*2i)
 + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*13
5104i)/(9009*b*d^2*(exp(a*2i + b*x*2i) - 1)^4) - ((exp(a*2i + b*x*2i) + 1)*
(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*13088i)/
(1287*b*d^2*(exp(a*2i + b*x*2i) - 1)^5) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(e
xp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*384i)/(143*b*d^
2*(exp(a*2i + b*x*2i) - 1)^6) - (55808*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a
*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(6435*b*d^2*(exp(a
*2i + b*x*2i)*1i - 1i)) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i
)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*7424i)/(1155*b*d^2*(exp(a*2i +
b*x*2i)*1i - 1i)^2) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i
 - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*18368i)/(2145*b*d^2*(exp(a*2i + b*x
*2i) - 1)) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2)*228736i)/(9009*b*d^2*(exp(a*2i + b*x*2i)*1i
- 1i)^4) - (17152*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i
))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(429*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)
^5) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2
i + b*x*2i) + 1))^(1/2)*4608i)/(143*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)^6) +
 (128*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i
 + b*x*2i) + 1))^(1/2))/(13*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)^7)
```

3.99 $\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} + \frac{\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{12bd^2}$$

```
[Out] -1/6*sin(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+1/3*sin(b*x+a)^3/b/d/(d*tan(b*x+a)
)^(1/2)-1/12*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Ellip
ticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
/d^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2676, 2678, 2681, 2653, 2720}

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{12bd^2} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}-\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Sin[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/6*Sin[a + b*x]/(b*d*Sqrt[d*Tan[a + b*x]]) + Sin[a + b*x]^3/(3*b*d*Sqrt[d
*Tan[a + b*x]]) + (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a +
 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(12*b*d^2)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}+\frac{\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx}{6d^2} \\
&= -\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}+\frac{\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx}{12d^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} \\
&\quad + \frac{\left(\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{12d^2\sqrt{\sin(a+bx)}} \\
&= -\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} \\
&\quad + \frac{\left(\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{12d^2} \\
&= -\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} \\
&\quad + \frac{\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{12bd^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.91

$$
\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\csc(a+bx)\left(\sqrt{\sec^2(a+bx)}\sin(4(a+bx)) + 4\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sqrt{\tan(}\right.}{24bd^2\sqrt{\sec^2(a+bx)}}
$$

```
[In] Integrate[Sin[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/24*(Csc[a + b*x]*(Sqrt[Sec[a + b*x]^2]*Sin[4*(a + b*x)] + 4*(-1)^(1/4)*E
llipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sqrt[Tan[a + b*x]])*
Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sec[a + b*x]^2])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.03 (sec) , antiderivative size = 1048, normalized size of antiderivative = 9.36

method	result	size
default	Expression too large to display	1048

```
[In] int(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/b*sec(b*x+a)*csc(b*x+a)*(6*I*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*
x+a)-csc(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*
x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(b*x+a)-6*I*(-csc(b*x+a)+1
+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^
(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin
(b*x+a)-8*2^(1/2)*cos(b*x+a)^4+8*sin(b*x+a)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(
-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((
1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-6*sin(b*x+a)*EllipticPi((1+csc(
b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(cot(b*x+a)-csc(b*x+a))^(1/
2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)-6*sin(b
*x+a)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(co
t(b*x+a)-csc(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+c
ot(b*x+a))^(1/2)+8*cos(b*x+a)^3*2^(1/2)+4*cos(b*x+a)^2*2^(1/2)+3*sin(b*x+a)
*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(
b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b
*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))*(-cos(b
*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-3*sin(b*x+a)*ln((2*sin(b*x+a)*(-co
t(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3
+cot(b*x+a)-csc(b*x+a))^(1/2)-cot(b*x+a)*cos(b*x+a)+2*cot(b*x+a)+2*cos(b*x+
a)+sin(b*x+a)-csc(b*x+a)-2)/(-1+cos(b*x+a)))*(-cos(b*x+a)*sin(b*x+a)/(cos(b
*x+a)+1)^2)^(1/2)+6*sin(b*x+a)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1
/2)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1
/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))+6*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)
^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)
^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*sin(b*x+a)-4*2^(1/2)*cos(b*x+a))*(
cos(b*x+a)+1)/(d*tan(b*x+a))^(1/2)/d*2^(1/2)
```

Fricas [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=\int \frac{\sin\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^2*tan(b
*x + a)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{{\sin\left(a+b\,x\right)}^3}{{\left(d\,\tan\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int(sin(a + b*x)^3/(d*tan(a + b*x))^(3/2),x)
[Out] int(sin(a + b*x)^3/(d*tan(a + b*x))^(3/2), x)
```

3.100 $\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=\frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{2bd\sqrt{d\tan(a+bx)}}$$

```
[Out] sin(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)-1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1
/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(
1/2)/b/d/(d*tan(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2682, 2649, 2653, 2720}

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=\frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2bd\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Sin[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] Sin[a + b*x]/(b*d*Sqrt[d*Tan[a + b*x]]) + (EllipticF[a - Pi/4 + b*x, 2]*Sec
[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*d*Sqrt[d*Tan[a + b*x]])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\sin(a+bx)}\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sqrt{\sin(a+bx)}}\,dx}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= \frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{2d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= \frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{\left(\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{2d\sqrt{d\tan(a+bx)}}\\
&= \frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{2bd\sqrt{d\tan(a+bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.59

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\cos(2(a+bx))\sec(a+bx)\left(\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\right.}{b\sqrt{\sec^2(a+bx)}(d\tan(a+bx))^{3/2}\,(-1}$$

```
[In] Integrate[Sin[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Cos[2*(a + b*x)]*Sec[a + b*x]*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*S
qrt[Tan[a + b*x]]], -1]*Sec[a + b*x]^2 - Sqrt[Sec[a + b*x]^2]*Sqrt[Tan[a +
b*x]])*Tan[a + b*x]^(3/2))/(b*Sqrt[Sec[a + b*x]^2]*(d*Tan[a + b*x])^(3/2)*(
-1 + Tan[a + b*x]^2))
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.91

method	result
default	$-\frac{\sec(bx+a)\csc(bx+a)\left(-\sin(bx+a)\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc}\right.\right.}{2b\sqrt{d\tan(bx+a)}\,d}$

```
[In] int(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*sec(b*x+a)*csc(b*x+a)*(-sin(b*x+a)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-c
sc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+
csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2*2^(1/2)-2^(1/2)*cos(
b*x+a))*(cos(b*x+a)+1)/(d*tan(b*x+a))^(1/2)/d*2^(1/2)
```

Fricas [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^2*tan(b*x + a)^2), x)
```

Sympy [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))**(3/2),x)
[Out] Integral(sin(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin(a+b\,x)}{(d\tan(a+b\,x))^{3/2}}\,dx$$

```
[In] int(sin(a + b*x)/(d*tan(a + b*x))^(3/2),x)
[Out] int(sin(a + b*x)/(d*tan(a + b*x))^(3/2), x)
```

3.101 $\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}-\frac{\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3bd^2}$$

```
[Out] -2/3*csc(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+1/3*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^
2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2
*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b/d^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2677, 2681, 2653, 2720}

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=-\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3bd^2}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Csc[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Csc[a + b*x])/(3*b*d*Sqrt[d*Tan[a + b*x]]) - (Csc[a + b*x]*EllipticF[a
- Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b*d^2)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}} - \frac{\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx}{3d^2} \\
&= -\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}} - \frac{\left(\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{3d^2\sqrt{\sin(a+bx)}} \\
&= -\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}} - \frac{\left(\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{3d^2} \\
&= -\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}} - \frac{\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3bd^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.34

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\cos(2(a+bx))\sec(a+bx)\sqrt{\sec^2(a+bx)}\left(\sqrt{\sec^2(a+bx)}-\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{ar}\right.\right.}{3b(d\tan(a+bx))^{3/2}\left(-1+\tan^2(a+\right.}$$

[In] Integrate[Csc[a + b*x]/(d*Tan[a + b*x])^(3/2),x]

[Out] (2*Cos[2*(a + b*x)]*Sec[a + b*x]*Sqrt[Sec[a + b*x]^2]*(Sqrt[Sec[a + b*x]^2] - (-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Tan[a + b*x]^(3/2)))/(3*b*(d*Tan[a + b*x])^(3/2)*(-1 + Tan[a + b*x]^2))

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.38

method	result
default	$-\frac{\left(\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)+\sec(b\right.}{3}$

[In] int(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/3/b/(d*tan(b*x+a))^(1/2)/d*((cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+csc(b*x+a)*2^(1/2))*2^(1/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.45

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\left(\cos(bx+a)^2-1\right)\sqrt{i\,d}F(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)+\left(\cos(bx\right.}{3\left(bd^2\,\text{co}\right.}$$

[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")

[Out] 1/3*((cos(b*x + a)^2 - 1)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + (cos(b*x + a)^2 - 1)*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) + 2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a))/(b*d^2*cos(b*x + a)^2 - b*d^2)

Sympy [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc{(a+bx)}}{(d\tan{(a+bx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))**(3/2),x)
[Out] Integral(csc(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc{(bx+a)}}{(d\tan{(bx+a)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc{(bx+a)}}{(d\tan{(bx+a)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{\sin{(a+b\,x)}\,\left(d\tan{(a+b\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(sin(a + b*x)*(d*tan(a + b*x))^(3/2)),x)
[Out] int(1/(sin(a + b*x)*(d*tan(a + b*x))^(3/2)), x)
```

3.102 $\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}} - \frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}} - \frac{2\csc(a+bx)\,\mathrm{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{21bd^2}$$

```
[Out] 2/21*csc(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)-2/7*csc(b*x+a)^3/b/d/(d*tan(b*x+a)
)^(1/2)+2/21*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Ellip
ticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
/d^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2677, 2679, 2681, 2653, 2720}

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\mathrm{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{21bd^2} - \frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}} + \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Csc[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*Csc[a + b*x])/(21*b*d*Sqrt[d*Tan[a + b*x]]) - (2*Csc[a + b*x]^3)/(7*b*d*
Sqrt[d*Tan[a + b*x]]) - (2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[S
in[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(21*b*d^2)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}-\frac{\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx}{7d^2}\\
&= \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}-\frac{2\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx}{21d^2}
\end{aligned}$$

$$= \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}} - \frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}} - \frac{\left(2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{21d^2\sqrt{\sin(a+bx)}}$$

$$= \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}} - \frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}} - \frac{\left(2\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{21d^2}$$

$$= \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}} - \frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}} - \frac{2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{21bd^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.21

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\csc^3(a+bx)\left((1+10\cos(2(a+bx))+\cos(4(a+bx)))\sec^2(a+bx)^{3/2}-8\sqrt[4]{-1}\,\mathrm{c}\right.}{42bd\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a-}}$$

```
[In] Integrate[Csc[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Csc[a + b*x]^3*((1 + 10*Cos[2*(a + b*x)] + Cos[4*(a + b*x)])*(Sec[a + b*x]
^2)^(3/2) - 8*(-1)^(1/4)*Cos[2*(a + b*x)]*EllipticF[I*ArcSinh[(-1)^(1/4)*Sq
rt[Tan[a + b*x]]], -1]*Tan[a + b*x]^(7/2)))/(42*b*d*Sqrt[Sec[a + b*x]^2]*Sq
rt[d*Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(123) = 246$.

Time = 0.86 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.08

method	result
default	$-\frac{\left(-3(\csc^7(bx+a))(1-\cos(bx+a))^8+16(\csc^2(bx+a))\sqrt{1+\csc(bx+a)-\cot(bx+a)}\sqrt{2-2\csc(bx+a)+2\cot(bx+a)}\sqrt{\cot(bx+a)-\csc}\right.}{168b(1-\cos(bx+a))\sqrt{(\csc^3(bx+a))(1-\cos(bx+a))^3-\csc(bx+a)+\cot(bx+a)}\sqrt{\csc(bx+a)(1-\cos(bx+}}$

```
[In] int(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/168/b/(1-cos(b*x+a))/(csc(b*x+a)^3*(1-cos(b*x+a))^3-csc(b*x+a)+cot(b*x+a
))^(1/2)/(csc(b*x+a)*(1-cos(b*x+a))*(csc(b*x+a)^2*(1-cos(b*x+a))^2-1))^(1/2
)/(csc(b*x+a)^2*(1-cos(b*x+a))^2-1)/(-d/(csc(b*x+a)^2*(1-cos(b*x+a))^2-1)*(
csc(b*x+a)-cot(b*x+a)))^(3/2)*(-3*csc(b*x+a)^7*(1-cos(b*x+a))^8+16*csc(b*x+
a)^2*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(2-2*csc(b*x+a)+2*cot(b*x+a))^(1/2)*(c
ot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2
^(1/2))*(1-cos(b*x+a))^3-2*csc(b*x+a)^5*(1-cos(b*x+a))^6+2*csc(b*x+a)*(1-co
s(b*x+a))^2+3*sin(b*x+a))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.46

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\left(\left(\cos\left(bx+a\right)^4-2\,\cos\left(bx+a\right)^2+1\right)\sqrt{i\,d}F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right.}{}$$

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(I*d)*elliptic_f(arcsin(c
os(b*x + a) + I*sin(b*x + a)), -1) + (cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1
)*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - (cos(b
*x + a)^3 + 2*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*d^2*cos(b
*x + a)^4 - 2*b*d^2*cos(b*x + a)^2 + b*d^2)
```

Sympy [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc^3\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)**3/(d*tan(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{{\sin\left(a+b\,x\right)}^3\,{\left(d\,\tan\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2)),x)
[Out] int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2)), x)
```

3.103 $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 257

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{5/2}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{5/2}} - \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{5/2}} + \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{5/2}} + \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3}$$

```
[Out] -3/64*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)*2^(1/2)+3/64
*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)*2^(1/2)-3/128*ln(
d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(5/2)*2^(1/2)+
3/128*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(5/2)
*2^(1/2)+1/16*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d^3-1/4*cos(b*x+a)^4*(d*t
an(b*x+a))^(1/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2671, 294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{5/2}} + \frac{3\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}bd^{5/2}} - \frac{3\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}bd^{5/2}} + \frac{3\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}bd^{5/2}} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} + \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3}$$

```
[In] Int[Sin[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*d^(5/
2)) + (3*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*
d^(5/2)) - (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b
*x]]])/(64*Sqrt[2]*b*d^(5/2)) + (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqr
t[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b*d^(5/2)) + (Cos[a + b*x]^2*Sqrt[d
*Tan[a + b*x]])/(16*b*d^3) - (Cos[a + b*x]^4*Sqrt[d*Tan[a + b*x]])/(4*b*d^3
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{d\text{Subst}\left(\int \frac{x^{3/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} + \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{8b}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\tan(a+bx)\right)}{32bd}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} + \frac{3\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{16bd}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} + \frac{3\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{32bd^2} + \frac{3\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{32bd^2}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} - \frac{3\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{5/2}} - \frac{3\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{5/2}} + \frac{3\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64bd^2} + \frac{3\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{64bd^2}$$

$$= -\frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{5/2}} + \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{5/2}} + \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} + \frac{3\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{5/2}} - \frac{3\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{5/2}}$$

$$= -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{5/2}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{5/2}} - \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{5/2}} + \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{5/2}} + \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.48

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{\csc(a+bx)\left(\sin(a+bx)+3\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-3\log\left(\cos(a+bx)+\sin(\right.\right.}{64bd^3}$$

```
[In] Integrate[Sin[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -1/64*(Csc[a + b*x]*(Sin[a + b*x] + 3*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*S
qrt[Sin[2*(a + b*x)]] - 3*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a +
 b*x)]]]*Sqrt[Sin[2*(a + b*x)]] + 2*Sin[3*(a + b*x)] + Sin[5*(a + b*x)])*Sq
rt[d*Tan[a + b*x]])/(b*d^3)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(197) = 394$.

Time = 15.09 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.35

method	result
default	$\frac{\csc(bx+a)(-1+\cos(bx+a))\left(16\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\,\sqrt{2}\,(\cos^4(bx+a))+16\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\,\sqrt{2}\,(\cos^3(bx+a))-4\sqrt{-\frac{\cos(bx+}{(\cos(}}\right.}{}$

```
[In] int(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/b*csc(b*x+a)*(-1+cos(b*x+a))*(16*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+
1)^2)^(1/2)*2^(1/2)*cos(b*x+a)^4+16*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^
2)^(1/2)*2^(1/2)*cos(b*x+a)^3-4*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(
1/2)*2^(1/2)*cos(b*x+a)^2-4*cos(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos
(b*x+a)+1)^2)^(1/2)+6*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(c
os(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+6*arctan((sin(b*x+a)*2
^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+co
s(b*x+a)))-3*ln((2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*co
t(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-cot(b*x+a)*
cos(b*x+a)+2*cot(b*x+a)+2*cos(b*x+a)+sin(b*x+a)-csc(b*x+a)-2)/(-1+cos(b*x+a
)))+3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3
*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-
csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a))))/(
-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*tan(b*x+a))^(1/2)/d^2*2^(
1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 956, normalized size of antiderivative = 3.72

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/256*(3*b*d^3*(-1/(b^4*d^10))^(1/4)*log(2*b^2*d^5*sqrt(-1/(b^4*d^10))*cos(
b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 + 2*(b^3*d^7*(-1/(b^4*d^10))^(3/4)
*cos(b*x + a)^2 + b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sq
rt(d*sin(b*x + a)/cos(b*x + a)) + 1) - 3*b*d^3*(-1/(b^4*d^10))^(1/4)*log(2*
b^2*d^5*sqrt(-1/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 -
```

```
2*(b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 + b*d^2*(-1/(b^4*d^10))^(1/
4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) + 3*I*
b*d^3*(-1/(b^4*d^10))^(1/4)*log(-2*b^2*d^5*sqrt(-1/(b^4*d^10))*cos(b*x + a)
*sin(b*x + a) - 2*cos(b*x + a)^2 - 2*(I*b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b
*x + a)^2 - I*b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d
*sin(b*x + a)/cos(b*x + a)) + 1) - 3*I*b*d^3*(-1/(b^4*d^10))^(1/4)*log(-2*b
^2*d^5*sqrt(-1/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 - 2
*(-I*b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 + I*b*d^2*(-1/(b^4*d^10))
^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) +
3*b*d^3*(-1/(b^4*d^10))^(1/4)*log(2*(b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b*x
+ a)^2 - b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(
b*x + a)/cos(b*x + a)) - 1) - 3*b*d^3*(-1/(b^4*d^10))^(1/4)*log(-2*(b^3*d^7
*(-1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 - b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x
 + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1) + 3*I*b*d^3*(-1/
(b^4*d^10))^(1/4)*log(-2*(I*b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 +
I*b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a
)/cos(b*x + a)) - 1) - 3*I*b*d^3*(-1/(b^4*d^10))^(1/4)*log(-2*(-I*b^3*d^7*(
-1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 - I*b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x
 + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1) - 16*(4*cos(b*x
+ a)^4 - cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*d^3)
```

Sympy [F]

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{\frac{5}{2}}}\,dx$$

[In] integrate(sin(b*x+a)**4/(d*tan(b*x+a))**(5/2),x)

[Out] Integral(sin(a + b*x)**4/(d*tan(a + b*x))**(5/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.85

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{6\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+6\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right.}{}$$

[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")

[Out] 1/128*(6*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d)) + 6*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(

```
d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 3*sqrt(2)*d^(5/2)*log(d*tan(b*x + a
) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 3*sqrt(2)*d^(5/2)*log(d*tan
(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) + 8*((d*tan(b*x + a))
^(5/2)*d^4 - 3*sqrt(d*tan(b*x + a))*d^6)/(d^4*tan(b*x + a)^4 + 2*d^4*tan(b*
x + a)^2 + d^4))/(b*d^5)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.96

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{3\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd^3} + \frac{3\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd^3} + \frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{128\,bd^3} - \frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{128\,bd^3} + \frac{\sqrt{d\tan(bx+a)}d^2\tan(bx+a)^2-3\sqrt{d\tan(bx+a)}d^2}{16\left(d^2\tan(bx+a)^2+d^2\right)^2bd}$$

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt
(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^3) + 3/64*sqrt(2)*sqrt(abs(d))*arctan(
-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/
(b*d^3) + 3/128*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*ta
n(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^3) - 3/128*sqrt(2)*sqrt(abs(d))*log
(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^
3) + 1/16*(sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)^2 - 3*sqrt(d*tan(b*x + a))
*d^2)/((d^2*tan(b*x + a)^2 + d^2)^2*b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin{(a+b\,x)}^4}{{(d\tan{(a+b\,x)})}^{5/2}}\,dx$$

```
[In] int(sin(a + b*x)^4/(d*tan(a + b*x))^(5/2),x)
[Out] int(sin(a + b*x)^4/(d*tan(a + b*x))^(5/2), x)
```

3.104 $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 227

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{5/2}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{5/2}} - \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{5/2}} + \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{5/2}} + \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3}$$

```
[Out] -3/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)*2^(1/2)+3/8*a
rctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)*2^(1/2)-3/16*ln(d^(
1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(5/2)*2^(1/2)+3/1
6*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(5/2)*2^(
1/2)+1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used

= {2671, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{5/2}} + \frac{3\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}bd^{5/2}} - \frac{3\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}bd^{5/2}} + \frac{3\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}bd^{5/2}} + \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3}$$

```
[In] Int[Sin[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^(5/2
)) + (3*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^
(5/2)) - (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x
]]])/(8*Sqrt[2]*b*d^(5/2)) + (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2
]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b*d^(5/2)) + (Cos[a + b*x]^2*Sqrt[d*Tan
[a + b*x]])/(2*b*d^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule steps

$$\text{integral} = \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\tan(a+bx)\right)}{4bd}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3} + \frac{3\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2bd}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3} + \frac{3\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4bd^2} + \frac{3\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4bd^2}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3} - \frac{3\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{5/2}} - \frac{3\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{5/2}} + \frac{3\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8bd^2} + \frac{3\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8bd^2}$$

$$= -\frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{5/2}} + \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{5/2}} + \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3} + \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{5/2}} - \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{5/2}}$$

$$= -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{5/2}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{5/2}} - \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{5/2}} + \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{5/2}} + \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\csc(a+bx)\left(\sin(a+bx)-3\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}\right.}{}$$

[In] Integrate[Sin[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]

[Out] (Csc[a + b*x]*(Sin[a + b*x] - 3*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt[Si
n[2*(a + b*x)]] + 3*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]
]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)])*Sqrt[d*Tan[a + b*x]])/(8*b*d^
3)

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(171) = 342.

Time = 14.34 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.32

method	result
default	$\frac{\csc(bx+a)(-1+\cos(bx+a))\left(-4\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\sqrt{2}\left(\cos^2(bx+a)\right)-4\cos(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}+6\arctan\left(\frac{\sin(bx}{}\right.\right.}{}$

[In] int(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/16/b*csc(b*x+a)*(-1+cos(b*x+a))*(-4*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1
)^2)^(1/2)*2^(1/2)*cos(b*x+a)^2-4*cos(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a
)/(cos(b*x+a)+1)^2)^(1/2)+6*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x
+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+6*arctan((sin(b*
x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/
(-1+cos(b*x+a)))+3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-c
ot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^
3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+co
s(b*x+a)))-3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-cot(b*x
+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(
b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+
a))))/(d*tan(b*x+a))^(1/2)/(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/
d^2*2^(1/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 943, normalized size of antiderivative = 4.15

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*b*d^3*(-1/(b^4*d^10))^(1/4)*log(2*b^2*d^5*sqrt(-1/(b^4*d^10))*cos(b
*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 + 2*(b^3*d^7*(-1/(b^4*d^10))^(3/4)*
cos(b*x + a)^2 + b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqr
t(d*sin(b*x + a)/cos(b*x + a)) + 1) - 3*b*d^3*(-1/(b^4*d^10))^(1/4)*log(2*b
^2*d^5*sqrt(-1/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 - 2
*(b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 + b*d^2*(-1/(b^4*d^10))^(1/4
)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) + 3*I*b
*d^3*(-1/(b^4*d^10))^(1/4)*log(-2*b^2*d^5*sqrt(-1/(b^4*d^10))*cos(b*x + a)*
sin(b*x + a) - 2*cos(b*x + a)^2 - 2*(I*b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b*
x + a)^2 - I*b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*
sin(b*x + a)/cos(b*x + a)) + 1) - 3*I*b*d^3*(-1/(b^4*d^10))^(1/4)*log(-2*b^
2*d^5*sqrt(-1/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 - 2*
(-I*b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 + I*b*d^2*(-1/(b^4*d^10))^
(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) + 3
*b*d^3*(-1/(b^4*d^10))^(1/4)*log(2*(b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b*x +
 a)^2 - b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b
*x + a)/cos(b*x + a)) - 1) - 3*b*d^3*(-1/(b^4*d^10))^(1/4)*log(-2*(b^3*d^7*
(-1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 - b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x
+ a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1) + 3*I*b*d^3*(-1/(
b^4*d^10))^(1/4)*log(-2*(I*b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 + I
*b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)
/cos(b*x + a)) - 1) - 3*I*b*d^3*(-1/(b^4*d^10))^(1/4)*log(-2*(-I*b^3*d^7*(-
1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 - I*b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x
+ a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1) + 16*sqrt(d*sin(b
*x + a)/cos(b*x + a))*cos(b*x + a)^2)/(b*d^3)
```

Sympy [F]

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)**2/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/(d*tan(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{6\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+6\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(b}\right.}{2\sqrt{d}}\right.}{}$$

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*(6*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
b*x + a)))/sqrt(d)) + 6*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 3*sqrt(2)*sqrt(d)*log(d*tan(b*x + a)
 + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 3*sqrt(2)*sqrt(d)*log(d*tan(
b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) + 8*sqrt(d*tan(b*x + a
))*d^2/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.97

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{3\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd^3} + \frac{3\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd^3} + \frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{16\,bd^3} - \frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{16\,bd^3} + \frac{\sqrt{d\tan(bx+a)}}{2\left(d^2\tan(bx+a)^2+d^2\right)bd}$$

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(
d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^3) + 3/8*sqrt(2)*sqrt(abs(d))*arctan(-1
/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b
*d^3) + 3/16*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b
*x + a))*sqrt(abs(d)) + abs(d))/(b*d^3) - 3/16*sqrt(2)*sqrt(abs(d))*log(d*t
an(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^3) +
 1/2*sqrt(d*tan(b*x + a))/((d^2*tan(b*x + a)^2 + d^2)*b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\left(d\tan\left(a+b\,x\right)\right)^{5/2}}\,dx$$

```
[In] int(sin(a + b*x)^2/(d*tan(a + b*x))^(5/2),x)
[Out] int(sin(a + b*x)^2/(d*tan(a + b*x))^(5/2), x)
```

3.105 $\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

```
[Out] -2/7*d/b/(d*tan(b*x+a))^(7/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 30}

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

```
[In] Int[Csc[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d)/(7*b*(d*Tan[a + b*x])^(7/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
```

```
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{d\text{Subst}\left(\int \frac{1}{x^{9/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

```
[In] Integrate[Csc[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
[Out] (-2*d)/(7*b*(d*Tan[a + b*x])^(7/2))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2d}{7b(d\tan(bx+a))^{\frac{7}{2}}}$	17
default	$-\frac{2d}{7b(d\tan(bx+a))^{\frac{7}{2}}}$	17

```
[In] int(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/7*d/b/(d*tan(b*x+a))^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.15

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)^4}{7\left(bd^3\cos(bx+a)^4 - 2\,bd^3\cos(bx+a)^2 + bd^3\right)}$$

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/7*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^4/(b*d^3*cos(b*x + a)^4
 - 2*b*d^3*cos(b*x + a)^2 + b*d^3)
```

Sympy [F]

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc^2{(a+bx)}}{(d\tan{(a+bx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)**2/(d*tan(b*x+a))**(5/2),x)
[Out] Integral(csc(a + b*x)**2/(d*tan(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2}{7\,(d\tan{(bx+a)})^{\frac{5}{2}}\,b\tan{(bx+a)}}$$

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
[Out] -2/7/((d*tan(b*x + a))^(5/2)*b*tan(b*x + a))
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2}{7\,\sqrt{d\tan{(bx+a)}}bd^2\tan{(bx+a)}^3}$$

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] -2/7/(sqrt(d*tan(b*x + a))*b*d^2*tan(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 8.18 (sec) , antiderivative size = 530, normalized size of antiderivative = 26.50

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{46\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{7\,b\,d^3\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)} + \frac{12\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{5\,b\,d^3\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^2} + \frac{24\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{35\,b\,d^3\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^3} - \frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,48\mathrm{i}}{7\,b\,d^3\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)} + \frac{144\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{35\,b\,d^3\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)^2} + \frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,144\mathrm{i}}{35\,b\,d^3\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)^3} - \frac{16\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{7\,b\,d^3\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)^4}$$

```
[In] int(1/(sin(a + b*x)^2*(d*tan(a + b*x))^(5/2)),x)
```

```
[Out] (46*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i +
 b*x*2i) + 1))^(1/2))/(7*b*d^3*(exp(a*2i + b*x*2i) - 1)) + (12*(exp(a*2i +
b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(
1/2))/(5*b*d^3*(exp(a*2i + b*x*2i) - 1)^2) + (24*(exp(a*2i + b*x*2i) + 1)*(
-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(35*b*d^
3*(exp(a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i +
b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*48i)/(7*b*d^3*(exp(a*2i +
 b*x*2i)*1i - 1i)) + (144*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)
*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(35*b*d^3*(exp(a*2i + b*x*2i)*1
i - 1i)^2) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2)*144i)/(35*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)
^3) - (16*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(
a*2i + b*x*2i) + 1))^(1/2))/(7*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)^4)
```

3.106 $\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2d^3}{11b(d\tan(a+bx))^{11/2}} - \frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

```
[Out] -2/11*d^3/b/(d*tan(b*x+a))^(11/2)-2/7*d/b/(d*tan(b*x+a))^(7/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 14}

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2d^3}{11b(d\tan(a+bx))^{11/2}} - \frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

```
[In] Int[Csc[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d^3)/(11*b*(d*Tan[a + b*x])^(11/2)) - (2*d)/(7*b*(d*Tan[a + b*x])^(7/2)
)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
```

```
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d\text{Subst}\left(\int \frac{d^2+x^2}{x^{13/2}}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= \frac{d\text{Subst}\left(\int \left(\frac{d^2}{x^{13/2}}+\frac{1}{x^{9/2}}\right)\,dx, x, d\tan(a+bx)\right)}{b}\\ &= -\frac{2d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{2(-9+2\cos(2(a+bx)))\cot^4(a+bx)\csc^2(a+bx)\sqrt{d\tan(a+bx)}}{77bd^3}$$

```
[In] Integrate[Csc[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
[Out] (2*(-9 + 2*Cos[2*(a + b*x)])*Cot[a + b*x]^4*Csc[a + b*x]^2*Sqrt[d*Tan[a + b
*x]])/(77*b*d^3)
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\frac{8\left(\cot^5(bx+a)\right)}{77}-\frac{2\left(\cot^3(bx+a)\right)\left(\csc^2(bx+a)\right)}{7}}{d^2\sqrt{d\tan(bx+a)}\,b}$	48

```
[In] int(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/77/b/d^2/(d*tan(b*x+a))^(1/2)*(4*cot(b*x+a)^5-11*cot(b*x+a)^3*csc(b*x+a)^
2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(35) = 70.

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.12

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(4\cos\left(bx+a\right)^6-11\cos\left(bx+a\right)^4\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{77\left(bd^3\cos\left(bx+a\right)^6-3\,bd^3\cos\left(bx+a\right)^4+3\,bd^3\cos\left(bx+a\right)^2-bd^3\right)}$$

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*cos(b*x + a)^6 - 11*cos(b*x + a)^4)*sqrt(d*sin(b*x + a)/cos(b*x +
a))/(b*d^3*cos(b*x + a)^6 - 3*b*d^3*cos(b*x + a)^4 + 3*b*d^3*cos(b*x + a)^2
 - b*d^3)
```

Sympy [F]

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc^4\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)**4/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Integral(csc(a + b*x)**4/(d*tan(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(11\,d^2\tan\left(bx+a\right)^2+7\,d^2\right)d}{77\,\left(d\tan\left(bx+a\right)\right)^{\frac{11}{2}}b}$$

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/77*(11*d^2*tan(b*x + a)^2 + 7*d^2)*d/((d*tan(b*x + a))^(11/2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(11\,d^3\tan\left(bx+a\right)^2+7\,d^3\right)}{77\,\sqrt{d\tan\left(bx+a\right)}bd^5\tan\left(bx+a\right)^5}$$

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] -2/77*(11*d^3*tan(b*x + a)^2 + 7*d^3)/(sqrt(d*tan(b*x + a))*b*d^5*tan(b*x +
 a)^5)
```

Mupad [B] (verification not implemented)

Time = 13.90 (sec) , antiderivative size = 831, normalized size of antiderivative = 19.33

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] int(1/(sin(a + b*x)^4*(d*tan(a + b*x))^(5/2)),x)
[Out] ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*
x*2i) + 1))^(1/2)*2048i)/(165*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)) - (7768*(
exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*
2i) + 1))^(1/2))/(945*b*d^3*(exp(a*2i + b*x*2i) - 1)^2) - (4232*(exp(a*2i +
 b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^
(1/2))/(495*b*d^3*(exp(a*2i + b*x*2i) - 1)^3) - (1328*(exp(a*2i + b*x*2i) +
 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(23
1*b*d^3*(exp(a*2i + b*x*2i) - 1)^4) - (160*(exp(a*2i + b*x*2i) + 1)*(-(d*(e
xp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(99*b*d^3*(exp
(a*2i + b*x*2i) - 1)^5) - (14456*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i +
b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(1155*b*d^3*(exp(a*2i +
b*x*2i) - 1)) - (86528*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i
 - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(10395*b*d^3*(exp(a*2i + b*x*2i)*1
i - 1i)^2) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2)*3904i)/(315*b*d^3*(exp(a*2i + b*x*2i)*1i - 1
i)^3) + (4160*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2))/(231*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)^4)
+ ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i +
b*x*2i) + 1))^(1/2)*1600i)/(99*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)^5) - (64*
(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x
*2i) + 1))^(1/2))/(11*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)^6)
```

3.107 $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=-\frac{2d^5}{15b(d\tan(a+bx))^{15/2}}-\frac{4d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

```
[Out] -2/15*d^5/b/(d*tan(b*x+a))^(15/2)-4/11*d^3/b/(d*tan(b*x+a))^(11/2)-2/7*d/b/
(d*tan(b*x+a))^(7/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 276}

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=-\frac{2d^5}{15b(d\tan(a+bx))^{15/2}}-\frac{4d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

```
[In] Int[Csc[a + b*x]^6/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d^5)/(15*b*(d*Tan[a + b*x])^(15/2)) - (4*d^3)/(11*b*(d*Tan[a + b*x])^(1
1/2)) - (2*d)/(7*b*(d*Tan[a + b*x])^(7/2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{d\text{Subst}\left(\int\frac{(d^2+x^2)^2}{x^{17/2}}\,dx,x,d\tan(a+bx)\right)}{b}\\&=\frac{d\text{Subst}\left(\int\left(\frac{d^4}{x^{17/2}}+\frac{2d^2}{x^{13/2}}+\frac{1}{x^{9/2}}\right)\,dx,x,d\tan(a+bx)\right)}{b}\\&=-\frac{2d^5}{15b(d\tan(a+bx))^{15/2}}-\frac{4d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int\frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=\frac{2(-117+44\cos(2(a+bx))-4\cos(4(a+bx)))\cot^4(a+bx)\csc^4(a+bx)\sqrt{d\tan(a+bx)}}{1155bd^3}$$

```
[In] Integrate[Csc[a + b*x]^6/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*(-117 + 44*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)])*Cot[a + b*x]^4*Csc[a +
 b*x]^4*Sqrt[d*Tan[a + b*x]])/(1155*b*d^3)
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{2(\cot^3(bx+a))(\csc^4(bx+a))(32(\cos^4(bx+a))-120(\cos^2(bx+a))+165)}{1155b\,d^2\sqrt{d\tan(bx+a)}}$	57

```
[In] int(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/1155/b*cot(b*x+a)^3*csc(b*x+a)^4*(32*cos(b*x+a)^4-120*cos(b*x+a)^2+165)/
d^2/(d*tan(b*x+a))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(53) = 106.

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.75

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(32\cos(bx+a)^8-120\cos(bx+a)^6+165\cos(bx+a)^4\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{1155\left(bd^3\cos(bx+a)^8-4\,bd^3\cos(bx+a)^6+6\,bd^3\cos(bx+a)^4-4\,bd^3\cos(bx+a)^2+bd^3\right)}$$

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(32*cos(b*x + a)^8 - 120*cos(b*x + a)^6 + 165*cos(b*x + a)^4)*sqrt(
d*sin(b*x + a)/cos(b*x + a))/(b*d^3*cos(b*x + a)^8 - 4*b*d^3*cos(b*x + a)^6
 + 6*b*d^3*cos(b*x + a)^4 - 4*b*d^3*cos(b*x + a)^2 + b*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**6/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(165\,d^4\tan(bx+a)^4+210\,d^4\tan(bx+a)^2+77\,d^4\right)d}{1155\,\left(d\tan(bx+a)\right)^{\frac{15}{2}}b}$$

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/1155*(165*d^4*tan(b*x + a)^4 + 210*d^4*tan(b*x + a)^2 + 77*d^4)*d/((d*ta
n(b*x + a))^(15/2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(165\,d^5\tan\left(bx+a\right)^4+210\,d^5\tan\left(bx+a\right)^2+77\,d^5\right)}{1155\,\sqrt{d\tan\left(bx+a\right)}bd^7\tan\left(bx+a\right)^7}$$

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] -2/1155*(165*d^5*tan(b*x + a)^4 + 210*d^5*tan(b*x + a)^2 + 77*d^5)/(sqrt(d*
tan(b*x + a))*b*d^7*tan(b*x + a)^7)
```

Mupad [B] (verification not implemented)

Time = 15.05 (sec) , antiderivative size = 1132, normalized size of antiderivative = 17.42

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] int(1/(sin(a + b*x)^6*(d*tan(a + b*x))^(5/2)),x)
[Out] (199232*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*
2i + b*x*2i) + 1))^(1/2))/(12285*b*d^3*(exp(a*2i + b*x*2i) - 1)) + (1581376
*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*
x*2i) + 1))^(1/2))/(135135*b*d^3*(exp(a*2i + b*x*2i) - 1)^2) + (4539104*(ex
p(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i
) + 1))^(1/2))/(225225*b*d^3*(exp(a*2i + b*x*2i) - 1)^3) + (1152*(exp(a*2i
+ b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))
^(1/2))/(35*b*d^3*(exp(a*2i + b*x*2i) - 1)^4) + (74528*(exp(a*2i + b*x*2i)
+ 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(2
145*b*d^3*(exp(a*2i + b*x*2i) - 1)^5) + (1088*(exp(a*2i + b*x*2i) + 1)*(-(d
*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(55*b*d^3*(
exp(a*2i + b*x*2i) - 1)^6) + (896*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i +
 b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(195*b*d^3*(exp(a*2i +
b*x*2i) - 1)^7) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1
i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*439808i)/(27027*b*d^3*(exp(a*2i + b*x*2
i)*1i - 1i)) + (1573888*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1
i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(135135*b*d^3*(exp(a*2i + b*x*2i)
*1i - 1i)^2) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))
/(exp(a*2i + b*x*2i) + 1))^(1/2)*4557824i)/(225225*b*d^3*(exp(a*2i + b*x*2i
)*1i - 1i)^3) - (7168*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i
- 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(165*b*d^3*(exp(a*2i + b*x*2i)*1i -
 1i)^4) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp
```

```
(a*2i + b*x*2i) + 1))^(1/2)*172288i)/(2145*b*d^3*(exp(a*2i + b*x*2i)*1i - 1
i)^5) + (5376*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2))/(55*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)^6) +
 ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2)*12544i)/(195*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)^7) - (25
6*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2))/(15*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)^8)
```

3.108 $\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=-\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}}-\frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}+\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{40bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] -3/40*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-1/
20*sin(b*x+a)^3/b/d/(d*tan(b*x+a))^(3/2)-3/70*sin(b*x+a)^5/b/d/(d*tan(b*x+a
))^(3/2)+1/7*sin(b*x+a)^7/b/d/(d*tan(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2676, 2678, 2681, 2652, 2719}

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=\frac{3\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{40bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}-\frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}}-\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}}$$

```
[In] Int[Sin[a + b*x]^7/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -1/20*Sin[a + b*x]^3/(b*d*(d*Tan[a + b*x])^(3/2)) - (3*Sin[a + b*x]^5)/(70*
b*d*(d*Tan[a + b*x])^(3/2)) + Sin[a + b*x]^7/(7*b*d*(d*Tan[a + b*x])^(3/2))
```

```
 + (3*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(40*b*d^2*Sqrt[Sin[2*a + 2
*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}+\frac{3\int\frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{14d^2}\\ &= -\frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}+\frac{3\int\frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{20d^2}\end{aligned}$$

$$= -\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}} - \frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{3\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{40d^2}$$

$$= -\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}} - \frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{\left(3\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{40d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$= -\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}} - \frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{(3\sin(a+bx))\int \sqrt{\sin(2a+2bx)}dx}{40d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$= -\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}} - \frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{40bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.86 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.85

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}dx = \frac{\sqrt{d\tan(a+bx)}\left(-\sqrt{\sec^2(a+bx)}(15\sin(a+bx)+29\sin(3(a+bx))+9\sin(5(a+\right.}{2}$$

```
[In] Integrate[Sin[a + b*x]^7/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (Sqrt[d*Tan[a + b*x]]*(-(Sqrt[Sec[a + b*x]^2]*(15*Sin[a + b*x] + 29*Sin[3*(
a + b*x)] + 9*Sin[5*(a + b*x)] - 5*Sin[7*(a + b*x)])) + 112*Hypergeometric2
F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Tan[a + b*x]))/(2240*b*d^3*
Sqrt[Sec[a + b*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(151) = 302$.

Time = 1.46 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.08

method	result
default	$\frac{\sec(bx+a)(\csc^2(bx+a))(-1+\cos(bx+a))(\cos(bx+a)+1)\left(40\sqrt{2}\,(\cos^8(bx+a))-108\sqrt{2}\,(\cos^6(bx+a))+82\sqrt{2}\,(\cos^4(bx+a))-21\sqrt{\cot(}\right.}{}$

```
[In] int(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/560/b*sec(b*x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))*(cos(b*x+a)+1)*(40*2^(1/2)*
cos(b*x+a)^8-108*2^(1/2)*cos(b*x+a)^6+82*2^(1/2)*cos(b*x+a)^4-21*(cot(b*x+a
)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+
a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)
+42*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b
*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/
2))*cos(b*x+a)-21*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^
(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(
1/2),1/2*2^(1/2))+42*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x
+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a)
)^(1/2),1/2*2^(1/2))+7*cos(b*x+a)^2*2^(1/2)-21*2^(1/2)*cos(b*x+a))/(d*tan(b
*x+a))^(1/2)/d^2*2^(1/2)
```

Fricas [F]

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^7}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^6 - 3*cos(b*x + a)^4 + 3*cos(b*x + a)^2 - 1)*sqrt(d
*tan(b*x + a))*sin(b*x + a)/(d^3*tan(b*x + a)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**7/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^7}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^7/(d*tan(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^7}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^7/(d*tan(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(a+b\,x)^7}{(d\tan(a+b\,x))^{5/2}}\,dx$$

```
[In] int(sin(a + b*x)^7/(d*tan(a + b*x))^(5/2),x)
[Out] int(sin(a + b*x)^7/(d*tan(a + b*x))^(5/2), x)
```

3.109 $\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}} + \frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{20bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] -3/20*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-1/
10*sin(b*x+a)^3/b/d/(d*tan(b*x+a))^(3/2)+1/5*sin(b*x+a)^5/b/d/(d*tan(b*x+a)
)^(3/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2676, 2678, 2681, 2652, 2719}

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{3\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{20bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}} - \frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}}$$

```
[In] Int[Sin[a + b*x]^5/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -1/10*Sin[a + b*x]^3/(b*d*(d*Tan[a + b*x])^(3/2)) + Sin[a + b*x]^5/(5*b*d*(
d*Tan[a + b*x])^(3/2)) + (3*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(20*
b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{3\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{10d^2}$$

$$= -\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}} + \frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{3\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{20d^2}$$

$$= -\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}} + \frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{\left(3\sqrt{\sin(a+bx)}\right)\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{20d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$= -\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}} + \frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{(3\sin(a+bx))\int\sqrt{\sin(2a+2bx)}\,dx}{20d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$= -\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}} + \frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{20bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d\tan(a+bx)}\left(-\sqrt{\sec^2(a+bx)}(\sin(3(a+bx))+\sin(5(a+bx)))+8\,\text{Hyperge}\right.}{80bd^3\sqrt{\sec^2(a+bx}}$$

```
[In] Integrate[Sin[a + b*x]^5/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (Sqrt[d*Tan[a + b*x]]*(-(Sqrt[Sec[a + b*x]^2]*(Sin[3*(a + b*x)] + Sin[5*(a
+ b*x)])) + 8*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x
]*Tan[a + b*x]))/(80*b*d^3*Sqrt[Sec[a + b*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(125) = 250.

Time = 1.18 (sec) , antiderivative size = 431, normalized size of antiderivative = 3.78

method	result
default	$-\frac{\sec(bx+a)\left(\csc^2(bx+a)\right)(-1+\cos(bx+a))(\cos(bx+a)+1)\left(4\sqrt{2}\left(\cos^6(bx+a)\right)-6\sqrt{2}\left(\cos^4(bx+a)\right)-6\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{}\right.}{}$

```
[In] int(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/40/b*sec(b*x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))*(cos(b*x+a)+1)*(4*2^(1/2)*c
os(b*x+a)^6-6*2^(1/2)*cos(b*x+a)^4-6*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(
b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b
*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+3*(cot(b*x+a)-csc(b*x+a))^(
1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*Ellip
ticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-6*(1+csc(b*x+a
```

```
)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a)
)^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+3*(cot(b*x+a
)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+
a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)
^2*2^(1/2)+3*2^(1/2)*cos(b*x+a))/(d*tan(b*x+a))^(1/2)/d^2*2^(1/2)
```

Fricas [F]

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*tan(b*x + a))*sin(b
*x + a)/(d^3*tan(b*x + a)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**5/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^5/(d*tan(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^5/(d*tan(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^5}{\left(d\tan\left(a+b\,x\right)\right)^{5/2}}\,dx$$

```
[In] int(sin(a + b*x)^5/(d*tan(a + b*x))^(5/2),x)
[Out] int(sin(a + b*x)^5/(d*tan(a + b*x))^(5/2), x)
```

3.110 $\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}} + \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] -1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+1/3
*sin(b*x+a)^3/b/d/(d*tan(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2676, 2681, 2652, 2719}

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

```
[In] Int[Sin[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] Sin[a + b*x]^3/(3*b*d*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 + b*x,
2]*Sin[a + b*x])/(2*b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}} + \frac{\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{2d^2} \\
&= \frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}} + \frac{\sqrt{\sin(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{2d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= \frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}} + \frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}\,dx}{2d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} \\
&= \frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}} + \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.77 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d\tan(a+bx)}\left(\sqrt{\sec^2(a+bx)}(\sin(a+bx)+\sin(3(a+bx)))+4\,\text{Hypergeomet}\right.}{12bd^3\sqrt{\sec^2(a+bx)}}$$

```
[In] Integrate[Sin[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

[Out] (Sqrt[d*Tan[a + b*x]]*(Sqrt[Sec[a + b*x]^2]*(Sin[a + b*x] + Sin[3*(a + b*x)]) + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Tan[a + b*x]))/(12*b*d^3*Sqrt[Sec[a + b*x]^2])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. 2(99) = 198.

Time = 0.94 (sec) , antiderivative size = 417, normalized size of antiderivative = 4.96

method	result
default	$\frac{\sec(bx+a)\left(\csc^2(bx+a)\right)(-1+\cos(bx+a))(\cos(bx+a)+1)\left(2\sqrt{2}\left(\cos^4(bx+a)\right)-3\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\right.}{}$

[In] int(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/12/b*sec(b*x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))*(cos(b*x+a)+1)*(2*2^(1/2)*cos(b*x+a)^4-3*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+6*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-3*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+6*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2*2^(1/2)-3*2^(1/2)*cos(b*x+a))/(d*tan(b*x+a))^(1/2)/d^2*2^(1/2)

Fricas [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")

[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^3*tan(b*x + a)^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**3/(d*tan(b*x+a))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^3}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^3}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(a+b\,x)^3}{(d\tan(a+b\,x))^{5/2}}\,dx$$

```
[In] int(sin(a + b*x)^3/(d*tan(a + b*x))^(5/2),x)
[Out] int(sin(a + b*x)^3/(d*tan(a + b*x))^(5/2), x)
```

3.111 $\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}-\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] 3*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x)
,2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-2*sin(
b*x+a)/b/d/(d*tan(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2677, 2681, 2652, 2719}

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=-\frac{3\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

```
[In] Int[Sin[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*Sin[a + b*x])/(b*d*(d*Tan[a + b*x])^(3/2)) - (3*EllipticE[a - Pi/4 + b*
x, 2]*Sin[a + b*x])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}-\frac{3\int\frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{d^2}\\
&= -\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}-\frac{\left(3\sqrt{\sin(a+bx)}\right)\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}-\frac{(3\sin(a+bx))\int\sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}-\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.49 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$
\int\frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=
-\frac{2\cos(a+bx)\left(1+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\tan^2(a+bx)\right)}{bd^2\sqrt{d\tan(a+bx)}}
$$

[In] Integrate[Sin[a + b*x]/(d*Tan[a + b*x])^(5/2),x]

[Out] (-2*Cos[a + b*x]*(1 + Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(b*d^2*Sqrt[d*Tan[a + b*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(97) = 194$.

Time = 0.95 (sec) , antiderivative size = 366, normalized size of antiderivative = 4.69

method	result
default	$\frac{\left(6\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)-3\sqrt{\cot(b}$

[In] int(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/2/b/(d*tan(b*x+a))^(1/2)/d^2*(6*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-3*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+6*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-3*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(b*x+a)-3*2^(1/2))*2^(1/2)

Fricas [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^3*tan(b*x + a)^3), x)

Sympy [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))**(5/2),x)
[Out] Integral(sin(a + b*x)/(d*tan(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)}{{\left(d\,\tan\left(a+b\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int(sin(a + b*x)/(d*tan(a + b*x))^(5/2),x)
[Out] int(sin(a + b*x)/(d*tan(a + b*x))^(5/2), x)
```

3.112 $\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 110

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}} + \frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{5bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] 6/5*cos(b*x+a)/b/d^2/(d*tan(b*x+a))^(1/2)-6/5*(sin(a+1/4*Pi+b*x)^2)^(1/2)/s
in(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/d^2/sin(
2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-2/5*csc(b*x+a)/b/d/(d*tan(b*x+a))^(3/
2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2677, 2681, 2650, 2652, 2719}

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}} + \frac{6\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} - \frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

```
[In] Int[Csc[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*Csc[a + b*x])/(5*b*d*(d*Tan[a + b*x])^(3/2)) + (6*Cos[a + b*x])/(5*b*d^
2*Sqrt[d*Tan[a + b*x]]) + (6*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(5*
b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} - \frac{3\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{5d^2}$$

$$= -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} - \frac{\left(3\sqrt{\sin(a+bx)}\right)\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$= -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}} + \frac{\left(6\sqrt{\sin(a+bx)}\right)\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$= -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}} + \frac{(6\sin(a+bx))\int\sqrt{\sin(2a+2bx)}\,dx}{5d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$= -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}} + \frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{5bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.40 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int\frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{2\left(2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sec^2(a+bx) - (3-4\csc^2(a+b\ldots\right.}{5bd^3\sqrt{\sec^2(a+bx)}\ldots}$$

```
[In] Integrate[Csc[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*(2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]^2 - (3
 - 4*Csc[a + b*x]^2 + Csc[a + b*x]^4)*Sqrt[Sec[a + b*x]^2])*Sin[a + b*x]*Sq
rt[d*Tan[a + b*x]])/(5*b*d^3*Sqrt[Sec[a + b*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(121) = 242.

Time = 0.96 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.39

method	result
default	$\frac{\left(-6\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)+3\sqrt{\cot(\ldots}\right.}{}$

```
[In] int(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/b/(d*tan(b*x+a))^(1/2)/d^2*(-6*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*
x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x
+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+3*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*
x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b
*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-6*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))
^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Ellip
```

```
ticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+3*sec(b*x+a)*(1+csc(b*x+a
)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a)
)^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+3*2^(1/2)-co
t(b*x+a)*csc(b*x+a)*2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.24

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{3\left(-i\cos\left(bx+a\right)^{2}+i\right)\sqrt{i\,d}E(\arcsin\left(\cos\left(bx+a\right)+i\sin\left(bx+a\right)\right)\mid -1)\sin\left(bx+a\right)+3\left(i\cos\left(bx+a\right)^{2}\right.}{}$$

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*(-I*cos(b*x + a)^2 + I)*sqrt(I*d)*elliptic_e(arcsin(cos(b*x + a) +
I*sin(b*x + a)), -1)*sin(b*x + a) + 3*(I*cos(b*x + a)^2 - I)*sqrt(-I*d)*ell
iptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + 3*(I*cos(
b*x + a)^2 - I)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)),
 -1)*sin(b*x + a) + 3*(-I*cos(b*x + a)^2 + I)*sqrt(-I*d)*elliptic_f(arcsin(
cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) - 2*(3*cos(b*x + a)^4 - 2*
cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/((b*d^3*cos(b*x + a)^2 -
 b*d^3)*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Integral(csc(a + b*x)/(d*tan(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{1}{\sin(a+b\,x)\,\left(d\tan(a+b\,x)\right)^{5/2}}\,dx$$

```
[In] int(1/(sin(a + b*x)*(d*tan(a + b*x))^(5/2)),x)
[Out] int(1/(sin(a + b*x)*(d*tan(a + b*x))^(5/2)), x)
```

3.113 $\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}} \, dx = \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} + \frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}} + \frac{4E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{15bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] 4/15*cos(b*x+a)/b/d^2/(d*tan(b*x+a))^(1/2)-4/15*(sin(a+1/4*Pi+b*x)^2)^(1/2)
/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/d^2/si
n(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2/15*csc(b*x+a)/b/d/(d*tan(b*x+a))^
(3/2)-2/9*csc(b*x+a)^3/b/d/(d*tan(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2677, 2679, 2681, 2650, 2652, 2719}

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}} \, dx = \frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}} + \frac{4\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{15bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} + \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}}$$

```
[In] Int[Csc[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*Csc[a + b*x])/(15*b*d*(d*Tan[a + b*x])^(3/2)) - (2*Csc[a + b*x]^3)/(9*b*
d*(d*Tan[a + b*x])^(3/2)) + (4*Cos[a + b*x])/(15*b*d^2*Sqrt[d*Tan[a + b*x]]
) + (4*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(15*b*d^2*Sqrt[Sin[2*a +
2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} - \frac{\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{3d^2}$$

$$= \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} - \frac{2\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{15d^2}$$

$$= \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} - \frac{\left(2\sqrt{\sin(a+bx)}\right)\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{15d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$= \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} + \frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}} + \frac{\left(4\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{15d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$= \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} + \frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}} + \frac{(4\sin(a+bx))\int \sqrt{\sin(2a+2bx)}\,dx}{15d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$= \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} + \frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}} + \frac{4E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{15bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.91 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{2\left(4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sec^2(a+bx)+(-6+3\csc^2(a\cdots\right.}{45bd^3\sqrt{s\cdots}}$$

```
[In] Integrate[Csc[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*(4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]^2 + (-
6 + 3*Csc[a + b*x]^2 + 8*Csc[a + b*x]^4 - 5*Csc[a + b*x]^6)*Sqrt[Sec[a + b*
x]^2])*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/(45*b*d^3*Sqrt[Sec[a + b*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(147) = 294.

Time = 1.00 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.80

method	result
default	$\frac{\left(-12\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)+6\sqrt{\cot}\right.}{}$

```
[In] int(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/45/b/(d*tan(b*x+a))^(1/2)/d^2*(-12*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(
b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b
*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+6*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(
b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc
(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-12*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+
a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*El
lipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+6*sec(b*x+a)*(1+csc(b*
x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x
+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+6*2^(1/2)
+3*cot(b*x+a)*csc(b*x+a)*2^(1/2)-5*cot(b*x+a)*csc(b*x+a)^3*2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.21

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx =$$

$$-\frac{2\left(3\left(-i\cos(bx+a)^4+2i\cos(bx+a)^2-i\right)\sqrt{i\,d}E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a)\right.}{}$$

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/45*(3*(-I*cos(b*x + a)^4 + 2*I*cos(b*x + a)^2 - I)*sqrt(I*d)*elliptic_e(
arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + 3*(I*cos(b*x + a)
^4 - 2*I*cos(b*x + a)^2 + I)*sqrt(-I*d)*elliptic_e(arcsin(cos(b*x + a) - I*
sin(b*x + a)), -1)*sin(b*x + a) + 3*(I*cos(b*x + a)^4 - 2*I*cos(b*x + a)^2
+ I)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*
x + a) + 3*(-I*cos(b*x + a)^4 + 2*I*cos(b*x + a)^2 - I)*sqrt(-I*d)*elliptic
_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) - (6*cos(b*x + a
)^6 - 15*cos(b*x + a)^4 + 4*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a
)))/((b*d^3*cos(b*x + a)^4 - 2*b*d^3*cos(b*x + a)^2 + b*d^3)*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc^3\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)**3/(d*tan(b*x+a))**(5/2),x)
[Out] Integral(csc(a + b*x)**3/(d*tan(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^3\,\left(d\tan\left(a+b\,x\right)\right)^{5/2}}\,dx$$

```
[In] int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2)),x)
[Out] int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2)), x)
```

3.114 $\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 68

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx=-\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}$$

```
[Out] -2/5*b*(a*sin(f*x+e))^(5/2)/f/(b*tan(f*x+e))^(1/2)-8/5*a^2*b*(a*sin(f*x+e))
^(1/2)/f/(b*tan(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2678, 2669}

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx=-\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}$$

```
[In] Int[(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-8*a^2*b*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[b*Tan[e + f*x]]) - (2*b*(a*Sin[e
+ f*x])^(5/2))/(5*f*Sqrt[b*Tan[e + f*x]])
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}+\frac{1}{5}(4a^2)\int\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx\\ &= -\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{a^2\sqrt{a\sin(e+fx)}(8\cot(e+fx)+\sin(2(e+fx)))\sqrt{b\tan(e+fx)}}{5f}$$

```
[In] Integrate[(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -1/5*(a^2*Sqrt[a*Sin[e + f*x]]*(8*Cot[e + f*x] + Sin[2*(e + f*x)])*Sqrt[b*T
an[e + f*x]])/f
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(56) = 112$.

Time = 1.86 (sec) , antiderivative size = 444, normalized size of antiderivative = 6.53

method	result
default	$\frac{\csc(fx+e)a^2\sqrt{b\tan(fx+e)}\,\sqrt{\sin(fx+e)a}\left(4\left(\cos^3(fx+e)\right)-5\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+}{(\cos(fx+e}}}{\cos(fx+e)+1}\right.\right.}{}$

```
[In] int((sin(f*x+e)*a)^(5/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/f*csc(f*x+e)*a^2*(b*tan(f*x+e))^(1/2)*(sin(f*x+e)*a)^(1/2)*(4*cos(f*x+
e)^3-5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)
```

```
/(cos(f*x+e)+1))*cos(f*x+e)+5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*
cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cos(f*x+e)-5*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+5*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+
1))-20*cos(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \frac{2\left(a^2\cos\left(fx+e\right)^3-5\,a^2\cos\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{5\,f\sin\left(fx+e\right)}$$

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] 2/5*(a^2*cos(f*x + e)^3 - 5*a^2*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*s
in(f*x + e)/cos(f*x + e))/(f*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(5/2)*(b*tan(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \int \left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{b\tan\left(fx+e\right)}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \frac{a^2\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\left(18\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}{10\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-1\right)}$$

```
[In] int((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(1/2),x)
```

```
[Out] (a^2*(a*sin(e + f*x))^(1/2)*(18*sin(2*e + 2*f*x) - sin(4*e + 4*f*x))*((b*si
n(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(10*f*(cos(2*e + 2*f*x) - 1)
)
```

3.115 $\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}} + \frac{4a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}$$

```
[Out] -2/3*b*(a*sin(f*x+e))^(3/2)/f/(b*tan(f*x+e))^(1/2)+4/3*a^2*(cos(1/2*f*x+1/2
*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f
*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2678, 2681, 2720}

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \frac{4a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}} - \frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}$$

```
[In] Int[(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*Sqrt[b*Tan[e + f*x]]) + (4*a^2*Sqrt[Cos[
e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e
 + f*x]])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}+\frac{1}{3}\left(2a^2\right)\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx\\ &= -\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}+\frac{\left(2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx}{3\sqrt{a\sin(e+fx)}}\\ &= -\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}+\frac{4a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 5.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{2ab\sqrt{a\sin(e+fx)}\left(-2\,\text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)+\sqrt[4]{\cos^2(e+fx)}\sin(e+fx)\right)}{3f\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
[Out] (-2*a*b*Sqrt[a*Sin[e + f*x]]*(-2*EllipticF[ArcSin[Sin[e + f*x]]/2, 2] + (Co
s[e + f*x]^2)^(1/4)*Sin[e + f*x]))/(3*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e
 + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.75 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.77

method	result
default	$-\frac{2\sqrt{\sin(fx+e)a}\,a\sqrt{b\tan(fx+e)}\left(2i\cot(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}F(i(\csc(fx+e)-\cot(fx+e)),i)+2i\csc(fx+e)\sqrt{\frac{\cos(fx}{\cos(fx+}}\right.}{3f}$

```
[In] int((sin(f*x+e)*a)^(3/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/f*(sin(f*x+e)*a)^(1/2)*a*(b*tan(f*x+e))^(1/2)*(2*I*cot(f*x+e)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-
cot(f*x+e)),I)+2*I*csc(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x
+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+cos(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.16

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx =$$
$$-\frac{2\left(\sqrt{a\sin(fx+e)}a\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)-\sqrt{2}\sqrt{-ab}a\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(f\right.}{3f}$$

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(a*sin(f*x + e))*a*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)
 - sqrt(2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x
 + e)) - sqrt(2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x + e) - I*s
in(f*x + e)))/f
```

Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(3/2)*(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int (a\sin(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int (a\sin(e+f\,x))^{3/2}\sqrt{b\tan(e+f\,x)}\,dx$$

```
[In] int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2),x)
[Out] int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2), x)
```

3.116 $\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 30

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx=-\frac{2b\sqrt{a\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

```
[Out] -2*b*(a*sin(f*x+e))^(1/2)/f/(b*tan(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2669}

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx=-\frac{2b\sqrt{a\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

```
[In] Int[Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
[Out] (-2*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Tan[e + f*x]])
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\text{integral}=-\frac{2b\sqrt{a\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = -\frac{2b\sqrt{a\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
[Out] (-2*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Tan[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.37

method	result
risch	$-\frac{2i\sqrt{-\frac{ib\left(e^{2i(fx+e)}-1\right)}{e^{2i(fx+e)}+1}}\,\sqrt{\sin(fx+e)a}\,\left(e^{2i(fx+e)}+1\right)}{\left(e^{2i(fx+e)}-1\right)f}$
default	$-\frac{\cot(fx+e)\left(4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-2\cos}{\cos(fx+e)+1}\right.\right.}{2f(\cos(fx+e)+1)\sqrt{-\frac{c}{(\cos}}}$

```
[In] int((sin(f*x+e)*a)^(1/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*I*(-I*b*(exp(2*I*(f*x+e))-1)/(exp(2*I*(f*x+e))+1))^(1/2)/(exp(2*I*(f*x+e
))-1)*(sin(f*x+e)*a)^(1/2)*(exp(2*I*(f*x+e))+1)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.57

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = -\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{f\sin(fx+e)}$$

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/(f*s
in(f*x + e))
```

Sympy [F]

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{a\sin\left(e+fx\right)}\sqrt{b\tan\left(e+fx\right)}\,dx$$

```
[In] integrate((a*sin(f*x+e))**(1/2)*(b*tan(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*sin(e + f*x))*sqrt(b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.00

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \frac{\sin\left(2\,e+2\,f\,x\right)\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{2\,\cos(e+f\,x)^2}}}{f\,\left({\cos\left(e+f\,x\right)}^2-1\right)}$$

```
[In] int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(1/2),x)
[Out] (sin(2*e + 2*f*x)*(a*sin(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(2*cos(e + f
*x)^2))^(1/2))/(f*(cos(e + f*x)^2 - 1))
```

3.117 $\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 50

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

```
[Out] 2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2
*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2681, 2720}

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

```
[In] Int[Sqrt[b*Tan[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sq
rt[a*Sin[e + f*x]])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx}{\sqrt{a\sin(e+fx)}}\\ &= \frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\cos(e+fx)\,\text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)\sqrt{b\tan(e+fx)}}{f\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}}$$

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
[Out] (2*Cos[e + f*x]*EllipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sqrt[b*Tan[e + f*x]])/
(f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

method	result	size
default	$-\frac{2iF(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{b\tan(fx+e)}}{f\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\sin(fx+e)a}}$	79

```
[In] int((b*tan(f*x+e))^(1/2)/(sin(f*x+e)*a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*I/f*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)/(1/(cos(f*x+e)+1))^(1/2)*(b*tan(f*x+e))^(1/2)/(sin(f*x+e)*a)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{\sqrt{2}\sqrt{-ab}\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+\sqrt{2}\sqrt{-ab}\text{weierstrassPInverse}(-4,0,$$
$$af$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
[Out] (sqrt(2)*sqrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e
)) + sqrt(2)*sqrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x
 + e)))/(a*f)
```

Sympy [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$$

```
[In] integrate((b*tan(f*x+e))**(1/2)/(a*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(b*tan(e + f*x))/sqrt(a*sin(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\tan(fx+e)}}{\sqrt{a\sin(fx+e)}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*tan(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{\sqrt{a\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(1/2),x)
[Out] int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(1/2), x)
```

3.118 $\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 107

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{af\sqrt{a\sin(e+fx)}} - \frac{\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{af\sqrt{a\sin(e+fx)}}$$

```
[Out] -arctan(cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/a/f/(a*sin(
f*x+e))^(1/2)-arctanh(cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/
2)/a/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2681, 12, 2645, 335, 218, 212, 209}

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{\sqrt{\cos(e+fx)}\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{b\tan(e+fx)}}{af\sqrt{a\sin(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{b\tan(e+fx)}}{af\sqrt{a\sin(e+fx)}}$$

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((ArcTan[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(a*f
*Sqrt[a*Sin[e + f*x]])) - (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*S
qrt[b*Tan[e + f*x]])/(a*f*Sqrt[a*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
```

```
f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int\frac{\csc(e+fx)}{a\sqrt{\cos(e+fx)}}\,dx}{\sqrt{a\sin(e+fx)}} \\
&= \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int\frac{\csc(e+fx)}{\sqrt{\cos(e+fx)}}\,dx}{a\sqrt{a\sin(e+fx)}} \\
&= -\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}(1-x^2)}\,dx,x,\cos(e+fx)\right)}{af\sqrt{a\sin(e+fx)}} \\
&= -\frac{\left(2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{1-x^4}\,dx,x,\sqrt{\cos(e+fx)}\right)}{af\sqrt{a\sin(e+fx)}} \\
&= -\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{\cos(e+fx)}\right)}{af\sqrt{a\sin(e+fx)}} \\
&\quad -\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{\cos(e+fx)}\right)}{af\sqrt{a\sin(e+fx)}} \\
&= -\frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{af\sqrt{a\sin(e+fx)}} \\
&\quad -\frac{\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{af\sqrt{a\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.67

$$\int\frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{b\left(\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right)+\text{arctanh}\left(\sqrt[4]{\cos^2(e+fx)}\right)\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
[Out] -((b*(ArcTan[(Cos[e + f*x]^2)^(1/4)] + ArcTanh[(Cos[e + f*x]^2)^(1/4)])*Sqr
t[a*Sin[e + f*x]])/(a^2*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]]))
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.53

method	result
default	$\frac{\cos(fx+e)\left(\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)-\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-2\cos(fx+e)+2}{\cos(fx+e)+1}\right)\right)\sqrt{b\tan(fx+e)}}{2f(\cos(fx+e)+1)\sqrt{\sin(fx+e)a}\,\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,a}$

```
[In] int((b*tan(f*x+e))^(1/2)/(sin(f*x+e)*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*cos(f*x+e)*(arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-ln(2*(2*
cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1)))*(b*tan(f*x+e))^(1/2)/(cos(f*x+e)
+1)/(sin(f*x+e)*a)^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(91) = 182$.

Time = 0.42 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.86

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \left[\frac{2\sqrt{-\frac{b}{a}}\arctan\left(\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{b}{a}}\cos(fx+e)}{(b\cos(fx+e)+b)\sin(fx+e)}\right)+\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos(fx+e)^3+4}{\cdots}\right.}{4\,af}\right.$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b/a)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*
x + e))*sqrt(-b/a)*cos(f*x + e)/((b*cos(f*x + e) + b)*sin(f*x + e))) + sqrt
(-b/a)*log(-(b*cos(f*x + e)^3 + 4*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/
cos(f*x + e))*sqrt(-b/a)*cos(f*x + e)*sin(f*x + e) - 5*b*cos(f*x + e)^2 - 5
*b*cos(f*x + e) + b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) +
1)))/(a*f), 1/4*(2*sqrt(b/a)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x +
 e)/cos(f*x + e))*sqrt(b/a)*cos(f*x + e)/((b*cos(f*x + e) - b)*sin(f*x + e)
)) + sqrt(b/a)*log((4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*
sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/a) - (b*cos(f*x + e)^2 + 6*b*cos(f
*x + e) + b)*sin(f*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x +
 e))))/(a*f)]
```

Sympy [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\tan\left(e+fx\right)}}{\left(a\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate((b*tan(f*x+e))**(1/2)/(a*sin(f*x+e))**(3/2),x)

[Out] Integral(sqrt(b*tan(e + f*x))/(a*sin(e + f*x))**(3/2), x)

Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(3/2), x)

Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{{\left(a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

[In] int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(3/2),x)

[Out] int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(3/2), x)

3.119 $\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 86

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = -\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] -b/a^2/f/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)+(cos(1/2*f*x+1/2*e)^2)^(
1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1
/2)*(b*tan(f*x+e))^(1/2)/a^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2679, 2681, 2720}

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}} - \frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(b/(a^2*f*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])) + (Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f*x
]])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{a^2 f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{2a^2}\\
&= -\frac{b}{a^2 f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx}{2a^2\sqrt{a\sin(e+fx)}}\\
&= -\frac{b}{a^2 f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{a^2 f\sqrt{a\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int\frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx=\frac{b\left(-\sqrt[4]{\cos^2(e+fx)}+\text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)\sin(e+fx)\right)}{a^2 f\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
[Out] (b*(-(Cos[e + f*x]^2)^(1/4) + EllipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sin[e +
f*x]))/(a^2*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*
x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.78

method	result
default	$\frac{\sqrt{b\tan(fx+e)}\left(i\sqrt{\frac{1}{\cos(fx+e)+1}}\cos(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)+i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx\right.}{f\,a^2\sqrt{\sin(fx+e)a}}$

```
[In] int((b*tan(f*x+e))^(1/2)/(sin(f*x+e)*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*tan(f*x+e))^(1/2)/a^2/(sin(f*x+e)*a)^(1/2)*(I*(1/(cos(f*x+e)+1))^(1/
2)*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc
(f*x+e)),I)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(cot(f*x+e)-csc(f*x+e)),I)-cot(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\sqrt{2}\cos\left(fx+e\right)^2-\sqrt{2}\right)\sqrt{-ab}\,\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx\right.}{}$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/2*((sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassPInverse(-4,
0, cos(f*x + e) + I*sin(f*x + e)) + (sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt
(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*sqrt(a
*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))/(a^3*f*cos(f
*x + e)^2 - a^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*tan(f*x+e))**(1/2)/(a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{{\left(a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(5/2),x)
[Out] int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(5/2), x)
```

3.120 $\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = -\frac{24a^2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{12a^2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{5f} - \frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

```
[Out] -24/5*a^2*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin
(1/2*f*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x
+e))^(1/2)-2/5*b*(a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2)/f+12/5*a^2*b*(a*
sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2678, 2674, 2681, 2719}

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = -\frac{24a^2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{12a^2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{5f} - \frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

```
[In] Int[(a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-24*a^2*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[Cos[
e + f*x]]*Sqrt[b*Tan[e + f*x]]) + (12*a^2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan
[e + f*x]])/(5*f) - (2*b*(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]])/(5*f)
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f} \\
&\quad + \frac{1}{5}\left(6a^2\right)\int\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx \\
&= \frac{12a^2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{5f} \\
&\quad - \frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f} - \frac{1}{5}\left(12a^2b^2\right)\int\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx \\
&= \frac{12a^2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{5f} - \frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f} \\
&\quad - \frac{\left(12a^2b^2\sqrt{a\sin(e+fx)}\right)\int\sqrt{\cos(e+fx)}\,dx}{5\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}
\end{aligned}$$

$$= -\frac{24a^2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{12a^2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{5f} - \frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.63 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \frac{a^2b\left(\cos^2(e+fx)^{3/4}(11+\cos(2(e+fx))) - 12\cos^2(e+fx)\,\text{Hypergeometric2}\right.}{5f\cos^2(e+fx)^{3/4}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^2*b*((Cos[e + f*x]^2)^(3/4)*(11 + Cos[2*(e + f*x)]) - 12*Cos[e + f*x]^2*
Hypergeometric2F1[1/4, 1/2, 3/2, Sin[e + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sqrt
[b*Tan[e + f*x]])/(5*f*(Cos[e + f*x]^2)^(3/4))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.94 (sec) , antiderivative size = 467, normalized size of antiderivative = 3.71

method	result
default	$-\frac{2\sin(fx+e)\left(12i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(\cot(fx+e)-\csc(fx+e)),i)(\cos^2(fx+e))-12i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\right.}{}$

```
[In] int((sin(f*x+e)*a)^(5/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/f*sin(f*x+e)*(12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I)*cos(f*x+e)^2-12*I*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-cs
c(f*x+e)),I)*cos(f*x+e)^2+24*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I)*cos(f*x+e)-24*I*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+
e)-csc(f*x+e)),I)*cos(f*x+e)+12*I*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I)*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-12*I*EllipticF(I*(
cot(f*x+e)-csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)+cos(f*x+e)^3*sin(f*x+e)+sin(f*x+e)*cos(f*x+e)^2-7*sin(f*x+e)*cos(
```

```
f*x+e)+5*sin(f*x+e))*(b*tan(f*x+e))^(1/2)*(sin(f*x+e)*a)^(1/2)*a^2*b/(cos(f
*x+e)-1)/(cos(f*x+e)+1)^2
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.01

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \frac{2\left(6\sqrt{2}\sqrt{-ab}a^2b\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx\right.}{}$$

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(6*sqrt(2)*sqrt(-a*b)*a^2*b*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 6*sqrt(2)*sqrt(-a*b)*a^2*b*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)))
+ (a^2*b*cos(f*x + e)^2 + 5*a^2*b)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)
/cos(f*x + e)))/f
```

Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(5/2)*(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (a\sin(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)
```

Giac [F]

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (a\sin(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (a\,\sin(e+f\,x))^{5/2}\,(b\tan(e+f\,x))^{3/2}\,dx$$

```
[In] int((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2), x)
```

3.121 $\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 68

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \frac{8a^2b\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}} - \frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}$$

```
[Out] -2/3*b*(a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2)/f+8/3*a^2*b*(b*tan(f*x+e))
^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2678, 2669}

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \frac{8a^2b\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}} - \frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}$$

```
[In] Int[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (8*a^2*b*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e + f*x]]) - (2*b*(a*Sin[e +
 f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(3*f)
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
```

```
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}+\frac{1}{3}\left(4a^2\right)\int\frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx\\ &=\frac{8a^2b\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

$$\int(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx=\frac{a^2b(7+\cos(2(e+fx)))\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
[Out] (a^2*b*(7 + Cos[2*(e + f*x)])*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e + f*x
]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(56) = 112.

Time = 1.79 (sec) , antiderivative size = 436, normalized size of antiderivative = 6.41

method	result
default	$\frac{\csc(fx+e)\sqrt{b\tan(fx+e)}\,ba\left(3\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)\cos(fx+e)-}{}$

```
[In] int((sin(f*x+e)*a)^(3/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*csc(f*x+e)*(b*tan(f*x+e))^(1/2)*b*a*(3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cos(f*x+e)-3*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)
+1))*cos(f*x+e)+4*cos(f*x+e)^2+3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln((2
*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+12)*(sin(f*x+e)*a)
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \frac{2\left(ab\cos\left(fx+e\right)^2+3\,ab\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{3\,f\sin\left(fx+e\right)}$$

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a*b*cos(f*x + e)^2 + 3*a*b)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/c
os(f*x + e))/(f*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(3/2)*(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \int (a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

Giac [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \int (a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \frac{a\,b\,(13\,\sin(e+f\,x)+\sin(3\,e+3\,f\,x))\,\sqrt{a\,\sin(e+f\,x)}\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{6\,f\,\sin(e+f\,x)^2}$$

```
[In] int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(3/2),x)
[Out] (a*b*(13*sin(e + f*x) + sin(3*e + 3*f*x))*(a*sin(e + f*x))^(1/2)*((b*sin(2*
e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(6*f*sin(e + f*x)^2)
```

3.122 $\int \sqrt{a \sin(e+fx)}(b \tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \sqrt{a \sin(e+fx)}(b \tan(e+fx))^{3/2}\,dx = -\frac{4b^2 E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a \sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b \tan(e+fx)}} + \frac{2b\sqrt{a \sin(e+fx)}\sqrt{b \tan(e+fx)}}{f}$$

```
[Out] -4*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*
x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1
/2)+2*b*(a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2674, 2681, 2719}

$$\int \sqrt{a \sin(e+fx)}(b \tan(e+fx))^{3/2}\,dx = \frac{2b\sqrt{a \sin(e+fx)}\sqrt{b \tan(e+fx)}}{f} - \frac{4b^2 E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a \sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b \tan(e+fx)}}$$

```
[In] Int[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-4*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[Cos[e + f*x
]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])/
f
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] && !(GtQ[m, 1] && !IntegerQ[(m - 1)/2])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f} - \left(2b^2\right)\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx\\ &= \frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f} - \frac{\left(2b^2\sqrt{a\sin(e+fx)}\right)\int\sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\ &= -\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.51 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \frac{2b\left(\cos^2(e+fx)^{3/4} - \cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\right)\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f\cos^2(e+fx)^{3/4}}$$

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*b*((Cos[e + f*x]^2)^(3/4) - Cos[e + f*x]^2*Hypergeometric2F1[1/4, 1/2, 3
/2, Sin[e + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(f*(Cos[e +
 f*x]^2)^(3/4))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.69

method	result
default	$-\frac{4\left(-\frac{b(\csc(fx+e)-\cot(fx+e))}{\left(\csc^2(fx+e)\right)(1-\cos(fx+e))^2-1}\right)^{\frac{3}{2}}\left(\left(\csc^2(fx+e)\right)(1-\cos(fx+e))^2-1\right)\left(i\sqrt{\left(\csc^2(fx+e)\right)(1-\cos(fx+e))^2+1}\,\sqrt{-(\csc^2(fx+e}\right.}{}$

[In] int((sin(f*x+e)*a)^(1/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] -4/f*(-b/(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(csc(f*x+e)-cot(f*x+e)))^(3/2)*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(I*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)-I*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)+csc(f*x+e)^3*(1-cos(f*x+e))^3)*(1/(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)*a*(csc(f*x+e)-cot(f*x+e)))^(1/2)/(1-cos(f*x+e))^2*sin(f*x+e)^2

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \frac{2\left(\sqrt{2}\sqrt{-ab}b\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))\right.}{}$$

[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 2*(sqrt(2)*sqrt(-a*b)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(2)*sqrt(-a*b)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + sqrt(a*sin(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e)))/f

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(1/2)*(b*tan(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int \sqrt{a\sin(fx+e)}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int \sqrt{a\,\sin\left(e+f\,x\right)}\,{\left(b\tan(e+f\,x)\right)}^{3/2}\,dx$$

```
[In] int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2), x)
```

3.123 $\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 30

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

```
[Out] 2*b*(b*tan(f*x+e))^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2669}

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

```
[In] Int[(b*Tan[e + f*x])^(3/2)/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (2*b*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[a*Sin[e + f*x]])
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/Sqrt[a*Sin[e + f*x]],x]
[Out] (2*b*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[a*Sin[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(26) = 52.

Time = 2.07 (sec) , antiderivative size = 268, normalized size of antiderivative = 8.93

method	result
default	$\frac{\left(\cos(fx+e)\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-2\cos(fx+e)+2}{\cos(fx+e)+1}\right)-\cos(fx+e)\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2 \cdots}{\cos(fx \cdots}\right.\right.}{2f(\cos(fx+e)+1)\sqrt{\sin(fx+e)a}\sqrt{- \cdots}}$

```
[In] int((b*tan(f*x+e))^(3/2)/(sin(f*x+e)*a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/f*(cos(f*x+e)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-cos(f*x
+e)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+4*cos(f*x+e)*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(b*tan(f
*x+e))^(1/2)*b/(cos(f*x+e)+1)/(sin(f*x+e)*a)^(1/2)/(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\sqrt{a\sin(fx+e)}\,b\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{af\sin(fx+e)}$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a*sin(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e))/(a*f*sin(f*x + e
))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((b*tan(f*x+e))**(3/2)/(a*sin(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{\sqrt{a\sin(fx+e)}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^(3/2)/sqrt(a*sin(f*x + e)), x)
```

Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{\sqrt{a\sin(fx+e)}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(3/2)/sqrt(a*sin(f*x + e)), x)
```

Mupad [B] (verification not implemented)

Time = 3.56 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\,b\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{2\cos(e+f\,x)^2}}}{f\,\sqrt{a\,\sin\left(e+f\,x\right)}}$$

```
[In] int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(1/2),x)
[Out] (2*b*((b*sin(2*e + 2*f*x))/(2*cos(e + f*x)^2))^(1/2))/(f*(a*sin(e + f*x))^(
1/2))
```

3.124 $\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 90

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}$$

```
[Out] -2*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*
x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/a^2/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e)
)^(1/2)+2*b*(a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/a^2/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2673, 2681, 2719}

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f} - \frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(a^2*f*Sqrt[Cos[e +
 f*x]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x
]])/(a^2*f)
```

Rule 2673

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Dist[b^2*((m + 2)/(a^2*(n - 1))), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f} - \frac{b^2\int\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{a^2}\\ &= \frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f} - \frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\int\sqrt{\cos(e+fx)}\,dx}{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\ &= -\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.55 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int\frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \frac{\left(2\cos(e+fx)\cos^2(e+fx)^{3/4} - \cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2(e\right.\right.}{af\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}}$$

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((2*Cos[e + f*x]*(Cos[e + f*x]^2)^(3/4) - Cos[e + f*x]^3*Hypergeometric2F1[
1/4, 1/2, 3/2, Sin[e + f*x]^2])*(b*Tan[e + f*x])^(3/2))/(a*f*(Cos[e + f*x]^
2)^(3/4)*Sqrt[a*Sin[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.66

method	result
default	$-\frac{2\left(i\sqrt{(\csc^2(fx+e))(1-\cos(fx+e))^2+1}\,\sqrt{-(\csc^2(fx+e))(1-\cos(fx+e))^2+1}\,F(i(\csc(fx+e)-\cot(fx+e)),i)-i\sqrt{(\csc^2(fx+e))(1-c}\right.}{}$

[In] int((b*tan(f*x+e))^(3/2)/(sin(f*x+e)*a)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/f*(I*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)-I*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)+csc(f*x+e)^3*(1-cos(f*x+e))^3+csc(f*x+e)-cot(f*x+e))*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(-b/(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(csc(f*x+e)-cot(f*x+e)))^(3/2)/(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^2/(1/(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)*a*(csc(f*x+e)-cot(f*x+e)))^(3/2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.14

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\sqrt{-ab}b\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(}{}$$

[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] (sqrt(2)*sqrt(-a*b)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(2)*sqrt(-a*b)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*sqrt(a*sin(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(a^2*f)

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*tan(f*x+e))**(3/2)/(a*sin(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{3/2}}{(a\,\sin(e+f\,x))^{3/2}}\,dx$$

```
[In] int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(3/2),x)
[Out] int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(3/2), x)
```

3.125 $\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{b^2\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{b^2\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] b^2*arctan(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3/f/cos(f*x+e)^(1/2)/(b
*tan(f*x+e))^(1/2)-b^2*arctanh(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3/f
/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2)+2*b*(b*tan(f*x+e))^(1/2)/a^2/f/(a*si
n(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2673, 2681, 12, 2645, 335, 304, 209, 212}

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{b^2\sqrt{a\sin(e+fx)}\arctan\left(\sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{b^2\sqrt{a\sin(e+fx)}\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (b^2*ArcTan[Sqrt[Cos[e + f*x]]*Sqrt[a*Sin[e + f*x]])/(a^3*f*Sqrt[Cos[e + f
*x]]*Sqrt[b*Tan[e + f*x]]) - (b^2*ArcTanh[Sqrt[Cos[e + f*x]]*Sqrt[a*Sin[e
+ f*x]])/(a^3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[b*Tan[
e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2673

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
```

```
(a^2*f*(n - 1))), x] - Dist[b^2*((m + 2)/(a^2*(n - 1))), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}+\frac{b^2\int\frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx}{a^2}\\
&= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}+\frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\int\frac{\sqrt{\cos(e+fx)}\csc(e+fx)}{a}\,dx}{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\
&= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}+\frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\int\sqrt{\cos(e+fx)}\csc(e+fx)\,dx}{a^3\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\
&= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt{x}}{1-x^2}\,dx,x,\cos(e+fx)\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\
&= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{\left(2b^2\sqrt{a\sin(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{1-x^4}\,dx,x,\sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\
&= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\
&= \frac{b^2\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\
&\quad-\frac{b^2\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.72

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{b\left(\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right)\cos^2(e+fx) - \operatorname{arctanh}\left(\sqrt[4]{\cos^2(e+fx)}\right)\cos^2(e+f\right.}{a^2 f\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}}$$

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (b*(ArcTan[(Cos[e + f*x]^2)^(1/4)]*Cos[e + f*x]^2 - ArcTanh[(Cos[e + f*x]^2
)^(1/4)]*Cos[e + f*x]^2 + 2*(Cos[e + f*x]^2)^(3/4))*Sqrt[b*Tan[e + f*x]])/(
a^2*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[a*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.52

method	result
default	$\frac{\left(4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+\cos(fx+e)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)+\cos(fx+e)\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{}{(c}}}{\cos(fx+e)+}\right.\right.}{2f(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sqrt{\sin(fx+e)a}\,a^2}$

```
[In] int((b*tan(f*x+e))^(3/2)/(sin(f*x+e)*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+cos(f*x+e)*arctan(
1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+cos(f*x+e)*ln(2*(2*cos(f*x+e)*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-co
s(f*x+e)+1)/(cos(f*x+e)+1))+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(b*tan(
f*x+e))^(1/2)*b/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(sin(f*
x+e)*a)^(1/2)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(125) = 250.

Time = 0.48 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.61

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \left[\frac{2\,ab\sqrt{-\frac{b}{a}}\arctan\left(\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{b}{a}}\cos(fx+e)}{(b\cos(fx+e)+b)\sin(fx+e)}\right)\sin(fx+e) + ab\sqrt{-\frac{b}{a}}\,\mathrm{l}}{}\right.$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*sqrt(-b/a)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/co
s(f*x + e))*sqrt(-b/a)*cos(f*x + e)/((b*cos(f*x + e) + b)*sin(f*x + e)))*si
n(f*x + e) + a*b*sqrt(-b/a)*log(-(b*cos(f*x + e)^3 - 4*sqrt(a*sin(f*x + e))
*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/a)*cos(f*x + e)*sin(f*x + e) - 5
*b*cos(f*x + e)^2 - 5*b*cos(f*x + e) + b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^
2 + 3*cos(f*x + e) + 1))*sin(f*x + e) + 8*sqrt(a*sin(f*x + e))*b*sqrt(b*sin
(f*x + e)/cos(f*x + e)))/(a^3*f*sin(f*x + e)), -1/4*(2*a*b*sqrt(b/a)*arctan
(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/a)*cos(f*x
 + e)/((b*cos(f*x + e) - b)*sin(f*x + e)))*sin(f*x + e) - a*b*sqrt(b/a)*log
((4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e
)/cos(f*x + e))*sqrt(b/a) - (b*cos(f*x + e)^2 + 6*b*cos(f*x + e) + b)*sin(f
*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x + e)
 - 8*sqrt(a*sin(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(a^3*f*sin(f
*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*tan(f*x+e))**(3/2)/(a*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(a\sin(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(a\sin(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx=\int \frac{{\left(b\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}}{{\left(a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(5/2),x)
[Out] int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(5/2), x)
```

3.126 $\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}} - \frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}} + \frac{8a^4E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{15f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] 8/15*a^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*
f*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^
(1/2)-4/15*a^2*b*(a*sin(f*x+e))^(5/2)/f/(b*tan(f*x+e))^(3/2)-2/9*b*(a*sin(f
*x+e))^(9/2)/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2678, 2681, 2719}

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{8a^4E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{15f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}} - \frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

```
[In] Int[(a*Sin[e + f*x])^(9/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-4*a^2*b*(a*Sin[e + f*x])^(5/2))/(15*f*(b*Tan[e + f*x])^(3/2)) - (2*b*(a*S
in[e + f*x])^(9/2))/(9*f*(b*Tan[e + f*x])^(3/2)) + (8*a^4*EllipticE[(e + f*
```

```
x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(15*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]
])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}+\frac{1}{3}\left(2a^2\right)\int\frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx\\
&= -\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}+\frac{1}{15}\left(4a^4\right)\int\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx\\
&= -\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}+\frac{\left(4a^4\sqrt{a\sin(e+fx)}\right)\int\sqrt{\cos(e+fx)}\,dx}{15\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\
&= -\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}+\frac{8a^4E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{15f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{a^4\left(\cos^2(e+fx)^{3/4}(-17+5\cos(2(e+fx)))+12\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2\right.\right.}{90f\cos^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(9/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (a^4*((Cos[e + f*x]^2)^(3/4)*(-17 + 5*Cos[2*(e + f*x)]) + 12*Hypergeometric
2F1[1/4, 1/2, 3/2, Sin[e + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sin[2*(e + f*x)])/
(90*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[b*Tan[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.08 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.77

method	result
default	$-\frac{2\sqrt{\sin(fx+e)a}\,a^4\left(12i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}-12i\cos(fx+e)E(i(\csc(fx+e)-\cot(\right.}{}$

```
[In] int((sin(f*x+e)*a)^(9/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/f*(sin(f*x+e)*a)^(1/2)*a^4/(cos(f*x+e)+1)/(b*tan(f*x+e))^(1/2)*(12*I*
cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)-12*I*cos(f*x+e)*EllipticE(I*(csc(f*x+e)-c
ot(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)-5*
cos(f*x+e)^4*sin(f*x+e)+24*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)-24*I*EllipticE(I*(csc(f
*x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(
1/2)-5*cos(f*x+e)^3*sin(f*x+e)+12*I*sec(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(
f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)-12*I*
sec(f*x+e)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)+11*sin(f*x+e)*cos(f*x+e)^2+11*sin(f*x+e)*
cos(f*x+e)-12*sin(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2\left(6\sqrt{2}\sqrt{-ab}a^4\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+6\sqrt{2}\sqrt{}\right.}{}$$

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(6*sqrt(2)*sqrt(-a*b)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 6*sqrt(2)*sqrt(-a*b)*a^4*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) -
(5*a^4*cos(f*x + e)^4 - 11*a^4*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b*
sin(f*x + e)/cos(f*x + e)))/(b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(9/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{9}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(9/2)/sqrt(b*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{9}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(9/2)/sqrt(b*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx$$

```
[In] int((a*sin(e + f*x))^(9/2)/(b*tan(e + f*x))^(1/2),x)
[Out] int((a*sin(e + f*x))^(9/2)/(b*tan(e + f*x))^(1/2), x)
```

3.127 $\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{8a^2b(a\sin(e+fx))^{3/2}}{21f(b\tan(e+fx))^{3/2}} - \frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}$$

```
[Out] -8/21*a^2*b*(a*sin(f*x+e))^(3/2)/f/(b*tan(f*x+e))^(3/2)-2/7*b*(a*sin(f*x+e)
)^(7/2)/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2678, 2669}

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{8a^2b(a\sin(e+fx))^{3/2}}{21f(b\tan(e+fx))^{3/2}} - \frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}$$

```
[In] Int[(a*Sin[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-8*a^2*b*(a*Sin[e + f*x])^(3/2))/(21*f*(b*Tan[e + f*x])^(3/2)) - (2*b*(a*S
in[e + f*x])^(7/2))/(7*f*(b*Tan[e + f*x])^(3/2))
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}+\frac{1}{7}\left(4a^2\right)\int\frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx\\ &= -\frac{8a^2b(a\sin(e+fx))^{3/2}}{21f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int\frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx=\frac{a^3\cos(e+fx)(-11+3\cos(2(e+fx)))\sqrt{a\sin(e+fx)}}{21f\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
[Out] (a^3*Cos[e + f*x]*(-11 + 3*Cos[2*(e + f*x)])*Sqrt[a*Sin[e + f*x]])/(21*f*Sq
rt[b*Tan[e + f*x]])
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{2a^3\sqrt{\sin(fx+e)a}\,\left(3\left(\cos^3(fx+e)\right)-7\cos(fx+e)\right)}{21f\sqrt{b\tan(fx+e)}}$	48

```
[In] int((sin(f*x+e)*a)^(7/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/21/f*a^3*(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2)*(3*cos(f*x+e)^3-7*cos(
f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{2\left(3\,a^3\cos\left(fx+e\right)^4-7\,a^3\cos\left(fx+e\right)^2\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{21\,bf\sin\left(fx+e\right)}$$

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] 2/21*(3*a^3*cos(f*x + e)^4 - 7*a^3*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqr
t(b*sin(f*x + e)/cos(f*x + e))/(b*f*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(7/2)/(b*tan(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.29

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{a^3\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\sqrt{-\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{2\,{\sin\left(e+f\,x\right)}^2-2}}\,\left(22\,\sin\left(e+f\,x\right)+19\,\sin\left(3\,e+3\,f\,x\right)-3\,\sin\left(5\,e+5\,f\,x\right)\right)}{168\,b\,f\,{\sin\left(e+f\,x\right)}^2}$$

```
[In] int((a*sin(e + f*x))^(7/2)/(b*tan(e + f*x))^(1/2),x)
[Out] -(a^3*(a*sin(e + f*x))^(1/2)*(-(b*sin(2*e + 2*f*x))/(2*sin(e + f*x)^2 - 2))
^(1/2)*(22*sin(e + f*x) + 19*sin(3*e + 3*f*x) - 3*sin(5*e + 5*f*x)))/(168*b
*f*sin(e + f*x)^2)
```

3.128 $\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}} + \frac{4a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] 4/5*a^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f
*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(
1/2)-2/5*b*(a*sin(f*x+e))^(5/2)/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2678, 2681, 2719}

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{4a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}$$

```
[In] Int[(a*Sin[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*b*(a*Sin[e + f*x])^(5/2))/(5*f*(b*Tan[e + f*x])^(3/2)) + (4*a^2*Ellipti
cE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan
[e + f*x]])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
```

```
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}+\frac{1}{5}\left(2a^2\right)\int\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx\\ &= -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}+\frac{\left(2a^2\sqrt{a\sin(e+fx)}\right)\int\sqrt{\cos(e+fx)}\,dx}{5\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\ &= -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}+\frac{4a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int\frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{a^2\left(\cos^2(e+fx)^{3/4}-\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\right)\sqrt{a\sin(e+fx)}\sin(2(e+fx))}{5f\cos^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -1/5*(a^2*((Cos[e + f*x]^2)^(3/4) - Hypergeometric2F1[1/4, 1/2, 3/2, Sin[e
+ f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sin[2*(e + f*x)])/(f*(Cos[e + f*x]^2)^(3/4)
*Sqrt[b*Tan[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.91

method	result
default	$\frac{2\sqrt{\sin(fx+e)a}\,a^2\left(2i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\cos(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)-2i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i($

```
[In] int((sin(f*x+e)*a)^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/f*(sin(f*x+e)*a)^(1/2)*a^2/(cos(f*x+e)+1)/(b*tan(f*x+e))^(1/2)*(2*I*cos
(f*x+e)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*(1/(cos(f*x+e)+1))^(1/2)-2*I*cos(f*x+e)*EllipticE(I*(cot(f*x+e)-csc(f
*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)+4*I*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(1/(
cos(f*x+e)+1))^(1/2)-4*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cot
(f*x+e)-csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+2*I*sec(f*x+e)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(1/(cos(f*x+e)
+1))^(1/2)-2*I*sec(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(co
t(f*x+e)-csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)-sin(f*x+e)*cos(f*x+e)^2-si
n(f*x+e)*cos(f*x+e)+2*sin(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.33

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx =$$
$$-\frac{2\left(\sqrt{a\sin\left(fx+e\right)}a^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos\left(fx+e\right)^2+\sqrt{2}\sqrt{-ab}a^2\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-\right.}{}$$

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(sqrt(a*sin(f*x + e))*a^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x +
e)^2 + sqrt(2)*sqrt(-a*b)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(2)*sqrt(-a*b)*a^2*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{5}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{5}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$$

```
[In] int((a*sin(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2),x)
[Out] int((a*sin(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2), x)
```

3.129 $\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

```
[Out] -2/3*b*(a*sin(f*x+e))^(3/2)/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2669}

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

```
[In] Int[(a*Sin[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*(b*Tan[e + f*x])^(3/2))
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(a \sin(e+fx))^{3/2}}{\sqrt{b \tan(e+fx)}}\,dx = -\frac{2b(a \sin(e+fx))^{3/2}}{3f(b \tan(e+fx))^{3/2}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
[Out] (-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*(b*Tan[e + f*x])^(3/2))
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{2\sqrt{\sin(fx+e)a}\,\cos(fx+e)a}{3f\sqrt{b\tan(fx+e)}}$	33

```
[In] int((sin(f*x+e)*a)^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3/f*(sin(f*x+e)*a)^(1/2)*cos(f*x+e)*a/(b*tan(f*x+e))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \frac{(a \sin(e+fx))^{3/2}}{\sqrt{b \tan(e+fx)}}\,dx = -\frac{2\sqrt{a \sin(fx+e)}a\sqrt{\frac{b \sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)^2}{3\,bf\sin(fx+e)}$$

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(a*sin(f*x + e))*a*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)^
2/(b*f*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.16

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{a\,\sqrt{a\,\sin(e+f\,x)}\,(\sin(e+f\,x)+\sin(3\,e+3\,f\,x))\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{6\,b\,f\,\sin(e+f\,x)^2}$$

```
[In] int((a*sin(e + f*x))^(3/2)/(b*tan(e + f*x))^(1/2),x)
[Out] -(a*(a*sin(e + f*x))^(1/2)*(sin(e + f*x) + sin(3*e + 3*f*x))*((b*sin(2*e +
2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(6*b*f*sin(e + f*x)^2)
```

3.130 $\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 50

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx=\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] 2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2
*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2681, 2719}

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx=\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Int[Sqrt[a*Sin[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[Cos[e + f*x]]*Sq
rt[b*Tan[e + f*x]])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{a\sin(e+fx)}\int\sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\ &= \frac{2E\left(\frac{1}{2}(e+fx)\middle|\,2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sqrt{a\sin(e+fx)}\sin(2(e+fx))}{2f\cos^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
[Out] (Hypergeometric2F1[1/4, 1/2, 3/2, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*Sin[
2*(e + f*x)])/(2*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[b*Tan[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.82 (sec) , antiderivative size = 306, normalized size of antiderivative = 6.12

method	result
default	$\frac{2\left(i\sqrt{(\csc^2(fx+e))(1-\cos(fx+e))^2+1}\,\sqrt{-(\csc^2(fx+e))(1-\cos(fx+e))^2+1}\,F(i(\csc(fx+e)-\cot(fx+e)),i)-i\sqrt{(\csc^2(fx+e))(1-\cos\cdots}\right.}{f\sqrt{-\frac{b(\csc(\cdots}{(\csc^2(fx+\cdots}}}$
risch	$-\frac{i\sqrt{2}\,\sqrt{-i\left(e^{2i(fx+e)}-1\right)a\,e^{-i(fx+e)}}}{f\sqrt{-\frac{ib\left(e^{2i(fx+e)}-1\right)}{e^{2i(fx+e)}+1}}}-\frac{i\left(-\frac{2\left(ab\,e^{2i(fx+e)}+ab\right)}{ab\sqrt{e^{i(fx+e)}\left(ab\,e^{2i(fx+e)}+ab\right)}}+\frac{i\sqrt{-i\left(e^{i(fx+e)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(fx+e)}-i\right)}\,\sqrt{ie^{i(fx+e)}}\left(-\cdots\right.}{\sqrt{ab\,e^{3i(fx\cdots}}}\cdots}{f\sqrt{\cdots}}$

```
[In] int((sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(I*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e)
)^2+1)^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)-I*(csc(f*x+e)^2*(1-cos(
f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*EllipticE(I*(cs
c(f*x+e)-cot(f*x+e)),I)+csc(f*x+e)^3*(1-cos(f*x+e))^3+cot(f*x+e)-csc(f*x+e)
)*(1/(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)*a*(csc(f*x+e)-cot(f*x+e)))^(1/2)/(-b
/(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(csc(f*x+e)-cot(f*x+e)))^(1/2)/(csc(f*x+
e)^2*(1-cos(f*x+e))^2-1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{\sqrt{2}\sqrt{-ab}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\sin{(fx+e)}))+\sqrt{2}\sqrt{-ab}\,\mathrm{w}}{bf}$$

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*sqrt(-a*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
f*x + e) + I*sin(f*x + e))) + sqrt(2)*sqrt(-a*b)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b*f)
```

Sympy [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{a\sin{(e+fx)}}}{\sqrt{b\tan{(e+fx)}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sin(e + f*x))/sqrt(b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{a\sin{(fx+e)}}}{\sqrt{b\tan{(fx+e)}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx=\int \frac{\sqrt{a\sin\left(fx+e\right)}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sin(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx=\int \frac{\sqrt{a\,\sin\left(e+f\,x\right)}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

```
[In] int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2),x)
[Out] int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2), x)
```

3.131 $\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] arctan(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a/f/cos(f*x+e)^(1/2)/(b*tan(f
*x+e))^(1/2)-arctanh(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a/f/cos(f*x+e)^
(1/2)/(b*tan(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2681, 12, 2645, 335, 304, 209, 212}

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{a\sin(e+fx)}\arctan\left(\sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\sqrt{a\sin(e+fx)}\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Int[1/(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (ArcTan[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(a*f*Sqrt[Cos[e + f*x]]*S
qrt[b*Tan[e + f*x]]) - (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(
a*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}\csc(e+fx)}{a}\,dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= \frac{\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}\csc(e+fx)\,dx}{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= -\frac{\sqrt{a\sin(e+fx)}\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, \cos(e+fx)\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= -\frac{\left(2\sqrt{a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= -\frac{\sqrt{a\sin(e+fx)}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{\sqrt{a\sin(e+fx)}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= \frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{\left(\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right) - \text{arctanh}\left(\sqrt[4]{\cos^2(e+fx)}\right)\right)\sin(2(e+fx))}{2f\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[1/(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
[Out] ((ArcTan[(Cos[e + f*x]^2)^(1/4)] - ArcTanh[(Cos[e + f*x]^2)^(1/4)])*Sin[2*(
e + f*x)])/(2*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e +
f*x]])
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{\sin(fx+e)\left(\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)+\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)\right)}{2f(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sqrt{\sin(fx+e)a}\sqrt{b\tan(fx+e)}}$	158

```
[In] int(1/(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*sin(f*x+e)*(arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+ln((2*co
s(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1)))/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)/(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(90) = 180$.

Time = 0.41 (sec) , antiderivative size = 419, normalized size of antiderivative = 3.95

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$$
$$= \left[\frac{2\sqrt{-ab}\arctan\left(\frac{2\sqrt{-ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{(ab\cos(fx+e)+ab)\sin(fx+e)}\right)-\sqrt{-ab}\log\left(-\frac{ab\cos(fx+e)^3-5ab\cos(fx+e)^2+4\sqrt{-ab}\sqrt{a\sin(fx+e)}\cdots}{\cos(fx+e)^3+\cdots}\right)}{4abf}\right.$$
$$-\frac{2\sqrt{ab}\arctan\left(\frac{2\sqrt{ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{(ab\cos(fx+e)-ab)\sin(fx+e)}\right)-\sqrt{ab}\log\left(\frac{4\sqrt{ab}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(\cdots}{\cos(\cdots}}\cdots}{\left(\cos(fx+e)^2-2\cos(\cdots\right)\cdots}\right)}{4abf}$$

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*(2*sqrt(-a*b)*arctan(2*sqrt(-a*b)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x
+ e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) + a*b)*sin(f*x + e))) -
sqrt(-a*b)*log(-(a*b*cos(f*x + e)^3 - 5*a*b*cos(f*x + e)^2 + 4*sqrt(-a*b)*s
qrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e) - 5*a*b*cos(f*x + e) + a*b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos
(f*x + e) + 1)))/(a*b*f), -1/4*(2*sqrt(a*b)*arctan(2*sqrt(a*b)*sqrt(a*sin(f
*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e)
```

```
- a*b)*sin(f*x + e))) - sqrt(a*b)*log((4*sqrt(a*b)*(cos(f*x + e)^2 + cos(f*
x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)) - (a*b*cos(f
*x + e)^2 + 6*a*b*cos(f*x + e) + a*b)*sin(f*x + e))/((cos(f*x + e)^2 - 2*co
s(f*x + e) + 1)*sin(f*x + e))))/(a*b*f)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sin\left(e+fx\right)}\sqrt{b\tan\left(e+fx\right)}}\,dx$$

```
[In] integrate(1/(a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
[Out] Integral(1/(sqrt(a*sin(e + f*x))*sqrt(b*tan(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\,\sin\left(e+f\,x\right)}\,\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

```
[In] int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(1/2)),x)
[Out] int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(1/2)), x)
```

3.132 $\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = -\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] -(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*
e),2^(1/2))*(a*sin(f*x+e))^(1/2)/a^2/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2
)-b*(a*sin(f*x+e))^(1/2)/a^2/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2679, 2681, 2719}

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = -\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Int[1/((a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] -((b*Sqrt[a*Sin[e + f*x]])/(a^2*f*(b*Tan[e + f*x])^(3/2))) - (EllipticE[(e
+ f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(a^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e +
 f*x]])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}-\frac{\int\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{2a^2}\\ &= -\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}-\frac{\sqrt{a\sin(e+fx)}\int\sqrt{\cos(e+fx)}\,dx}{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\ &= -\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.63 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int\frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = -\frac{b\sqrt{a\sin(e+fx)}\left(2\cos^2(e+fx)^{3/4}+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sin^2(e+fx)\right)}{2a^2f\cos^2(e+fx)^{3/4}(b\tan(e+fx))^{3/2}}$$

```
[In] Integrate[1/((a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] -1/2*(b*Sqrt[a*Sin[e + f*x]]*(2*(Cos[e + f*x]^2)^(3/4) + Hypergeometric2F1[
1/4, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x]^2))/(a^2*f*(Cos[e + f*x]^2)^(3/
4)*(b*Tan[e + f*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.16

method	result
default	$-\frac{i\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(\csc(fx+e)-\cot(fx+e)),i)-i\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)}{}$

[In] int(1/(sin(f*x+e)*a)^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/f/(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2)/a*(I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+I*tan(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-I*tan(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+1)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.80

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)^2+\left(\sqrt{2}\cos(fx+e)^2-\right.}{}$$

[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")

[Out] 1/2*(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)^2 + (sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + (sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(a^2*b*f*cos(f*x + e)^2 - a^2*b*f)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

Giac [F]

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\,\sin(e+f\,x))^{3/2}\,\sqrt{b\tan(e+f\,x)}}\,dx$$

```
[In] int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2)),x)
[Out] int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2)), x)
```

3.133 $\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}} + \frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] 1/4*arctan(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3/f/cos(f*x+e)^(1/2)/(b
*tan(f*x+e))^(1/2)-1/4*arctanh(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3/f
/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2)-1/2*b/a^2/f/(a*sin(f*x+e))^(1/2)/(b*
tan(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2679, 2681, 12, 2645, 335, 304, 209, 212}

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{a\sin(e+fx)}\arctan\left(\sqrt{\cos(e+fx)}\right)}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\sqrt{a\sin(e+fx)}\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

```
[In] Int[1/((a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] -1/2*b/(a^2*f*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)) + (ArcTan[Sqrt[C
os[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(4*a^3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[
```

```
e + f*x]]) - (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(4*a^3*f*Sq
rt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
```

```
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}+\frac{\int\frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx}{4a^2}\\
&= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}+\frac{\sqrt{a\sin(e+fx)}\int\frac{\sqrt{\cos(e+fx)}\csc(e+fx)}{a}\,dx}{4a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\
&= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}+\frac{\sqrt{a\sin(e+fx)}\int\sqrt{\cos(e+fx)}\csc(e+fx)\,dx}{4a^3\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\
&= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}-\frac{\sqrt{a\sin(e+fx)}\text{Subst}\left(\int\frac{\sqrt{x}}{1-x^2}\,dx,x,\cos(e+fx)\right)}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\
&= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\\
&\quad-\frac{\sqrt{a\sin(e+fx)}\text{Subst}\left(\int\frac{x^2}{1-x^4}\,dx,x,\sqrt{\cos(e+fx)}\right)}{2a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\
&= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\\
&\quad-\frac{\sqrt{a\sin(e+fx)}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{\cos(e+fx)}\right)}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{\sqrt{a\sin(e+fx)}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{\cos(e+fx)}\right)}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\
&= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}+\frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\
&\quad-\frac{\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{-4\cos^2(e+fx)^{3/4}\cot(e+fx)+\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right)\sin(2(}{8a^2f\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+}}$$

```
[In] Integrate[1/((a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (-4*(Cos[e + f*x]^2)^(3/4)*Cot[e + f*x] + ArcTan[(Cos[e + f*x]^2)^(1/4)]*Si
n[2*(e + f*x)] - ArcTanh[(Cos[e + f*x]^2)^(1/4)]*Sin[2*(e + f*x)])/(8*a^2*f
*(Cos[e + f*x]^2)^(3/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(120) = 240.

Time = 1.15 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.00

method	result
default	$-\frac{\csc(fx+e)\left(\cos(fx+e)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)+\cos(fx+e)\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx}{\cos(fx+e)+1}\right.\right.}{8f\sqrt{b\tan(fx+e)}}$

```
[In] int(1/(sin(f*x+e)*a)^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/f*csc(f*x+e)*(cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2))+cos(f*x+e)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+4*cos(f*x+e)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2))-ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1)))/(b*tan(f*x+e))
^(1/2)/(sin(f*x+e)*a)^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(120) = 240.

Time = 0.48 (sec) , antiderivative size = 605, normalized size of antiderivative = 4.14

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \left[\frac{2\sqrt{-ab}\left(\cos(fx+e)^2-1\right)\arctan\left(\frac{2\sqrt{-ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos}{(ab\cos(fx+e)+ab)\sin(fx+e)}\right.}{} \right.$$

$$-\frac{2\sqrt{ab}\left(\cos(fx+e)^2-1\right)\arctan\left(\frac{2\sqrt{ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{(ab\cos(fx+e)-ab)\sin(fx+e)}\right)\sin(fx+e)-\sqrt{ab}\left(\cos(fx+e)^2-1\right.}{1}$$

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)

[Out] [1/16*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(-a*b)*sqrt(a*sin(f*x
 + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) +
a*b)*sin(f*x + e)))*sin(f*x + e) - sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(-(a*
b*cos(f*x + e)^3 - 5*a*b*cos(f*x + e)^2 + 4*sqrt(-a*b)*sqrt(a*sin(f*x + e))
*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 5*a*b*cos(f*
x + e) + a*b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) + 1))*sin
(f*x + e) + 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*
x + e)^2)/((a^3*b*f*cos(f*x + e)^2 - a^3*b*f)*sin(f*x + e)), -1/16*(2*sqrt(
a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(a*b)*sqrt(a*sin(f*x + e))*sqrt(b*si
n(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) - a*b)*sin(f*x + e
)))*sin(f*x + e) - sqrt(a*b)*(cos(f*x + e)^2 - 1)*log((4*sqrt(a*b)*(cos(f*x
 + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x +
e)) - (a*b*cos(f*x + e)^2 + 6*a*b*cos(f*x + e) + a*b)*sin(f*x + e))/((cos(f
*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x + e) - 8*sqrt(a*sin(
f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)^2)/((a^3*b*f*cos(f
*x + e)^2 - a^3*b*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)

[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

Giac [F]

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$$

```
[In] int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(1/2)),x)
[Out] int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(1/2)), x)
```

3.134 $\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{64a^6\sqrt{a\sin(e+fx)}}{585bf\sqrt{b\tan(e+fx)}} - \frac{16a^4(a\sin(e+fx))^{5/2}}{585bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

```
[Out] -16/585*a^4*(a*sin(f*x+e))^(5/2)/b/f/(b*tan(f*x+e))^(1/2)-2/117*a^2*(a*sin(
f*x+e))^(9/2)/b/f/(b*tan(f*x+e))^(1/2)+2/13*(a*sin(f*x+e))^(13/2)/b/f/(b*ta
n(f*x+e))^(1/2)-64/585*a^6*(a*sin(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2676, 2678, 2669}

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{64a^6\sqrt{a\sin(e+fx)}}{585bf\sqrt{b\tan(e+fx)}} - \frac{16a^4(a\sin(e+fx))^{5/2}}{585bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

```
[In] Int[(a*Sin[e + f*x])^(13/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-64*a^6*Sqrt[a*Sin[e + f*x]])/(585*b*f*Sqrt[b*Tan[e + f*x]]) - (16*a^4*(a*
Sin[e + f*x])^(5/2))/(585*b*f*Sqrt[b*Tan[e + f*x]]) - (2*a^2*(a*Sin[e + f*x
])^(9/2))/(117*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e + f*x])^(13/2))/(13*
b*f*Sqrt[b*Tan[e + f*x]])
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}+\frac{a^2\int(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}\,dx}{13b^2}\\
&= -\frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{(8a^4)\int(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx}{117b^2}\\
&= -\frac{16a^4(a\sin(e+fx))^{5/2}}{585bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}+\frac{(32a^6)\int\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx}{585b^2}\\
&= -\frac{64a^6\sqrt{a\sin(e+fx)}}{585bf\sqrt{b\tan(e+fx)}}-\frac{16a^4(a\sin(e+fx))^{5/2}}{585bf\sqrt{b\tan(e+fx)}}\\
&\quad-\frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.46

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^6\cos^2(e+fx)(-551+340\cos(2(e+fx))-45\cos(4(e+fx)))\sqrt{a\sin(e+fx)}}{2340bf\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(13/2)/(b*Tan[e + f*x])^(3/2),x]
[Out] (a^6*Cos[e + f*x]^2*(-551 + 340*Cos[2*(e + f*x)] - 45*Cos[4*(e + f*x)])*Sqr
t[a*Sin[e + f*x]])/(2340*b*f*Sqrt[b*Tan[e + f*x]])
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.43

method	result	size
default	$-\frac{2a^6\sqrt{\sin(fx+e)a}\,(45(\cos^6(fx+e))-130(\cos^4(fx+e))+117(\cos^2(fx+e)))}{585f\sqrt{b\tan(fx+e)}\,b}$	63

```
[In] int((sin(f*x+e)*a)^(13/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/585/f*a^6*(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2)/b*(45*cos(f*x+e)^6-1
30*cos(f*x+e)^4+117*cos(f*x+e)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.58

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(45\,a^6\cos\left(fx+e\right)^7-130\,a^6\cos\left(fx+e\right)^5+117\,a^6\cos\left(fx+e\right)^3\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{585\,b^2f\sin\left(fx+e\right)}$$

```
[In] integrate((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] -2/585*(45*a^6*cos(f*x + e)^7 - 130*a^6*cos(f*x + e)^5 + 117*a^6*cos(f*x +
e)^3)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))/(b^2*f*sin(f*x
 + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(13/2)/(b*tan(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{13}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(13/2)/(b*tan(f*x + e))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.03

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{\left(\cos(7e+7fx)-\sin(7e+7fx)\ 1\mathrm{i}\right)\sqrt{\frac{b\,(\sin(2e+2fx)-\cos(2e+2fx)\,1\mathrm{i}+1\mathrm{i})}{\cos(2e+2fx)+1+\sin(2e+2fx)\,1\mathrm{i}}}\left(\frac{a^6\cos(3e}{}\right.}{}$$

```
[In] int((a*sin(e + f*x))^(13/2)/(b*tan(e + f*x))^(3/2),x)
[Out] ((cos(7*e + 7*f*x) - sin(7*e + 7*f*x)*1i)*((b*(sin(2*e + 2*f*x) - cos(2*e +
 2*f*x)*1i + 1i))/(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))^(1/2)*((a^6
*cos(3*e + 3*f*x)*(a*sin(e + f*x))^(1/2)*(cos(7*e + 7*f*x) + sin(7*e + 7*f*
x)*1i)*217i)/(9360*b^2*f) - (a^6*cos(5*e + 5*f*x)*(a*sin(e + f*x))^(1/2)*(c
os(7*e + 7*f*x) + sin(7*e + 7*f*x)*1i)*41i)/(1872*b^2*f) + (a^6*cos(7*e + 7
*f*x)*(a*sin(e + f*x))^(1/2)*(cos(7*e + 7*f*x) + sin(7*e + 7*f*x)*1i)*1i)/(
208*b^2*f) + (a^6*cos(e + f*x)*(a*sin(e + f*x))^(1/2)*(cos(7*e + 7*f*x) + s
in(7*e + 7*f*x)*1i)*1991i)/(9360*b^2*f))*1i)/(2*sin(e + f*x))
```

3.135 $\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 109

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{8a^4\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{5/2}}{45bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

```
[Out] -2/45*a^2*(a*sin(f*x+e))^(5/2)/b/f/(b*tan(f*x+e))^(1/2)+2/9*(a*sin(f*x+e))^
(9/2)/b/f/(b*tan(f*x+e))^(1/2)-8/45*a^4*(a*sin(f*x+e))^(1/2)/b/f/(b*tan(f*x
+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2676, 2678, 2669}

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{8a^4\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{5/2}}{45bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

```
[In] Int[(a*Sin[e + f*x])^(9/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-8*a^4*Sqrt[a*Sin[e + f*x]])/(45*b*f*Sqrt[b*Tan[e + f*x]]) - (2*a^2*(a*Sin
[e + f*x])^(5/2))/(45*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e + f*x])^(9/2)
)/(9*b*f*Sqrt[b*Tan[e + f*x]])
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}+\frac{a^2\int(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx}{9b^2}\\
&=-\frac{2a^2(a\sin(e+fx))^{5/2}}{45bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}+\frac{(4a^4)\int\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx}{45b^2}\\
&=-\frac{8a^4\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{5/2}}{45bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.52

$$\int\frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx=\frac{a^4\cos^2(e+fx)(-13+5\cos(2(e+fx)))\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(9/2)/(b*Tan[e + f*x])^(3/2),x]
[Out] (a^4*Cos[e + f*x]^2*(-13 + 5*Cos[2*(e + f*x)])*Sqrt[a*Sin[e + f*x]])/(45*b*
f*Sqrt[b*Tan[e + f*x]])
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{2a^{4}\sqrt{\sin(fx+e)a}\,(5(\cos^{4}(fx+e))-9(\cos^{2}(fx+e)))}{45f\sqrt{b\tan(fx+e)}\,b}$	53

[In] int((sin(f*x+e)*a)^(9/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/45/f*a^4*(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2)/b*(5*cos(f*x+e)^4-9*co
s(f*x+e)^2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.65

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\left(5\,a^{4}\cos\left(fx+e\right)^{5}-9\,a^{4}\cos\left(fx+e\right)^{3}\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{45\,b^{2}f\sin\left(fx+e\right)}$$

[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 2/45*(5*a^4*cos(f*x + e)^5 - 9*a^4*cos(f*x + e)^3)*sqrt(a*sin(f*x + e))*sqr
t(b*sin(f*x + e)/cos(f*x + e))/(b^2*f*sin(f*x + e))

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

[In] integrate((a*sin(f*x+e))**(9/2)/(b*tan(f*x+e))**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{9}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(9/2)/(b*tan(f*x + e))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^4\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}\,\left(47\,\sin\left(2\,e+2\,f\,x\right)+16\,\sin\left(4\,e+4\,f\,x\right)-5\,\sin\left(6\,e+6\,f\,x\right)\right)}{360\,b^2\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-1\right)}$$

```
[In] int((a*sin(e + f*x))^(9/2)/(b*tan(e + f*x))^(3/2),x)
[Out] (a^4*(a*sin(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(
1/2)*(47*sin(2*e + 2*f*x) + 16*sin(4*e + 4*f*x) - 5*sin(6*e + 6*f*x)))/(360
*b^2*f*(cos(2*e + 2*f*x) - 1))
```

3.136 $\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{5/2}}$$

```
[Out] -2/5*b*(a*sin(f*x+e))^(5/2)/f/(b*tan(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2669}

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{5/2}}$$

```
[In] Int[(a*Sin[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*b*(a*Sin[e + f*x])^(5/2))/(5*f*(b*Tan[e + f*x])^(5/2))
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx=-\frac{2a^2\cos^2(e+fx)\sqrt{a\sin(e+fx)}}{5bf\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
[Out] (-2*a^2*Cos[e + f*x]^2*Sqrt[a*Sin[e + f*x]])/(5*b*f*Sqrt[b*Tan[e + f*x]])
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

method	result	size
default	$-\frac{2\left(\cos^2(fx+e)\right)a^2\sqrt{\sin(fx+e)a}}{5fb\sqrt{b\tan(fx+e)}}$	40

```
[In] int((sin(f*x+e)*a)^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/5/f*cos(f*x+e)^2*a^2*(sin(f*x+e)*a)^(1/2)/b/(b*tan(f*x+e))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx=-\frac{2\sqrt{a\sin\left(fx+e\right)}a^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos\left(fx+e\right)^3}{5\,b^2f\sin\left(fx+e\right)}$$

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] -2/5*sqrt(a*sin(f*x + e))*a^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e
)^3/(b^2*f*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.53

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^2\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\left(2\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)\right)\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{10\,b^2\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-1\right)}$$

```
[In] int((a*sin(e + f*x))^(5/2)/(b*tan(e + f*x))^(3/2),x)
[Out] (a^2*(a*sin(e + f*x))^(1/2)*(2*sin(2*e + 2*f*x) + sin(4*e + 4*f*x))*((b*sin
(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(10*b^2*f*(cos(2*e + 2*f*x) -
 1))
```

3.137 $\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{a\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}} - \frac{a\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] 2*(a*sin(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)-a*arctan(cos(f*x+e)^(1/2))*
cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)-a*arctanh(
cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e)
)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2675, 2681, 12, 2645, 335, 218, 212, 209}

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{a\sqrt{\cos(e+fx)}\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}} - \frac{a\sqrt{\cos(e+fx)}\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}} + \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

```
[In] Int[Sqrt[a*Sin[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*Sqrt[a*Sin[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]]) - (a*ArcTan[Sqrt[Cos[e
+ f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[a*Sin[e + f*x
]]) - (a*ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]
])/(b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2675

```
Int[Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]/((b_.)*tan[(e_.) + (f_.)*(x_)])^(3/
2), x_Symbol] :> Simp[2*(Sqrt[a*Sin[e + f*x]]/(b*f*Sqrt[b*Tan[e + f*x]])),
```

```
x] + Dist[a^2/b^2, Int[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(3/2), x], x]
/; FreeQ[{a, b, e, f}, x]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}+\frac{a^2\int\frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx}{b^2}\\
&= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}+\frac{\left(a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int\frac{\csc(e+fx)}{a\sqrt{\cos(e+fx)}}\,dx}{b^2\sqrt{a\sin(e+fx)}}\\
&= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}+\frac{\left(a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int\frac{\csc(e+fx)}{\sqrt{\cos(e+fx)}}\,dx}{b^2\sqrt{a\sin(e+fx)}}\\
&= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{\left(a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}(1-x^2)}\,dx,x,\cos(e+fx)\right)}{b^2f\sqrt{a\sin(e+fx)}}\\
&= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{\left(2a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{1-x^4}\,dx,x,\sqrt{\cos(e+fx)}\right)}{b^2f\sqrt{a\sin(e+fx)}}\\
&= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{\left(a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{\cos(e+fx)}\right)}{b^2f\sqrt{a\sin(e+fx)}}\\
&\quad-\frac{\left(a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{\cos(e+fx)}\right)}{b^2f\sqrt{a\sin(e+fx)}}\\
&= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{a\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}\\
&\quad-\frac{a\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{\left(-\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right)-\operatorname{arctanh}\left(\sqrt[4]{\cos^2(e+fx)}\right)+2\sqrt[4]{\cos^2(e+fx)}\right)\sqrt{a}}{bf\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-ArcTan[(Cos[e + f*x]^2)^(1/4)] - ArcTanh[(Cos[e + f*x]^2)^(1/4)] + 2*(Co
s[e + f*x]^2)^(1/4))*Sqrt[a*Sin[e + f*x]])/(b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt
[b*Tan[e + f*x]])
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.47

method	result
default	$\frac{\left(4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)-\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{c}{(cos}}}{\cos(fx+e)+1}\right.}{2f(\cos(fx+e)+1)\sqrt{b\tan(fx+e)}\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,b}$

```
[In] int((sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)+arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-ln((2
*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1)))*(sin(f*x+e)*a)^(1/2)/(cos(f*x+e
)+1)/(b*tan(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(121) = 242.

Time = 0.48 (sec) , antiderivative size = 529, normalized size of antiderivative = 3.75

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \left[\frac{2b\sqrt{-\frac{a}{b}}\arctan\left(\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{a}{b}}\cos(fx+e)}{(a\cos(fx+e)+a)\sin(fx+e)}\right)\sin(fx+e)+b\sqrt{-\frac{a}{b}}\log\left(\right.}{}\right.$$

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*sqrt(-a/b)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(
f*x + e))*sqrt(-a/b)*cos(f*x + e)/((a*cos(f*x + e) + a)*sin(f*x + e)))*sin(
f*x + e) + b*sqrt(-a/b)*log(-(a*cos(f*x + e)^3 + 4*sqrt(a*sin(f*x + e))*sqr
t(b*sin(f*x + e)/cos(f*x + e))*sqrt(-a/b)*cos(f*x + e)*sin(f*x + e) - 5*a*c
os(f*x + e)^2 - 5*a*cos(f*x + e) + a)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 +
3*cos(f*x + e) + 1))*sin(f*x + e) + 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x +
 e)/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin(f*x + e)), 1/4*(2*b*sqrt(a/b)*ar
ctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(a/b)*cos
(f*x + e)/((a*cos(f*x + e) - a)*sin(f*x + e)))*sin(f*x + e) + b*sqrt(a/b)*l
og((4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x +
 e)/cos(f*x + e))*sqrt(a/b) - (a*cos(f*x + e)^2 + 6*a*cos(f*x + e) + a)*sin
(f*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x +
e) + 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))
/(b^2*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sin{(e+fx)}}}{(b\tan{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
[Out] Integral(sqrt(a*sin(e + f*x))/(b*tan(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sin{(fx+e)}}}{(b\tan{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sin{(fx+e)}}}{(b\tan{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sin(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\,\sin\left(e+f\,x\right)}}{\left(b\tan\left(e+f\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(3/2), x)
```

3.138 $\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 151

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} + \frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{4ab^2f\sqrt{a\sin(e+fx)}} + \frac{\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{4ab^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] -1/2/b/f/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2)+1/4*arctan(cos(f*x+e)^(1
/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/a/b^2/f/(a*sin(f*x+e))^(1/2)+1/4
*arctanh(cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/a/b^2/f/(a
*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2677, 2681, 12, 2645, 335, 218, 212, 209}

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{\cos(e+fx)}\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{b\tan(e+fx)}}{4ab^2f\sqrt{a\sin(e+fx)}} + \frac{\sqrt{\cos(e+fx)}\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{b\tan(e+fx)}}{4ab^2f\sqrt{a\sin(e+fx)}} - \frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

```
[In] Int[1/((a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -1/2*1/(b*f*(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]) + (ArcTan[Sqrt[Cos
[e + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(4*a*b^2*f*Sqrt[a*Sin[
e + f*x]]) + (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e +
 f*x]])/(4*a*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
```

```
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx}{4b^2}\\
&= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}-\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{\csc(e+fx)}{a\sqrt{\cos(e+fx)}}\,dx}{4b^2\sqrt{a\sin(e+fx)}}\\
&= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}-\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{\csc(e+fx)}{\sqrt{\cos(e+fx)}}\,dx}{4ab^2\sqrt{a\sin(e+fx)}}\\
&= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(1-x^2)}\,dx,x,\cos(e+fx)\right)}{4ab^2f\sqrt{a\sin(e+fx)}}\\
&= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\sqrt{\cos(e+fx)}\right)}{2ab^2f\sqrt{a\sin(e+fx)}}\\
&= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{\cos(e+fx)}\right)}{4ab^2f\sqrt{a\sin(e+fx)}}\\
&\quad+\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{\cos(e+fx)}\right)}{4ab^2f\sqrt{a\sin(e+fx)}}
\end{aligned}
$$

$$= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} + \frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{4ab^2f\sqrt{a\sin(e+fx)}} + \frac{\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{4ab^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.68

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{\left(\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right) + \operatorname{arctanh}\left(\sqrt[4]{\cos^2(e+fx)}\right) - 2\sqrt[4]{\text{co}}\right.}{4bf\sqrt[4]{\cos^2(e+fx)}(a\sin(e+fx))^{3/2}\sqrt{b}}$$

```
[In] Integrate[1/((a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((ArcTan[(Cos[e + f*x]^2)^(1/4)] + ArcTanh[(Cos[e + f*x]^2)^(1/4)] - 2*(Cos
[e + f*x]^2)^(1/4)*Csc[e + f*x]^2)*Sin[e + f*x]^2)/(4*b*f*(Cos[e + f*x]^2)^
(1/4)*(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(125) = 250.

Time = 1.10 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.91

method	result
default	$\frac{\csc(fx+e)\left(\cos(fx+e)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)-\cos(fx+e)\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+}{\cos(fx+e)+1}\right.\right.}{8f\sqrt{b\tan(fx+e)}\sqrt{\sin(fx-}}$

```
[In] int(1/(sin(f*x+e)*a)^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*csc(f*x+e)*(cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
))-cos(f*x+e)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-arctan(1/2/(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*
x+e)+1))-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))/(b*tan(f*x+e))^(1/2)/(sin(
f*x+e)*a)^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/a/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(125) = 250$.

Time = 0.47 (sec) , antiderivative size = 608, normalized size of antiderivative = 4.03

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \left[-\frac{2\sqrt{-ab}\left(\cos\left(fx+e\right)^2-1\right)\arctan\left(\frac{2\sqrt{-ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx}{\cos(fx}}}{(ab\cos(fx+e)+ab)\sin(}\right.}{}\right.$$

```
[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(-a*b)*sqrt(a*sin(f*
x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) +
 a*b)*sin(f*x + e)))*sin(f*x + e) + sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(-(a
*b*cos(f*x + e)^3 - 5*a*b*cos(f*x + e)^2 + 4*sqrt(-a*b)*sqrt(a*sin(f*x + e)
)*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 5*a*b*cos(f
*x + e) + a*b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) + 1))*si
n(f*x + e) - 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f
*x + e))/((a^2*b^2*f*cos(f*x + e)^2 - a^2*b^2*f)*sin(f*x + e)), -1/16*(2*sq
rt(a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(a*b)*sqrt(a*sin(f*x + e))*sqrt(b
*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) - a*b)*sin(f*x
+ e)))*sin(f*x + e) - sqrt(a*b)*(cos(f*x + e)^2 - 1)*log(-(4*sqrt(a*b)*(cos
(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*
x + e)) + (a*b*cos(f*x + e)^2 + 6*a*b*cos(f*x + e) + a*b)*sin(f*x + e))/((c
os(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x + e) - 8*sqrt(a*
sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))/((a^2*b^2*f*c
os(f*x + e)^2 - a^2*b^2*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)

[Out] integrate(1/((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)

Giac [F]

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate(1/((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\,\sin(e+f\,x))^{3/2}\,(b\tan(e+f\,x))^{3/2}}\,dx$$

[In] int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(3/2)),x)

[Out] int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(3/2)), x)

3.139 $\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}+\frac{8a^6\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{77b^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] -4/77*a^4*(a*sin(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(1/2)-2/77*a^2*(a*sin(f*x
+e))^(7/2)/b/f/(b*tan(f*x+e))^(1/2)+2/11*(a*sin(f*x+e))^(11/2)/b/f/(b*tan(f
*x+e))^(1/2)+8/77*a^6*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellip
ticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/b^2/
f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2676, 2678, 2681, 2720}

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{8a^6\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{77b^2f\sqrt{a\sin(e+fx)}}-\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

```
[In] Int[(a*Sin[e + f*x])^(11/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-4*a^4*(a*Sin[e + f*x])^(3/2))/(77*b*f*Sqrt[b*Tan[e + f*x]]) - (2*a^2*(a*S
in[e + f*x])^(7/2))/(77*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e + f*x])^(11
```

```
/2))/(11*b*f*Sqrt[b*Tan[e + f*x]]) + (8*a^6*Sqrt[Cos[e + f*x]]*EllipticF[(e
 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(77*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}+\frac{a^2\int(a\sin(e+fx))^{7/2}\sqrt{b\tan(e+fx)}\,dx}{11b^2}\\
&=-\frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}+\frac{(6a^4)\int(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx}{77b^2}\\
&=-\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}+\frac{(4a^6)\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{77b^2}
\end{aligned}$$

$$= -\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}} + \frac{\left(4a^6\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{77b^2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}} + \frac{8a^6\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{77b^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.71

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^5\left(\sqrt[4]{\cos^2(e+fx)}(-22\cos(e+fx)-17\cos(3(e+fx))+7\cos(5(e+fx)))+\right.}{616f\sqrt[4]{\cos^2(e+fx}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(11/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^5*((Cos[e + f*x]^2)^(1/4)*(-22*Cos[e + f*x] - 17*Cos[3*(e + f*x)] + 7*Co
s[5*(e + f*x)]) + 64*Cot[e + f*x]*EllipticF[ArcSin[Sin[e + f*x]]/2, 2])*Sqr
t[a*Sin[e + f*x]]*Tan[e + f*x]^2)/(616*f*(Cos[e + f*x]^2)^(1/4)*(b*Tan[e +
f*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.37 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.07

method	result
default	$\frac{2i\sec(fx+e)\csc(fx+e)\left(7i(\cos^6(fx+e))-7i(\cos^5(fx+e))-13i(\cos^4(fx+e))+13i(\cos^3(fx+e))-4\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos}{\cos(}}\right.}{77f\sqrt{b\tan(fx+e)}\,b}$

```
[In] int((sin(f*x+e)*a)^(11/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/77*I/f*sec(f*x+e)*csc(f*x+e)*(7*I*cos(f*x+e)^6-7*I*cos(f*x+e)^5-13*I*cos(
f*x+e)^4+13*I*cos(f*x+e)^3-4*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+4*I*cos(f*x+
e)^2-4*I*cos(f*x+e))*(sin(f*x+e)*a)^(1/2)*a^5*(cos(f*x+e)+1)/(b*tan(f*x+e))
^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\left(2\sqrt{2}\sqrt{-ab}a^5 \text{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e)})+2\sqrt{2}\cdots\right.}{}$$

```
[In] integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/77*(2*sqrt(2)*sqrt(-a*b)*a^5*weierstrassPInverse(-4, 0, cos(f*x + e) + I*
sin(f*x + e)) + 2*sqrt(2)*sqrt(-a*b)*a^5*weierstrassPInverse(-4, 0, cos(f*x
 + e) - I*sin(f*x + e)) + (7*a^5*cos(f*x + e)^5 - 13*a^5*cos(f*x + e)^3 + 4
*a^5*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)))/
(b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(11/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin{(fx+e)})^{\frac{11}{2}}}{(b\tan{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(11/2)/(b*tan(f*x + e))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\left(a\,\sin\left(e+f\,x\right)\right)^{11/2}}{\left(b\tan\left(e+f\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((a*sin(e + f*x))^(11/2)/(b*tan(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(11/2)/(b*tan(e + f*x))^(3/2), x)
```

3.140 $\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}} + \frac{4a^4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{21b^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] -2/21*a^2*(a*sin(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(1/2)+2/7*(a*sin(f*x+e))^
(7/2)/b/f/(b*tan(f*x+e))^(1/2)+4/21*a^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/
2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*tan(
f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2676, 2678, 2681, 2720}

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{4a^4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{21b^2f\sqrt{a\sin(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

```
[In] Int[(a*Sin[e + f*x])^(7/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*(a*Sin[e + f*x])^(3/2))/(21*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e
 + f*x])^(7/2))/(7*b*f*Sqrt[b*Tan[e + f*x]]) + (4*a^4*Sqrt[Cos[e + f*x]]*El
```

```
lipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(21*b^2*f*Sqrt[a*Sin[e + f*x]
])
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}+\frac{a^2\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx}{7b^2}\\
&= -\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}+\frac{(2a^4)\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{21b^2}\\
&= -\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{\left(2a^4\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{21b^2\sqrt{a\sin(e+fx)}}
\end{aligned}
$$

$$= -\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}} + \frac{4a^4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{21b^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.75

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^3\sqrt{a\sin(e+fx)}\left(8\,\text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)+\sqrt[4]{\cos^2(e+fx)}(5\sin(e\right.}{42bf\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(7/2)/(b*Tan[e + f*x])^(3/2),x]
[Out] (a^3*Sqrt[a*Sin[e + f*x]]*(8*EllipticF[ArcSin[Sin[e + f*x]]/2, 2] + (Cos[e
+ f*x]^2)^(1/4)*(5*Sin[e + f*x] - 3*Sin[3*(e + f*x)])))/(42*b*f*(Cos[e + f*
x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.80 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.97

method	result
default	$\frac{\left(\frac{3}{1985}-\frac{i}{41685}\right)\sec(fx+e)\csc(fx+e)\left(126i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}F(i(\cot(fx+e)-\csc(fx+e)),i)\sin(fx+e)+3i(\cos^4(fx+e))-3\right.}{}$

```
[In] int((sin(f*x+e)*a)^(7/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] (3/1985-1/41685*I)/f*sec(f*x+e)*csc(f*x+e)*(126*I*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*si
n(f*x+e)+3*I*cos(f*x+e)^4-3*I*cos(f*x+e)^3-2*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*sin(f*x
+e)+189*cos(f*x+e)^4-2*I*cos(f*x+e)^2-189*cos(f*x+e)^3+2*I*cos(f*x+e)-126*c
os(f*x+e)^2+126*cos(f*x+e))*(sin(f*x+e)*a)^(1/2)*a^3*(cos(f*x+e)+1)/(b*tan(
f*x+e))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\left(\sqrt{2}\sqrt{-ab}a^3\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+\sqrt{2}\sqrt{-\right.}{}$$

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(sqrt(2)*sqrt(-a*b)*a^3*weierstrassPInverse(-4, 0, cos(f*x + e) + I*si
n(f*x + e)) + sqrt(2)*sqrt(-a*b)*a^3*weierstrassPInverse(-4, 0, cos(f*x + e
) - I*sin(f*x + e)) - (3*a^3*cos(f*x + e)^3 - 2*a^3*cos(f*x + e))*sqrt(a*si
n(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(7/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(7/2)/(b*tan(f*x + e))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\left(a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(b\tan\left(e+f\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((a*sin(e + f*x))^(7/2)/(b*tan(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(7/2)/(b*tan(e + f*x))^(3/2), x)
```

3.141 $\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}} + \frac{2a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3b^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] 2/3*(a*sin(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(1/2)+2/3*a^2*(cos(1/2*f*x+1/2*
e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*
x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2676, 2681, 2720}

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3b^2f\sqrt{a\sin(e+fx)}} + \frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}$$

```
[In] Int[(a*Sin[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(a*Sin[e + f*x])^(3/2))/(3*b*f*Sqrt[b*Tan[e + f*x]]) + (2*a^2*Sqrt[Cos[e
 + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*b^2*f*Sqrt[a*Si
n[e + f*x]])
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}+\frac{a^2\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{3b^2}\\ &= \frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}+\frac{\left(a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx}{3b^2\sqrt{a\sin(e+fx)}}\\ &= \frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}+\frac{2a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3b^2f\sqrt{a\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int\frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx=\frac{2a\sqrt{a\sin(e+fx)}\left(\text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)+\sqrt[4]{\cos^2(e+fx)}\sin(e+f$$

$$3bf\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}$$

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*Sin[e + f*x]]*(EllipticF[ArcSin[Sin[e + f*x]]/2, 2] + (Cos[e +
f*x]^2)^(1/4)*Sin[e + f*x]))/(3*b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e + f
*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.41

method	result
default	$\frac{2i\sec(fx+e)\csc(fx+e)\left(\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)\sin(fx+e)+i(\cos^2(fx+e))-i\cos(fx+e)\right)\sqrt{s}}{3f\sqrt{b\tan(fx+e)}\,b}$

```
[In] int((sin(f*x+e)*a)^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I/f*sec(f*x+e)*csc(f*x+e)*((1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*sin(f*x+e)+I*cos(f*x+
e)^2-I*cos(f*x+e))*(sin(f*x+e)*a)^(1/2)*a*(cos(f*x+e)+1)/(b*tan(f*x+e))^(1/
2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.12

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\sqrt{a\sin(fx+e)}a\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)+\sqrt{2}\sqrt{-ab}a\text{weierstrassPInverse}(-4}{}$$

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(a*sin(f*x + e))*a*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e
) + sqrt(2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*
x + e)) + sqrt(2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x + e) - I*
sin(f*x + e)))/(b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\,\sin(e+f\,x))^{3/2}}{(b\tan(e+f\,x))^{3/2}}\,dx$$

```
[In] int((a*sin(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2), x)
```

3.142 $\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 86

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] -1/b/f/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)-(cos(1/2*f*x+1/2*e)^2)^(1/
2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2
)*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2677, 2681, 2720}

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = -\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}} - \frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Int[1/(Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -(1/(b*f*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])) - (Sqrt[Cos[e + f*x]]*
EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[a*Sin[e + f*x]]
)
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{2b^2}\\ &= -\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx}{2b^2\sqrt{a\sin(e+fx)}}\\ &= -\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int\frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx=\frac{-\sqrt[4]{\cos^2(e+fx)}-\text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)\sin(e+fx)}{bf\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[1/(Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (-(Cos[e + f*x]^2)^(1/4) - EllipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sin[e + f*x
])/(b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.77

method	result
default	$\frac{i\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)+i\tan(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)}{f\sqrt{b\tan(fx+e)}\,\sqrt{\sin(fx+e)a}\,b}$

[In] int(1/(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/f/(b*tan(f*x+e))^(1/2)/(sin(f*x+e)*a)^(1/2)/b*(I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*sin(f*x+e)+I*tan(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)-1)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = -\frac{\left(\sqrt{2}\cos\left(fx+e\right)^2-\sqrt{2}\right)\sqrt{-ab}\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+\left(\sqrt{2}\cos\left(fx+\right.\right.}{2\left(ab^2f\,c\right.}$$

[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")

[Out] -1/2*((sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + (sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))/(a*b^2*f*cos(f*x + e)^2 - a*b^2*f)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a \sin(e+fx)}(b \tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{a \sin(e+fx)}(b \tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a \sin(fx+e)}\,(b \tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
[Out] integrate(1/(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a \sin(e+fx)}(b \tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a \sin(fx+e)}\,(b \tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a \sin(e+fx)}(b \tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a\,\sin(e+f\,x)}\,(b \tan(e+f\,x))^{3/2}}\,dx$$

```
[In] int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2)),x)
[Out] int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2)), x)
```

3.143 $\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}+\frac{1}{6a^2bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} -\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{6a^2b^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] -1/3/b/f/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2)+1/6/a^2/b/f/(a*sin(f*x+e
))^(1/2)/(b*tan(f*x+e))^(1/2)-1/6*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+
1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e)
)^(1/2)/a^2/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2677, 2679, 2681, 2720}

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{6a^2b^2f\sqrt{a\sin(e+fx)}} +\frac{1}{6a^2bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

```
[In] Int[1/((a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -1/3*1/(b*f*(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) + 1/(6*a^2*b*f*Sqr
t[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (Sqrt[Cos[e + f*x]]*EllipticF[(e
+ f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(6*a^2*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx}{6b^2} \\ &= -\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} \\ &\quad + \frac{1}{6a^2bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{12a^2b^2}\end{aligned}$$

$$= -\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} + \frac{1}{6a^2bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}dx}{12a^2b^2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} + \frac{1}{6a^2bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{6a^2b^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.74

$$\int\frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt[4]{\cos^2(e+fx)}(1-2\csc^2(e+fx)) - \text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e\right.}{6a^2bf\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e}$$

```
[In] Integrate[1/((a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
[Out] ((Cos[e + f*x]^2)^(1/4)*(1 - 2*Csc[e + f*x]^2) - EllipticF[ArcSin[Sin[e + f
*x]]/2, 2]*Sin[e + f*x])/(6*a^2*b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f
*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.91 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.32

method	result
default	$-\frac{i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)\sin(fx+e)+i\tan(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\,F(i(\cot(fx+}{6f\sqrt{b\tan(fx+e)}\sqrt{\sin(fx+e)a}\,a^2b}$

```
[In] int(1/(sin(f*x+e)*a)^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/6/f/(b*tan(f*x+e))^(1/2)/(sin(f*x+e)*a)^(1/2)/a^2/b*(I*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e
)),I)*sin(f*x+e)+I*tan(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x
+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)+cot(f*x+e)^2+csc(f*x+e
)^2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.54

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{\left(\sqrt{2}\cos\left(fx+e\right)^4-2\,\sqrt{2}\cos\left(fx+e\right)^2+\sqrt{2}\right)\sqrt{-ab}\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\dots}{\dots}$$

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/12*((sqrt(2)*cos(f*x + e)^4 - 2*sqrt(2)*cos(f*x + e)^2 + sqrt(2))*sqrt(-
a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + (sqrt(2)*c
os(f*x + e)^4 - 2*sqrt(2)*cos(f*x + e)^2 + sqrt(2))*sqrt(-a*b)*weierstrassP
Inverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(cos(f*x + e)^3 + cos(f*x
 + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(a^3*b^2*f*c
os(f*x + e)^4 - 2*a^3*b^2*f*cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(b\tan\left(e+f\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2)),x)
[Out] int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2)), x)
```

3.144 $\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\begin{aligned}\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx &= -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}\\ &+\frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}+\frac{1}{12a^4bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\\ &-\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{12a^4b^2f\sqrt{a\sin(e+fx)}}\end{aligned}$$

```
[Out] -1/5/b/f/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2)+1/30/a^2/b/f/(a*sin(f*x+
e))^(5/2)/(b*tan(f*x+e))^(1/2)+1/12/a^4/b/f/(a*sin(f*x+e))^(1/2)/(b*tan(f*x
+e))^(1/2)-1/12*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(s
in(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/a^4/b^2/f/
(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2677, 2679, 2681, 2720}

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{12a^4b^2f\sqrt{a\sin(e+fx)}} + \frac{1}{12a^4bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

```
[In] Int[1/((a*Sin[e + f*x])^(9/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -1/5*1/(b*f*(a*Sin[e + f*x])^(9/2)*Sqrt[b*Tan[e + f*x]]) + 1/(30*a^2*b*f*(a
*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) + 1/(12*a^4*b*f*Sqrt[a*Sin[e + f
*x]]*Sqrt[b*Tan[e + f*x]]) - (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*
Sqrt[b*Tan[e + f*x]])/(12*a^4*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx}{10b^2}\\
&= -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx}{12a^2b^2}\\
&= -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}+\frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{1}{12a^4bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{24a^4b^2}\\
&= -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}+\frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{1}{12a^4bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\\
&\quad-\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{24a^4b^2\sqrt{a\sin(e+fx)}}\\
&= -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}+\frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{1}{12a^4bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\\
&\quad-\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{12a^4b^2f\sqrt{a\sin(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt[4]{\cos^2(e+fx)}(5+2\csc^2(e+fx)-12\csc^4(e+fx))-5\,\mathrm{Ellip}}{60a^4bf\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}}$$

```
[In] Integrate[1/((a*Sin[e + f*x])^(9/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[e + f*x]^2)^(1/4)*(5 + 2*Csc[e + f*x]^2 - 12*Csc[e + f*x]^4) - 5*Elli
pticF[ArcSin[Sin[e + f*x]]/2, 2]*Sin[e + f*x])/(60*a^4*b*f*(Cos[e + f*x]^2)
^(1/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.54 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

method	result
default	$-\frac{5i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}F(i(\cot(fx+e)-\csc(fx+e)),i)\sin(fx+e)+5i\tan(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}F(i(\cot(f}{60f\sqrt{\sin(fx+e)a}\sqrt{b\tan(fx+e)}\,a^4b}$

```
[In] int(1/(sin(f*x+e)*a)^(9/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60/f/(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2)/a^4/b*(5*I*EllipticF(I*(c
ot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1
))^(1/2)*sin(f*x+e)+5*I*tan(f*x+e)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)-5*cot(f*x+e)^4+12
*cot(f*x+e)^2*csc(f*x+e)^2+5*csc(f*x+e)^4)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.58

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx =$$

$$-\frac{5\left(\sqrt{2}\cos\left(fx+e\right)^6-3\sqrt{2}\cos\left(fx+e\right)^4+3\sqrt{2}\cos\left(fx+e\right)^2-\sqrt{2}\right)\sqrt{-ab}\mathrm{weierstrassPInverse}(-4,0,\cos}{}$$

```
[In] integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/120*(5*(sqrt(2)*cos(f*x + e)^6 - 3*sqrt(2)*cos(f*x + e)^4 + 3*sqrt(2)*co
s(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e)
+ I*sin(f*x + e)) + 5*(sqrt(2)*cos(f*x + e)^6 - 3*sqrt(2)*cos(f*x + e)^4 +
3*sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassPInverse(-4, 0, c
os(f*x + e) - I*sin(f*x + e)) + 2*(5*cos(f*x + e)^5 - 12*cos(f*x + e)^3 - 5
*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(a^5
*b^2*f*cos(f*x + e)^6 - 3*a^5*b^2*f*cos(f*x + e)^4 + 3*a^5*b^2*f*cos(f*x +
e)^2 - a^5*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*sin(f*x+e))**(9/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{9}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e))^(9/2)*(b*tan(f*x + e))^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{{\left(a\,\sin\left(e+f\,x\right)\right)}^{9/2}\,{\left(b\tan\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

```
[In] int(1/((a*sin(e + f*x))^(9/2)*(b*tan(e + f*x))^(3/2)),x)
[Out] int(1/((a*sin(e + f*x))^(9/2)*(b*tan(e + f*x))^(3/2)), x)
```

3.145 $\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx=\frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{4/}}{17df}$$

```
[Out] 6/17*(cos(f*x+e)^2)^(3/4)*hypergeom([3/4, 17/12],[29/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2682, 2657}

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx=\frac{6\cos^2(e+fx)^{3/4}(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{1}\right.}{17df}$$

```
[In] Int[(b*Sin[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, 17/12, 29/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2))/(17*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\right)\int\frac{(b\sin(e+fx))^{11/6}}{\sqrt{\cos(e+fx)}}\,dx}{d(b\sin(e+fx))^{3/2}}$$

$$= \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}}{17df}$$

Mathematica [A] (verified)

Time = 10.77 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int(b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}\sin(2(e+fx}{17f\sqrt[4]{\cos^2(e+fx)}}$$

```
[In] Integrate[(b*Sin[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
[Out] (3*Hypergeometric2F1[3/4, 17/12, 29/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(4
/3)*Sin[2*(e + f*x)]*Sqrt[d*Tan[e + f*x]])/(17*f*(Cos[e + f*x]^2)^(1/4))
```

Maple [F]

$$\int(b\sin(fx+e))^{\frac{4}{3}}\sqrt{d\tan(fx+e)}\,dx$$

```
[In] int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
[Out] int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\sin{(fx+e)})^{\frac{4}{3}}\sqrt{d\tan{(fx+e)}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*sin(f*x+e))**(4/3)*(d*tan(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\sin{(fx+e)})^{\frac{4}{3}}\sqrt{d\tan{(fx+e)}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(4/3)*sqrt(d*tan(f*x + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\,\sin\left(e+f\,x\right))^{4/3}\,\sqrt{d\tan\left(e+f\,x\right)}\,dx$$

```
[In] int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(1/2),x)
[Out] int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(1/2), x)
```

3.146 $\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}}{11df}$$

```
[Out] 6/11*(cos(f*x+e)^2)^(3/4)*hypergeom([3/4, 11/12],[23/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2682, 2657}

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \frac{6\cos^2(e+fx)^{3/4}\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)}{11df}$$

```
[In] Int[(b*Sin[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, 11/12, 23/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2))/(11*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\right)\int\frac{(b\sin(e+fx))^{5/6}}{\sqrt{\cos(e+fx)}}\,dx}{d(b\sin(e+fx))^{3/2}}$$

$$=\frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}}{11df}$$

Mathematica [A] (verified)

Time = 10.62 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int\sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx$$

$$=\frac{3\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\sin(2(e+fx))\sqrt{d\tan(e+fx)}}{11f\sqrt[4]{\cos^2(e+fx)}}$$

```
[In] Integrate[(b*Sin[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
[Out] (3*Hypergeometric2F1[3/4, 11/12, 23/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(1
/3)*Sin[2*(e + f*x)]*Sqrt[d*Tan[e + f*x]])/(11*f*(Cos[e + f*x]^2)^(1/4))
```

Maple [F]

$$\int(b\sin(fx+e))^{\frac{1}{3}}\sqrt{d\tan(fx+e)}dx$$

```
[In] int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
[Out] int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \sqrt[3]{b \sin(e+fx)}\sqrt{d \tan(e+fx)}\, dx = \int (b \sin(fx+e))^{\frac{1}{3}} \sqrt{d \tan(fx+e)}\, dx$$

[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)

Sympy [F]

$$\int \sqrt[3]{b \sin(e+fx)}\sqrt{d \tan(e+fx)}\, dx = \int \sqrt[3]{b \sin(e+fx)}\sqrt{d \tan(e+fx)}\, dx$$

[In] integrate((b*sin(f*x+e))**(1/3)*(d*tan(f*x+e))**(1/2),x)

[Out] Integral((b*sin(e + f*x))**(1/3)*sqrt(d*tan(e + f*x)), x)

Maxima [F]

$$\int \sqrt[3]{b \sin(e+fx)}\sqrt{d \tan(e+fx)}\, dx = \int (b \sin(fx+e))^{\frac{1}{3}} \sqrt{d \tan(fx+e)}\, dx$$

[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)

Giac [F(-2)]

Exception generated.

$$\int \sqrt[3]{b \sin(e+fx)}\sqrt{d \tan(e+fx)}\, dx = \text{Exception raised: TypeError}$$

[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen &

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\,\sin(e+f\,x))^{1/3}\,\sqrt{d\tan(e+f\,x)}\,dx$$

```
[In] int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(1/2),x)
[Out] int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(1/2), x)
```

3.147 $\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{19}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{7df\sqrt[3]{b\sin(e+fx)}}$$

```
[Out] 6/7*(cos(f*x+e)^2)^(3/4)*hypergeom([7/12, 3/4],[19/12],sin(f*x+e)^2)*(d*tan
(f*x+e))^(3/2)/d/f/(b*sin(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2682, 2657}

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{6\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{19}{12},\sin^2(e+fx)\right)}{7df\sqrt[3]{b\sin(e+fx)}}$$

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[7/12, 3/4, 19/12, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(3/2))/(7*d*f*(b*Sin[e + f*x])^(1/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\right)\int\frac{\sqrt[6]{b\sin(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx}{d(b\sin(e+fx))^{3/2}}\\ &= \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{19}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{7df\sqrt[3]{b\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 10.58 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int\frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx\\ &=\frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{19}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{7df\sqrt[3]{b\sin(e+fx)}}\end{aligned}$$

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[7/12, 3/4, 19/12, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(3/2))/(7*d*f*(b*Sin[e + f*x])^(1/3))
```

Maple [F]

$$\int \frac{\sqrt{d\tan(fx+e)}}{(b\sin(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x)
[Out] int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sqrt{d\tan(fx+e)}}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx$$

```
[In] integrate((d*tan(f*x+e))**(1/2)/(b*sin(f*x+e))**(1/3),x)
[Out] Integral(sqrt(d*tan(e + f*x))/(b*sin(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sqrt{d\tan(fx+e)}}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sin(f*x + e))^(1/3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{\left(b\,\sin\left(e+f\,x\right)\right)^{1/3}}\,dx$$

```
[In] int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(1/3),x)
```

```
[Out] int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(1/3), x)
```

3.148 $\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{13}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{df(b\sin(e+fx))^{4/3}}$$

```
[Out] 6*(cos(f*x+e)^2)^(3/4)*hypergeom([1/12, 3/4],[13/12],sin(f*x+e)^2)*(d*tan(f
*x+e))^(3/2)/d/f/(b*sin(f*x+e))^(4/3)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2682, 2657}

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{6\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{13}{12},\sin^2(e+fx)\right)}{df(b\sin(e+fx))^{4/3}}$$

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(4/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[1/12, 3/4, 13/12, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(3/2))/(d*f*(b*Sin[e + f*x])^(4/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\right)\int \frac{1}{\sqrt{\cos(e+fx)}(b\sin(e+fx))^{5/6}}\,dx}{d(b\sin(e+fx))^{3/2}}$$
$$= \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{13}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{df(b\sin(e+fx))^{4/3}}$$

Mathematica [A] (verified)

Time = 10.71 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{13}{12},\sin^2(e+fx)\right)\sin(2(e+fx))\sqrt{d\tan(e+fx)}}{f\sqrt[4]{\cos^2(e+fx)}(b\sin(e+fx))^{4/3}}$$

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(4/3),x]
[Out] (3*Hypergeometric2F1[1/12, 3/4, 13/12, Sin[e + f*x]^2]*Sin[2*(e + f*x)]*Sqr
t[d*Tan[e + f*x]])/(f*(Cos[e + f*x]^2)^(1/4)*(b*Sin[e + f*x])^(4/3))
```

Maple [F]

$$\int \frac{\sqrt{d\tan(fx+e)}}{(b\sin(fx+e))^{\frac{4}{3}}}dx$$

```
[In] int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x)
[Out] int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x)
```

Fricas [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x, algorithm="fricas")

[Out] integral(-(b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b^2*cos(f*x + e)^2 - b^2), x)

Sympy [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan\left(e+fx\right)}}{\left(b\sin\left(e+fx\right)\right)^{\frac{4}{3}}}\,dx$$

[In] integrate((d*tan(f*x+e))**(1/2)/(b*sin(f*x+e))**(4/3),x)

[Out] Integral(sqrt(d*tan(e + f*x))/(b*sin(e + f*x))**(4/3), x)

Maxima [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x, algorithm="maxima")

[Out] integrate(sqrt(d*tan(f*x + e))/(b*sin(f*x + e))^(4/3), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueDone

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{\left(b\,\sin\left(e+f\,x\right)\right)^{4/3}}\,dx$$

```
[In] int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(4/3),x)
[Out] int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(4/3), x)
```

3.149 $\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \frac{6\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{23}{12},\frac{35}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{4}}{23df}$$

```
[Out] 6/23*(cos(f*x+e)^2)^(5/4)*hypergeom([5/4, 23/12],[35/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(4/3)*(d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2682, 2657}

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \frac{6\cos^2(e+fx)^{5/4}(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\right.}{23df}$$

```
[In] Int[(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, 23/12, 35/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(5/2))/(23*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\right)\int\frac{(b\sin(e+fx))^{17/6}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{d(b\sin(e+fx))^{5/2}}$$

$$= \frac{6\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{23}{12},\frac{35}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{5/2}}{23df}$$

Mathematica [A] (verified)

Time = 10.75 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx =$$
$$-\frac{2d\left(-1+\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)\right)(b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}}{f}$$

```
[In] Integrate[(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
[Out] (-2*d*(-1 + (Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, 11/12, 23/12, Sin
[e + f*x]^2])*(b*Sin[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]])/f
```

Maple [F]

$$\int(b\sin(fx+e))^{\frac{4}{3}}(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
[Out] int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\sin(fx+e))^{\frac{4}{3}}(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*d*sin(f*x + e)*tan(f
*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((b*sin(f*x+e))**(4/3)*(d*tan(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\sin(fx+e))^{\frac{4}{3}}(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(4/3)*(d*tan(f*x + e))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\,\sin(e+f\,x))^{4/3}\,(d\tan(e+f\,x))^{3/2}\,dx$$

```
[In] int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(3/2),x)
[Out] int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(3/2), x)
```

3.150 $\int \sqrt[3]{b \sin(e+fx)}(d \tan(e+fx))^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt[3]{b \sin(e+fx)}(d \tan(e+fx))^{3/2} dx = \frac{6 \cos^2(e+fx)^{5/4} \text{Hypergeometric2F1}\left(\frac{5}{4},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right) \sqrt[3]{b \sin(e+fx)}(d \tan(e+fx))^{5/2}}{17df}$$

```
[Out] 6/17*(cos(f*x+e)^2)^(5/4)*hypergeom([5/4, 17/12],[29/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(1/3)*(d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\int \sqrt[3]{b \sin(e+fx)}(d \tan(e+fx))^{3/2} dx = \frac{6 \cos^2(e+fx)^{5/4} \sqrt[3]{b \sin(e+fx)}(d \tan(e+fx))^{5/2} \text{Hypergeometric2F1}\left(\frac{5}{4},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)}{17df}$$

```
[In] Int[(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, 17/12, 29/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(5/2))/(17*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\right)\int\frac{(b\sin(e+fx))^{11/6}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{d(b\sin(e+fx))^{5/2}}$$

$$= \frac{6\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{5/2}}{17df}$$

Mathematica [A] (verified)

Time = 10.64 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = -\frac{2d\left(-1+\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{12},\frac{17}{12},\sin^2(e+fx)\right)\right)\sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}}{f}$$

```
[In] Integrate[(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
[Out] (-2*d*(-1 + (Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, 5/12, 17/12, Sin[
e + f*x]^2])*(b*Sin[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]])/f
```

Maple [F]

$$\int(b\sin(fx+e))^{\frac{1}{3}}(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
[Out] int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \sqrt[3]{b \sin(e+fx)}(d \tan(e+fx))^{3/2} dx = \int (b \sin(fx+e))^{\frac{1}{3}} (d \tan(fx+e))^{\frac{3}{2}} dx$$

```
[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{b \sin(e+fx)}(d \tan(e+fx))^{3/2} dx = \text{Timed out}$$

```
[In] integrate((b*sin(f*x+e))**(1/3)*(d*tan(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt[3]{b \sin(e+fx)}(d \tan(e+fx))^{3/2} dx = \int (b \sin(fx+e))^{\frac{1}{3}} (d \tan(fx+e))^{\frac{3}{2}} dx$$

```
[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt[3]{b \sin(e+fx)}(d \tan(e+fx))^{3/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int (b\,\sin(e+f\,x))^{1/3}\,(d\tan(e+f\,x))^{3/2}\,dx$$

```
[In] int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(3/2),x)
[Out] int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(3/2), x)
```

3.151 $\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{6\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{13}{12},\frac{5}{4},\frac{25}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{13df\sqrt[3]{b\sin(e+fx)}}$$

```
[Out] 6/13*(cos(f*x+e)^2)^(5/4)*hypergeom([13/12, 5/4],[25/12],sin(f*x+e)^2)*(d*t
an(f*x+e))^(5/2)/d/f/(b*sin(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2682, 2657}

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{6\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{13}{12},\frac{5}{4},\frac{25}{12},\sin^2(e+fx)\right)}{13df\sqrt[3]{b\sin(e+fx)}}$$

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[13/12, 5/4, 25/12, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(5/2))/(13*d*f*(b*Sin[e + f*x])^(1/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
```

```
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\right)\int\frac{(b\sin(e+fx))^{7/6}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{d(b\sin(e+fx))^{5/2}}$$
$$=\frac{6\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{13}{12},\frac{5}{4},\frac{25}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{13df\sqrt[3]{b\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 10.63 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int\frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = -\frac{2d\left(-1+\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{1}{4},\frac{13}{12},\sin^2(e+fx)\right)\right)\sqrt{d\tan(e+fx)}}{f\sqrt[3]{b\sin(e+fx)}}$$

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(1/3),x]
[Out] (-2*d*(-1 + (Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/12, 1/4, 13/12, Sin[
e + f*x]^2])*Sqrt[d*Tan[e + f*x]])/(f*(b*Sin[e + f*x])^(1/3))
```

Maple [F]

$$\int\frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x)
[Out] int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b*sin(
f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((d*tan(f*x+e))**(3/2)/(b*sin(f*x+e))**(1/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sin(f*x + e))^(1/3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx=\int \frac{(d\tan(e+f\,x))^{3/2}}{(b\,\sin\left(e+f\,x\right))^{1/3}}\,dx$$

```
[In] int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(1/3),x)
[Out] int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(1/3), x)
```

3.152 $\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{6\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{19}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{7df(b\sin(e+fx))^{4/3}}$$

```
[Out] 6/7*(cos(f*x+e)^2)^(5/4)*hypergeom([7/12, 5/4],[19/12],sin(f*x+e)^2)*(d*tan
(f*x+e))^(5/2)/d/f/(b*sin(f*x+e))^(4/3)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2682, 2657}

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{6\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{19}{12},\sin^2(e+fx)\right)}{7df(b\sin(e+fx))^{4/3}}$$

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(4/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[7/12, 5/4, 19/12, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(5/2))/(7*d*f*(b*Sin[e + f*x])^(4/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\right)\int\frac{\sqrt[6]{b\sin(e+fx)}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{d(b\sin(e+fx))^{5/2}}$$

$$= \frac{6\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{19}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{7df(b\sin(e+fx))^{4/3}}$$

Mathematica [A] (verified)

Time = 10.83 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int\frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx =$$

$$-\frac{2d\left(-7+4\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)\right)(b\sin(e+fx))^{2/3}\sqrt{d\tan(e+fx}}{7b^2f}$$

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(4/3),x]
```

```
[Out] (-2*d*(-7 + 4*(Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, 7/12, 19/12, Si
n[e + f*x]^2])*(b*Sin[e + f*x])^(2/3)*Sqrt[d*Tan[e + f*x]])/(7*b^2*f)
```

Maple [F]

$$\int\frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{4}{3}}}dx$$

```
[In] int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x)
```

```
[Out] int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x)
```

Fricas [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x, algorithm="fricas")
[Out] integral(-(b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b^2*c
os(f*x + e)^2 - b^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \text{Timed out}$$

```
[In] integrate((d*tan(f*x+e))**(3/2)/(b*sin(f*x+e))**(4/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sin(f*x + e))^(4/3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(e+f\,x))^{3/2}}{(b\,\sin(e+f\,x))^{4/3}}\,dx$$

```
[In] int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(4/3),x)
[Out] int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(4/3), x)
```

3.153 $\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \frac{6\cos^2(e+fx)^{7/6}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{7/3}}{17df}$$

```
[Out] 6/17*(cos(f*x+e)^2)^(7/6)*hypergeom([7/6, 17/12],[29/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(1/2)*(d*tan(f*x+e))^(7/3)/d/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \frac{6\cos^2(e+fx)^{7/6}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)}{17df}$$

```
[In] Int[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 17/12, 29/12, Sin[e + f*x]
^2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(7/3))/(17*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(b\cos^{\frac{7}{3}}(e+fx)(d\tan(e+fx))^{7/3}\right)\int \frac{(b\sin(e+fx))^{11/6}}{\cos^{\frac{4}{3}}(e+fx)}\,dx}{d(b\sin(e+fx))^{7/3}}$$

$$= \frac{6\cos^2(e+fx)^{7/6}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{7/3}}{17df}$$

Mathematica [A] (verified)

Time = 11.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = -\frac{3d\left(-1+\text{Hypergeometric2F1}\left(\frac{5}{12},\frac{5}{4},\frac{17}{12},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\right)\sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}}{f}$$

```
[In] Integrate[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
[Out] (-3*d*(-1 + Hypergeometric2F1[5/12, 5/4, 17/12, -Tan[e + f*x]^2]*(Sec[e + f
*x]^2)^(1/4))*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(1/3))/f
```

Maple [F]

$$\int \sqrt{b\sin(fx+e)}\,(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

```
[In] int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
[Out] int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

Fricas [F]

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\sin\left(fx+e\right)}(d\tan\left(fx+e\right))^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3)*d*tan(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \text{Timed out}$$

```
[In] integrate((b*sin(f*x+e))**(1/2)*(d*tan(f*x+e))**(4/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\sin\left(fx+e\right)}(d\tan\left(fx+e\right))^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

Giac [F]

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\sin\left(fx+e\right)}(d\tan\left(fx+e\right))^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx=\int \sqrt{b\,\sin\left(e+f\,x\right)}\,\left(d\tan(e+f\,x)\right)^{4/3}\,dx$$

```
[In] int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(4/3),x)
[Out] int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(4/3), x)
```

3.154 $\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{6\cos^2(e+fx)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}}{11df}$$

```
[Out] 6/11*(cos(f*x+e)^2)^(2/3)*hypergeom([2/3, 11/12],[23/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3)/d/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2682, 2657}

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{6\cos^2(e+fx)^{2/3}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)}{11df}$$

```
[In] Int[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[2/3, 11/12, 23/12, Sin[e + f*x]
^2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(11*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(b\cos^{\frac{4}{3}}(e+fx)(d\tan(e+fx))^{4/3}\right)\int\frac{(b\sin(e+fx))^{5/6}}{\sqrt[3]{\cos(e+fx)}}\,dx}{d(b\sin(e+fx))^{4/3}}$$

$$=\frac{6\cos^2(e+fx)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}}{11df}$$

Mathematica [A] (verified)

Time = 10.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int\sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx$$
$$=\frac{6\,\text{Hypergeometric2F1}\left(\frac{11}{12},\frac{5}{4},\frac{23}{12},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}}{11df}$$

```
[In] Integrate[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
[Out] (6*Hypergeometric2F1[11/12, 5/4, 23/12, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(
1/4)*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(11*d*f)
```

Maple [F]

$$\int\sqrt{b\sin(fx+e)}\,(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
[Out] int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

Sympy [F]

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sin\left(e+fx\right)}\sqrt[3]{d\tan\left(e+fx\right)}\,dx$$

```
[In] integrate((b*sin(f*x+e))**(1/2)*(d*tan(f*x+e))**(1/3),x)
[Out] Integral(sqrt(b*sin(e + f*x))*(d*tan(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sin(e+fx)}\,\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\,\sin\left(e+f\,x\right)}\,\left(d\tan(e+f\,x)\right)^{1/3}\,dx$$

```
[In] int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(1/3),x)
[Out] int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(1/3), x)
```

3.155 $\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{6\sqrt[3]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{2/3}}{7df}$$

```
[Out] 6/7*(cos(f*x+e)^2)^(1/3)*hypergeom([1/3, 7/12],[19/12],sin(f*x+e)^2)*(b*sin
(f*x+e))^(1/2)*(d*tan(f*x+e))^(2/3)/d/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{6\sqrt[3]{\cos^2(e+fx)}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)}{7df}$$

```
[In] Int[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 7/12, 19/12, Sin[e + f*x]^
2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(7*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(b\cos^{\frac{2}{3}}(e+fx)(d\tan(e+fx))^{2/3}\right)\int\sqrt[3]{\cos(e+fx)}\sqrt[6]{b\sin(e+fx)}\,dx}{d(b\sin(e+fx))^{2/3}}$$

$$= \frac{6\sqrt[3]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{2/3}}{7df}$$

Mathematica [A] (verified)

Time = 10.64 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int\frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx$$

$$= \frac{6\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{19}{12},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{2/3}}{7df}$$

```
[In] Integrate[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*Hypergeometric2F1[7/12, 5/4, 19/12, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1
/4)*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(7*d*f)
```

Maple [F]

$$\int \frac{\sqrt{b\sin\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}dx$$

[In] int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)

[Out] int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)

Fricas [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")

[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)/(d*tan(f*x + e)), x)

Sympy [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\sin\left(e+fx\right)}}{\sqrt[3]{d\tan\left(e+fx\right)}}\,dx$$

[In] integrate((b*sin(f*x+e))**(1/2)/(d*tan(f*x+e))**(1/3),x)

[Out] Integral(sqrt(b*sin(e + f*x))/(d*tan(e + f*x))**(1/3), x)

Maxima [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")

[Out] integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(1/3), x)

Giac [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\,\sin\left(e+f\,x\right)}}{\left(d\tan\left(e+f\,x\right)\right)^{1/3}}\,dx$$

```
[In] int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3),x)
[Out] int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3), x)
```

3.156 $\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{6\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{12},\frac{13}{12},\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}}{df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

```
[Out] 6*hypergeom([-1/6, 1/12],[13/12],sin(f*x+e)^2)*(b*sin(f*x+e))^(1/2)/d/f/(co
s(f*x+e)^2)^(1/6)/(d*tan(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2682, 2657}

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{6\sqrt{b\sin(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{12},\frac{13}{12},\sin^2(e+fx)\right)}{df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

```
[In] Int[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (6*Hypergeometric2F1[-1/6, 1/12, 13/12, Sin[e + f*x]^2]*Sqrt[b*Sin[e + f*x]
])/(d*f*(Cos[e + f*x]^2)^(1/6)*(d*Tan[e + f*x])^(1/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(b\sqrt[3]{b\sin(e+fx)}\right)\int\frac{\cos^{\frac{4}{3}}(e+fx)}{(b\sin(e+fx))^{5/6}}\,dx}{d\sqrt[3]{\cos(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$
$$= \frac{6\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{12},\frac{13}{12},\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}}{df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

Mathematica [A] (verified)

Time = 31.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.95

$$\int\frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{3\sqrt[4]{\sec^2(e+fx)}\sqrt{b\sin(e+fx)}\left(13\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{1}{4},\frac{13}{12},-\tan^2(e+fx\right.\right.}{}$$

```
[In] Integrate[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (3*(Sec[e + f*x]^2)^(1/4)*Sqrt[b*Sin[e + f*x]]*(13*Hypergeometric2F1[1/12,
1/4, 13/12, -Tan[e + f*x]^2] + 13*Hypergeometric2F1[1/12, 5/4, 13/12, -Tan[
e + f*x]^2] - Hypergeometric2F1[13/12, 5/4, 25/12, -Tan[e + f*x]^2]*Tan[e +
 f*x]^2))/(13*d*f*(d*Tan[e + f*x])^(1/3))
```

Maple [F]

$$\int\frac{\sqrt{b\sin(fx+e)}}{(d\tan(fx+e))^{\frac{4}{3}}}dx$$

```
[In] int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

```
[Out] int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

Fricas [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")

[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)/(d^2*tan(f*x + e)^2), x)

Sympy [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sin\left(e+fx\right)}}{\left(d\tan\left(e+fx\right)\right)^{\frac{4}{3}}}\,dx$$

[In] integrate((b*sin(f*x+e))**(1/2)/(d*tan(f*x+e))**(4/3),x)

[Out] Integral(sqrt(b*sin(e + f*x))/(d*tan(e + f*x))**(4/3), x)

Maxima [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")

[Out] integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(4/3), x)

Giac [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")

[Out] integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(4/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\,\sin\left(e+f\,x\right)}}{{\left(d\tan\left(e+f\,x\right)\right)}^{4/3}}\,dx$$

```
[In] int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3),x)
[Out] int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3), x)
```

3.157 $\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx=\frac{6\cos^2(e+fx)^{7/6}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{23}{12},\frac{35}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{7/3}}{23df}$$

```
[Out] 6/23*(cos(f*x+e)^2)^(7/6)*hypergeom([7/6, 23/12],[35/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(3/2)*(d*tan(f*x+e))^(7/3)/d/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2682, 2657}

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx=\frac{6\cos^2(e+fx)^{7/6}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{23}{12},\frac{35}{12},\sin^2(e+fx)\right)}{23df}$$

```
[In] Int[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 23/12, 35/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(7/3))/(23*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(b\cos^{\frac{7}{3}}(e+fx)(d\tan(e+fx))^{7/3}\right)\int\frac{(b\sin(e+fx))^{17/6}}{\cos^{\frac{4}{3}}(e+fx)}\,dx}{d(b\sin(e+fx))^{7/3}}$$

$$= \frac{6\cos^2(e+fx)^{7/6}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{23}{12},\frac{35}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{7/3}}{23df}$$

Mathematica [A] (verified)

Time = 10.77 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.33

$$\int(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \frac{3d\left(-\text{Hypergeometric2F1}\left(\frac{11}{12},\frac{7}{4},\frac{23}{12},-\tan^2(e+fx)\right)\sec^2(e+fx)+\sqrt[4]{\sec^2}\right.}{f\sqrt[4]{\sec^2(e+fx)}}$$

```
[In] Integrate[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
[Out] (3*d*(-(Hypergeometric2F1[11/12, 7/4, 23/12, -Tan[e + f*x]^2]*Sec[e + f*x]^
2) + (Sec[e + f*x]^2)^(1/4))*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3))
/(f*(Sec[e + f*x]^2)^(1/4))
```

Maple [F]

$$\int(b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

```
[In] int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
[Out] int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

Fricas [F]

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3)*b*d*sin(f*x + e)*tan(f
*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \text{Timed out}$$

```
[In] integrate((b*sin(f*x+e))**(3/2)*(d*tan(f*x+e))**(4/3),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

Giac [F]

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (b\,\sin(e+f\,x))^{3/2}\,(d\tan(e+f\,x))^{4/3}\,dx$$

```
[In] int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(4/3),x)
[Out] int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(4/3), x)
```

3.158 $\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{6\cos^2(e+fx)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{3/}}{17df}$$

```
[Out] 6/17*(cos(f*x+e)^2)^(2/3)*hypergeom([2/3, 17/12],[29/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3)/d/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2682, 2657}

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{6\cos^2(e+fx)^{2/3}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{1}{1}\right.}{17df}$$

```
[In] Int[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[2/3, 17/12, 29/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3))/(17*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(b\cos^{\frac{4}{3}}(e+fx)(d\tan(e+fx))^{4/3}\right)\int\frac{(b\sin(e+fx))^{11/6}}{\sqrt[3]{\cos(e+fx)}}\,dx}{d(b\sin(e+fx))^{4/3}}$$

$$= \frac{6\cos^2(e+fx)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}}{17df}$$

Mathematica [A] (verified)

Time = 10.69 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{6\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{17}{12},\frac{7}{4},\frac{29}{12},-\tan^2(e+fx)\right)\sec^2(e+fx)^{7/4}(}{17bf}$$

```
[In] Integrate[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
[Out] (6*Cos[e + f*x]*Hypergeometric2F1[17/12, 7/4, 29/12, -Tan[e + f*x]^2]*(Sec[
e + f*x]^2)^(7/4)*(b*Sin[e + f*x])^(5/2)*(d*Tan[e + f*x])^(1/3))/(17*b*f)
```

Maple [F]

$$\int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
[Out] int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3)*b*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*sin(f*x+e))**(3/2)*(d*tan(f*x+e))**(1/3),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(3/2)*(d*tan(f*x + e))^(1/3), x)
```

Giac [F]

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx=\int \left(b\,\sin\left(e+f\,x\right)\right)^{3/2}\,\left(d\tan(e+f\,x)\right)^{1/3}\,dx$$

```
[In] int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(1/3),x)
[Out] int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(1/3), x)
```

3.159 $\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{6\sqrt[3]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{12},\frac{25}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{2/3}}{13df}$$

```
[Out] 6/13*(cos(f*x+e)^2)^(1/3)*hypergeom([1/3, 13/12],[25/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(3/2)*(d*tan(f*x+e))^(2/3)/d/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2682, 2657}

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{6\sqrt[3]{\cos^2(e+fx)}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{12},\frac{25}{12},\sin^2(e+fx)\right)}{13df}$$

```
[In] Int[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 13/12, 25/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(2/3))/(13*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
```

```
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(b\cos^{\frac{2}{3}}(e+fx)(d\tan(e+fx))^{2/3}\right)\int\sqrt[3]{\cos(e+fx)}(b\sin(e+fx))^{7/6}\,dx}{d(b\sin(e+fx))^{2/3}}$$

$$= \frac{6\sqrt[3]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{12},\frac{25}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{2/3}}{13df}$$

Mathematica [A] (verified)

Time = 10.86 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int\frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{2d\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{13}{12},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)(b\sin(e}{3f(d\tan(e+fx))^{4/3}}$$

```
[In] Integrate[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
[Out] (2*d*(-1 + Hypergeometric2F1[1/12, 3/4, 13/12, -Tan[e + f*x]^2]*(Sec[e + f*
x]^2)^(3/4))*(b*Sin[e + f*x])^(3/2))/(3*f*(d*Tan[e + f*x])^(4/3))
```

Maple [F]

$$\int\frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
[Out] int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sin(f*x + e)/(d*tan(
f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((b*sin(f*x+e))**(3/2)/(d*tan(f*x+e))**(1/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\,\sin\left(e+f\,x\right))^{3/2}}{(d\tan\left(e+f\,x\right))^{1/3}}\,dx$$

```
[In] int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3),x)
[Out] int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3), x)
```

3.160 $\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx=\frac{6\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}}{7df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

```
[Out] 6/7*hypergeom([-1/6, 7/12],[19/12],sin(f*x+e)^2)*(b*sin(f*x+e))^(3/2)/d/f/(
cos(f*x+e)^2)^(1/6)/(d*tan(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2682, 2657}

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx=\frac{6(b\sin(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)}{7df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

```
[In] Int[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (6*Hypergeometric2F1[-1/6, 7/12, 19/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(3
/2))/(7*d*f*(Cos[e + f*x]^2)^(1/6)*(d*Tan[e + f*x])^(1/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(b\sqrt[3]{b\sin(e+fx)}\right)\int \cos^{\frac{4}{3}}(e+fx)\sqrt[6]{b\sin(e+fx)}\,dx}{d\sqrt[3]{\cos(e+fx)}\sqrt[3]{d\tan(e+fx)}}\\ &= \frac{6\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}}{7df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 10.70 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{2\left(7+2\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{19}{12},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)(b\sin(e+}{21df\sqrt[3]{d\tan(e+fx)}}$$

```
[In] Integrate[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
[Out] (2*(7 + 2*Hypergeometric2F1[7/12, 3/4, 19/12, -Tan[e + f*x]^2]*(Sec[e + f*x
]^2)^(3/4))*(b*Sin[e + f*x])^(3/2))/(21*d*f*(d*Tan[e + f*x])^(1/3))
```

Maple [F]

$$\int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}dx$$

```
[In] int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
[Out] int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

Fricas [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sin(f*x + e)/(d^2*ta
n(f*x + e)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \text{Timed out}$$

```
[In] integrate((b*sin(f*x+e))**(3/2)/(d*tan(f*x+e))**(4/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

Giac [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\,\sin\left(e+f\,x\right))^{3/2}}{\left(d\tan\left(e+f\,x\right)\right)^{4/3}}\,dx$$

```
[In] int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3),x)
[Out] int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3), x)
```

3.161 $\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{4+m}{2},\frac{6+m}{2},\sin^2(e+fx)\right)(a\sin(e+fx))^{4+m}}{a^4 f(4+m)}$$

```
[Out] hypergeom([2, 2+1/2*m],[3+1/2*m],sin(f*x+e)^2)*(a*sin(f*x+e))^(4+m)/a^4/f/(
4+m)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2672, 371}

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx = \frac{(a\sin(e+fx))^{m+4}\,\text{Hypergeometric2F1}\left(2,\frac{m+4}{2},\frac{m+6}{2},\sin^2(e+fx)\right)}{a^4 f(m+4)}$$

```
[In] Int[(a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] (Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x]
)^(4 + m))/(a^4*f*(4 + m))
```

<u>Rule 371</u>

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^{3+m}}{(a^2-x^2)^2}\,dx, x, a\sin(e+fx)\right)}{f}$$

$$= \frac{\text{Hypergeometric2F1}\left(2, \frac{4+m}{2}, \frac{6+m}{2}, \sin^2(e+fx)\right)(a\sin(e+fx))^{4+m}}{a^4 f(4+m)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int (a\sin(e+fx))^m\tan^3(e+fx)\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(2, \frac{4+m}{2}, 1+\frac{4+m}{2}, \sin^2(e+fx)\right)\sin^4(e+fx)(a\sin(e+fx))^m}{f(4+m)}$$

```
[In] Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
[Out] (Hypergeometric2F1[2, (4 + m)/2, 1 + (4 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x
]^4*(a*Sin[e + f*x])^m)/(f*(4 + m))
```

Maple [F]

$$\int (\sin(fx+e)\,a)^m\left(\tan^3(fx+e)\right)dx$$

```
[In] int((sin(f*x+e)*a)^m*tan(f*x+e)^3,x)
[Out] int((sin(f*x+e)*a)^m*tan(f*x+e)^3,x)
```

Fricas [F]

$$\int (a\sin(e+fx))^m\tan^3(e+fx)\,dx=\int (a\sin(fx+e))^m\tan(fx+e)^3\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e))^m*tan(f*x + e)^3, x)
```

Sympy [F]

$$\int (a\sin(e+fx))^m\tan^3(e+fx)\,dx=\int (a\sin(e+fx))^m\tan^3(e+fx)\,dx$$

```
[In] integrate((a*sin(f*x+e))**m*tan(f*x+e)**3,x)
[Out] Integral((a*sin(e + f*x))**m*tan(e + f*x)**3, x)
```

Maxima [F]

$$\int (a\sin(e+fx))^m\tan^3(e+fx)\,dx=\int (a\sin(fx+e))^m\tan(fx+e)^3\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^3, x)
```

Giac [F]

$$\int (a\sin(e+fx))^m\tan^3(e+fx)\,dx=\int (a\sin(fx+e))^m\tan(fx+e)^3\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m\tan^3(e+fx)\,dx = \int \tan(e+f\,x)^3\,(a\,\sin(e+f\,x))^m\,dx$$

```
[In] int(tan(e + f*x)^3*(a*sin(e + f*x))^m,x)
[Out] int(tan(e + f*x)^3*(a*sin(e + f*x))^m, x)
```

3.162 $\int (a\sin(e+fx))^m \tan(e+fx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int (a\sin(e+fx))^m \tan(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\sin^2(e+fx)\right)(a\sin(e+fx))^{2+m}}{a^2f(2+m)}$$

```
[Out] hypergeom([1, 1+1/2*m],[2+1/2*m],sin(f*x+e)^2)*(a*sin(f*x+e))^(2+m)/a^2/f/(
2+m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2672, 371}

$$\int (a\sin(e+fx))^m \tan(e+fx)\,dx = \frac{(a\sin(e+fx))^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},\sin^2(e+fx)\right)}{a^2f(m+2)}$$

```
[In] Int[(a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] (Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x]
)^(2 + m))/(a^2*f*(2 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^{1+m}}{a^2-x^2}\,dx, x, a\sin(e+fx)\right)}{f}$$
$$= \frac{\text{Hypergeometric2F1}\left(1, \frac{2+m}{2}, \frac{4+m}{2}, \sin^2(e+fx)\right)(a\sin(e+fx))^{2+m}}{a^2 f(2+m)}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int (a\sin(e+fx))^m \tan(e+fx)\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(1, \frac{2+m}{2}, 1+\frac{2+m}{2}, \sin^2(e+fx)\right)\sin^2(e+fx)(a\sin(e+fx))^m}{f(2+m)}$$

```
[In] Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x],x]
[Out] (Hypergeometric2F1[1, (2 + m)/2, 1 + (2 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x
]^2*(a*Sin[e + f*x])^m)/(f*(2 + m))
```

Maple [F]

$$\int (\sin(fx+e)\,a)^m \tan(fx+e)\,dx$$

```
[In] int((sin(f*x+e)*a)^m*tan(f*x+e),x)
[Out] int((sin(f*x+e)*a)^m*tan(f*x+e),x)
```

Fricas [F]

$$\int (a\sin(e+fx))^m\tan(e+fx)\,dx = \int (a\sin(fx+e))^m\tan(fx+e)\ dx$$

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
[Out] integral((a*sin(f*x + e))^m*tan(f*x + e), x)
```

Sympy [F]

$$\int (a\sin(e+fx))^m\tan(e+fx)\,dx = \int (a\sin(e+fx))^m\tan(e+fx)\,dx$$

```
[In] integrate((a*sin(f*x+e))**m*tan(f*x+e),x)
[Out] Integral((a*sin(e + f*x))**m*tan(e + f*x), x)
```

Maxima [F]

$$\int (a\sin(e+fx))^m\tan(e+fx)\,dx = \int (a\sin(fx+e))^m\tan(fx+e)\ dx$$

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e), x)
```

Giac [F]

$$\int (a\sin(e+fx))^m\tan(e+fx)\,dx = \int (a\sin(fx+e))^m\tan(fx+e)\ dx$$

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m\tan(e+fx)\,dx = \int \tan(e+f\,x)\,\left(a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

```
[In] int(tan(e + f*x)*(a*sin(e + f*x))^m,x)
[Out] int(tan(e + f*x)*(a*sin(e + f*x))^m, x)
```

3.163 $\int \cot(e+fx)(a\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \frac{(a\sin(e+fx))^m}{fm}$$

```
[Out] (a*sin(f*x+e))^m/f/m
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2672, 30}

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \frac{(a\sin(e+fx))^m}{fm}$$

```
[In] Int[Cot[e + f*x]*(a*Sin[e + f*x])^m,x]
```

```
[Out] (a*Sin[e + f*x])^m/(f*m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^{-1+m}\, dx, x, a\sin(e+fx)\right)}{f}$$
$$= \frac{(a\sin(e+fx))^m}{fm}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(a\sin(e+fx))^m\, dx = \frac{(a\sin(e+fx))^m}{fm}$$

```
[In] Integrate[Cot[e + f*x]*(a*Sin[e + f*x])^m,x]
[Out] (a*Sin[e + f*x])^m/(f*m)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{(\sin(fx+e)a)^m}{fm}$
default	$\frac{(\sin(fx+e)a)^m}{fm}$
risch	$\frac{\left(e^{i(fx+e)}\right)^{-m}\left(e^{2i(fx+e)}-1\right)^m\left(\frac{1}{2}\right)^m a^m e^{-\frac{i\pi m\left(\text{csgn}(i\sin(fx+e)a)^3+\text{csgn}(i\sin(fx+e)a)^2\,\text{csgn}(\sin(fx+e)a)-\text{csgn}(\sin(fx+e)a)^3\right.}{}}}{}$

```
[In] int(cot(f*x+e)*(sin(f*x+e)*a)^m,x,method=_RETURNVERBOSE)
[Out] (sin(f*x+e)*a)^m/f/m
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(a\sin(e+fx))^m\, dx = \frac{(a\sin(fx+e))^m}{fm}$$

```
[In] integrate(cot(f*x+e)*(a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] (a*sin(f*x + e))^m/(f*m)
```

Sympy [F]

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(e+fx))^m \cot(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)*(a*sin(f*x+e))**m,x)
[Out] Integral((a*sin(e + f*x))**m*cot(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \frac{a^m \sin(fx+e)^m}{fm}$$

```
[In] integrate(cot(f*x+e)*(a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] a^m*sin(f*x + e)^m/(f*m)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \frac{(a\sin(fx+e))^m}{fm}$$

```
[In] integrate(cot(f*x+e)*(a*sin(f*x+e))^m,x, algorithm="giac")
[Out] (a*sin(f*x + e))^m/(f*m)
```

Mupad [B] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \frac{(a\sin(e+f\,x))^m}{f\,m}$$

```
[In] int(cot(e + f*x)*(a*sin(e + f*x))^m,x)
[Out] (a*sin(e + f*x))^m/(f*m)
```

3.164 $\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a^2(a\sin(e+fx))^{-2+m}}{f(2-m)} - \frac{(a\sin(e+fx))^m}{fm}$$

```
[Out] -a^2*(a*sin(f*x+e))^(-2+m)/f/(2-m)-(a*sin(f*x+e))^m/f/m
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2672, 14}

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a^2(a\sin(e+fx))^{m-2}}{f(2-m)} - \frac{(a\sin(e+fx))^m}{fm}$$

```
[In] Int[Cot[e + f*x]^3*(a*Sin[e + f*x])^m,x]
[Out] -((a^2*(a*Sin[e + f*x])^(-2 + m))/(f*(2 - m))) - (a*Sin[e + f*x])^m/(f*m)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int x^{-3+m}(a^2-x^2)\,dx, x, a\sin(e+fx)\right)}{f} \\ &= \frac{\text{Subst}\left(\int (a^2x^{-3+m}-x^{-1+m})\,dx, x, a\sin(e+fx)\right)}{f} \\ &= -\frac{a^2(a\sin(e+fx))^{-2+m}}{f(2-m)}-\frac{(a\sin(e+fx))^m}{fm} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = \frac{(2-m+m\csc^2(e+fx))\,(a\sin(e+fx))^m}{f(-2+m)m}$$

```
[In] Integrate[Cot[e + f*x]^3*(a*Sin[e + f*x])^m,x]
[Out] ((2 - m + m*Csc[e + f*x]^2)*(a*Sin[e + f*x])^m)/(f*(-2 + m)*m)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.19 (sec) , antiderivative size = 2751, normalized size of antiderivative = 59.80

method	result	size
risch	Expression too large to display	2751

```
[In] int(cot(f*x+e)^3*(sin(f*x+e)*a)^m,x,method=_RETURNVERBOSE)
[Out] -1/(-2+m)/f/(exp(2*I*(f*x+e))-1)^2/m*exp(I*(f*x+e))^(-m)*(exp(2*I*(f*x+e))-
1)^m*(1/2)^m*a^m*(m*exp(-1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(
e))^3*Pi)*exp(-1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2*csgn
(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))*Pi)*exp(1/2*I*m*csgn(a*sin(f*x)*cos(e
)+a*cos(f*x)*sin(e))^3*Pi)*exp(1/2*I*m*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*si
n(e))^2*csgn(I*a)*Pi)*exp(-1/2*I*m*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e)
)^2*Pi*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)))*exp(-1/2*I*m*csgn(a*sin(f*x)*
cos(e)+a*cos(f*x)*sin(e))*csgn(I*a)*Pi*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)
))*exp(1/2*I*m*Pi*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^3)*exp(1/2*I*Pi*m*c
sgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2*csgn(I*exp(-I*(f*x+e))))*exp(1/2*I*P
i*m*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2*csgn(I*exp(2*I*(f*x+e))-I))*exp
```

(1/2*I*Pi*m*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))*csgn(I*exp(-I*(f*x+e)))*c
sgn(I*exp(2*I*(f*x+e))-I))*exp(1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x
)*sin(e))^2*Pi)*exp(1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))*c
sgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))*Pi)*exp(-1/2*I*Pi*m)*exp(4*I*f*x)*
exp(4*I*e)-2*exp(-1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^3*P
i)*exp(-1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2*csgn(a*sin(
f*x)*cos(e)+a*cos(f*x)*sin(e))*Pi)*exp(1/2*I*m*csgn(a*sin(f*x)*cos(e)+a*cos
(f*x)*sin(e))^3*Pi)*exp(1/2*I*m*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^2
*csgn(I*a)*Pi)*exp(-1/2*I*m*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^2*Pi*
csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)))*exp(-1/2*I*m*csgn(a*sin(f*x)*cos(e)+
a*cos(f*x)*sin(e))*csgn(I*a)*Pi*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)))*exp(
1/2*I*m*Pi*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^3)*exp(1/2*I*Pi*m*csgn(sin
(f*x)*cos(e)+cos(f*x)*sin(e))^2*csgn(I*exp(-I*(f*x+e))))*exp(1/2*I*Pi*m*csg
n(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2*csgn(I*exp(2*I*(f*x+e))-I))*exp(1/2*I*
Pi*m*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))*csgn(I*exp(-I*(f*x+e)))*csgn(I*e
xp(2*I*(f*x+e))-I))*exp(1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e
))^2*Pi)*exp(1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))*csgn(a*s
in(f*x)*cos(e)+a*cos(f*x)*sin(e))*Pi)*exp(-1/2*I*Pi*m)*exp(4*I*f*x)*exp(4*I
*e)+2*m*exp(-1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^3*Pi)*ex
p(-1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2*csgn(a*sin(f*x)*
cos(e)+a*cos(f*x)*sin(e))*Pi)*exp(1/2*I*m*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)
*sin(e))^3*Pi)*exp(1/2*I*m*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^2*csgn
(I*a)*Pi)*exp(-1/2*I*m*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^2*Pi*csgn(
sin(f*x)*cos(e)+cos(f*x)*sin(e)))*exp(-1/2*I*m*csgn(a*sin(f*x)*cos(e)+a*cos
(f*x)*sin(e))*csgn(I*a)*Pi*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)))*exp(1/2*I
*m*Pi*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^3)*exp(1/2*I*Pi*m*csgn(sin(f*x)
*cos(e)+cos(f*x)*sin(e))^2*csgn(I*exp(-I*(f*x+e))))*exp(1/2*I*Pi*m*csgn(sin
(f*x)*cos(e)+cos(f*x)*sin(e))^2*csgn(I*exp(2*I*(f*x+e))-I))*exp(1/2*I*Pi*m*
csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))*csgn(I*exp(-I*(f*x+e)))*csgn(I*exp(2*
I*(f*x+e))-I))*exp(1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2*
Pi)*exp(1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))*csgn(a*sin(f*
x)*cos(e)+a*cos(f*x)*sin(e))*Pi)*exp(-1/2*I*Pi*m)*exp(2*I*f*x)*exp(2*I*e)+4
*exp(-1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^3*Pi)*exp(-1/2*
I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2*csgn(a*sin(f*x)*cos(e)+
a*cos(f*x)*sin(e))*Pi)*exp(1/2*I*m*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e)
)^3*Pi)*exp(1/2*I*m*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^2*csgn(I*a)*P
i)*exp(-1/2*I*m*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^2*Pi*csgn(sin(f*x
)*cos(e)+cos(f*x)*sin(e)))*exp(-1/2*I*m*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*s
in(e))*csgn(I*a)*Pi*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)))*exp(1/2*I*m*Pi*c
sgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^3)*exp(1/2*I*Pi*m*csgn(sin(f*x)*cos(e)
+cos(f*x)*sin(e))^2*csgn(I*exp(-I*(f*x+e))))*exp(1/2*I*Pi*m*csgn(sin(f*x)*c
os(e)+cos(f*x)*sin(e))^2*csgn(I*exp(2*I*(f*x+e))-I))*exp(1/2*I*Pi*m*csgn(si
n(f*x)*cos(e)+cos(f*x)*sin(e))*csgn(I*exp(-I*(f*x+e)))*csgn(I*exp(2*I*(f*x+
e))-I))*exp(1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2*Pi)*exp
(1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))*csgn(a*sin(f*x)*cos(

```
e)+a*cos(f*x)*sin(e))*Pi)*exp(-1/2*I*Pi*m)*exp(2*I*f*x)*exp(2*I*e)+m*exp(-1
/2*I*Pi*m*(-csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^3+csgn(a*sin(f*x)*cos
(e)+a*cos(f*x)*sin(e))^2*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))+csgn(I*a*sin
(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e
))-csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2*csgn(I*exp(2*I*(f*x+e))-I)+csgn(
I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^3-csgn(sin(f*x)*cos(e)+cos(f*x)*si
n(e))^3-csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^2*csgn(I*a)+csgn(a*sin(f*
x)*cos(e)+a*cos(f*x)*sin(e))*csgn(I*a)*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)
)-csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2*csgn(I*exp(-I*(f*x+e)))-csgn(sin(
f*x)*cos(e)+cos(f*x)*sin(e))*csgn(I*exp(-I*(f*x+e)))*csgn(I*exp(2*I*(f*x+e)
)-I)-csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))*csgn(a*sin(f*x)*cos(e)+a
*cos(f*x)*sin(e))-csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2+1))-2*exp
(-1/2*I*Pi*m*(-csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^3+csgn(a*sin(f*x)*
cos(e)+a*cos(f*x)*sin(e))^2*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))+csgn(I*a*
sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*si
n(e))-csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2*csgn(I*exp(2*I*(f*x+e))-I)+cs
gn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^3-csgn(sin(f*x)*cos(e)+cos(f*x)
*sin(e))^3-csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^2*csgn(I*a)+csgn(a*sin
(f*x)*cos(e)+a*cos(f*x)*sin(e))*csgn(I*a)*csgn(sin(f*x)*cos(e)+cos(f*x)*sin
(e))-csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2*csgn(I*exp(-I*(f*x+e)))-csgn(s
in(f*x)*cos(e)+cos(f*x)*sin(e))*csgn(I*exp(-I*(f*x+e)))*csgn(I*exp(2*I*(f*x
+e))-I)-csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))*csgn(a*sin(f*x)*cos(e
)+a*cos(f*x)*sin(e))-csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2+1)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = \frac{\left((m-2)\cos\left(fx+e\right)^2+2\right)\left(a\sin\left(fx+e\right)\right)^m}{fm^2-\left(fm^2-2\,fm\right)\cos\left(fx+e\right)^2-2\,fm}$$

```
[In] integrate(cot(f*x+e)^3*(a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] ((m - 2)*cos(f*x + e)^2 + 2)*(a*sin(f*x + e))^m/(f*m^2 - (f*m^2 - 2*f*m)*co
s(f*x + e)^2 - 2*f*m)
```

Sympy [F]

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(e+fx))^m \cot^3(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)**3*(a*sin(f*x+e))**m,x)
[Out] Integral((a*sin(e + f*x))**m*cot(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = -\frac{\frac{a^m\sin(fx+e)^m}{m} - \frac{a^m\sin(fx+e)^m}{(m-2)\sin(fx+e)^2}}{f}$$

```
[In] integrate(cot(f*x+e)^3*(a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] -(a^m*sin(f*x + e)^m/m - a^m*sin(f*x + e)^m/((m - 2)*sin(f*x + e)^2))/f
```

Giac [F]

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m \cot(fx+e)^3\,dx$$

```
[In] integrate(cot(f*x+e)^3*(a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^3, x)
```

Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.98

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = -\frac{(a\sin(e+fx))^m\left(m-4\sin(2e+2fx)^2+m\left(2\sin(2e+2fx)^2-1\right)+16\sin(e+fx)^2\right)}{f\,m\,\left(2\sin(2e+2fx)^2-8\sin(e+fx)^2\right)\,(m-2)}$$

```
[In] int(cot(e + f*x)^3*(a*sin(e + f*x))^m,x)
[Out] -((a*sin(e + f*x))^m*(m - 4*sin(2*e + 2*f*x)^2 + m*(2*sin(2*e + 2*f*x)^2 -
1) + 16*sin(e + f*x)^2))/(f*m*(2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)^2)*(m
- 2))
```

3.165 $\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a^4(a\sin(e+fx))^{-4+m}}{f(4-m)} + \frac{2a^2(a\sin(e+fx))^{-2+m}}{f(2-m)} + \frac{(a\sin(e+fx))^m}{fm}$$

```
[Out] -a^4*(a*sin(f*x+e))^(-4+m)/f/(4-m)+2*a^2*(a*sin(f*x+e))^(-2+m)/f/(2-m)+(a*s
in(f*x+e))^m/f/m
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2672, 276}

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a^4(a\sin(e+fx))^{m-4}}{f(4-m)} + \frac{2a^2(a\sin(e+fx))^{m-2}}{f(2-m)} + \frac{(a\sin(e+fx))^m}{fm}$$

```
[In] Int[Cot[e + f*x]^5*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a^4*(a*Sin[e + f*x])^(-4 + m))/(f*(4 - m))) + (2*a^2*(a*Sin[e + f*x])^(-
2 + m))/(f*(2 - m)) + (a*Sin[e + f*x])^m/(f*m)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^{-5+m}(a^2-x^2)^2\,dx, x, a\sin(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int (a^4x^{-5+m}-2a^2x^{-3+m}+x^{-1+m})\,dx, x, a\sin(e+fx)\right)}{f}\\ &= -\frac{a^4(a\sin(e+fx))^{-4+m}}{f(4-m)}+\frac{2a^2(a\sin(e+fx))^{-2+m}}{f(2-m)}+\frac{(a\sin(e+fx))^m}{fm}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = \frac{(8-6m+m^2-2(-4+m)m\csc^2(e+fx)+(-2+m)m\csc^4(e+fx))\,(a\sin(e+fx))^m}{f(-4+m)(-2+m)m}$$

```
[In] Integrate[Cot[e + f*x]^5*(a*Sin[e + f*x])^m,x]
[Out] ((8 - 6*m + m^2 - 2*(-4 + m)*m*Csc[e + f*x]^2 + (-2 + m)*m*Csc[e + f*x]^4)*
(a*Sin[e + f*x])^m)/(f*(-4 + m)*(-2 + m)*m)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.02 (sec) , antiderivative size = 6931, normalized size of antiderivative = 96.26

method	result	size
risch	Expression too large to display	6931

```
[In] int(cot(f*x+e)^5*(sin(f*x+e)*a)^m,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.56

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = \frac{\left((m^2-6\,m+8)\cos\left(fx+e\right)^4+4\,(m-4)\cos\left(fx+e\right)^2+8\right)\left(a\sin\left(fx+e\right)\right)^m}{(fm^3-6\,fm^2+8\,fm)\cos\left(fx+e\right)^4+fm^3-6\,fm^2-2\,(fm^3-6\,fm^2+8\,fm)\cos\left(fx+e\right)^2+8\,fm}$$

```
[In] integrate(cot(f*x+e)^5*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] ((m^2 - 6*m + 8)*cos(f*x + e)^4 + 4*(m - 4)*cos(f*x + e)^2 + 8)*(a*sin(f*x
+ e))^m/((f*m^3 - 6*f*m^2 + 8*f*m)*cos(f*x + e)^4 + f*m^3 - 6*f*m^2 - 2*(f*
m^3 - 6*f*m^2 + 8*f*m)*cos(f*x + e)^2 + 8*f*m)
```

Sympy [F]

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = \int \left(a\sin\left(e+fx\right)\right)^m\cot^5\left(e+fx\right)\,dx$$

```
[In] integrate(cot(f*x+e)**5*(a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*cot(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = \frac{\frac{a^m\sin(fx+e)^m}{m}-\frac{2\,a^m\sin(fx+e)^m}{(m-2)\sin(fx+e)^2}+\frac{a^m\sin(fx+e)^m}{(m-4)\sin(fx+e)^4}}{f}$$

```
[In] integrate(cot(f*x+e)^5*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] (a^m*sin(f*x + e)^m/m - 2*a^m*sin(f*x + e)^m/((m - 2)*sin(f*x + e)^2) + a^m
*sin(f*x + e)^m/((m - 4)*sin(f*x + e)^4))/f
```

Giac [F]

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m \cot(fx+e)^5\,dx$$

```
[In] integrate(cot(f*x+e)^5*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^5, x)
```

Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.04

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx =$$

$$-\frac{(a\,\sin(e+f\,x))^m\,\left(2\sin(2\,e+2\,f\,x)^2+\sin(4\,e+4\,f\,x)\;1\mathrm{i}-1\right)\,\left(-\frac{2\,\left(2\sin(2\,e+2\,f\,x)^2-1\right)\,\left(-2\sin(2\,e+2\,f\,x)^2+\mathrm{si}\right.}{f\,m}\right.}{}$$

```
[In] int(cot(e + f*x)^5*(a*sin(e + f*x))^m,x)
```

```
[Out] -((a*sin(e + f*x))^m*(sin(4*e + 4*f*x)*1i + 2*sin(2*e + 2*f*x)^2 - 1)*(((si
n(4*e + 4*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1)*(6*m^2 - 4*m + 48))/(f*m*(m^2
 - 6*m + 8)) - (2*(2*sin(2*e + 2*f*x)^2 - 1)*(sin(4*e + 4*f*x)*1i - 2*sin(2
*e + 2*f*x)^2 + 1))/(f*m) + (2*(2*sin(e + f*x)^2 - 1)*(sin(4*e + 4*f*x)*1i
- 2*sin(2*e + 2*f*x)^2 + 1)*(8*m - 4*m^2 + 32))/(f*m*(m^2 - 6*m + 8))))/(16
*sin(e + f*x)^4)
```

3.166 $\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx = \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{5+m}{2},\frac{7+m}{2},\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{5+m}}{a^5 f(5+m)}$$

```
[Out] hypergeom([5/2, 5/2+1/2*m],[7/2+1/2*m],sin(f*x+e)^2)*sec(f*x+e)*(a*sin(f*x+
e))^(5+m)*(cos(f*x+e)^2)^(1/2)/a^5/f/(5+m)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2680, 2657}

$$\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx = \frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)(a\sin(e+fx))^{m+5}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+5}{2},\frac{m+7}{2},\sin^2(e+fx)\right)}{a^5 f(m+5)}$$

```
[In] Int[(a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[5/2, (5 + m)/2, (7 + m)/2, Sin[e +
f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(5 + m))/(a^5*f*(5 + m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2680

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Dist[1/a^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /;
 FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\int \sec^4(e+fx)(a\sin(e+fx))^{4+m}\,dx}{a^4}$$
$$= \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{5+m}{2},\frac{7+m}{2},\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{5+m}}{a^5f(5+m)}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int (a\sin(e+fx))^m\tan^4(e+fx)\,dx$$
$$= \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{5+m}{2},\frac{7+m}{2},\sin^2(e+fx)\right)\sin^4(e+fx)(a\sin(e+fx))^m\tan(e+fx)}{f(5+m)}$$

```
[In] Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[5/2, (5 + m)/2, (7 + m)/2, Sin[e +
f*x]^2]*Sin[e + f*x]^4*(a*Sin[e + f*x])^m*Tan[e + f*x])/(f*(5 + m))
```

Maple [F]

$$\int (\sin(fx+e)\,a)^m\left(\tan^4(fx+e)\right)dx$$

```
[In] int((sin(f*x+e)*a)^m*tan(f*x+e)^4,x)
[Out] int((sin(f*x+e)*a)^m*tan(f*x+e)^4,x)
```

Fricas [F]

$$\int (a\sin(e+fx))^m\tan^4(e+fx)\,dx=\int (a\sin(fx+e))^m\tan(fx+e)^4\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e))^m*tan(f*x + e)^4, x)
```

Sympy [F]

$$\int (a\sin(e+fx))^m\tan^4(e+fx)\,dx=\int (a\sin(e+fx))^m\tan^4(e+fx)\,dx$$

```
[In] integrate((a*sin(f*x+e))**m*tan(f*x+e)**4,x)
[Out] Integral((a*sin(e + f*x))**m*tan(e + f*x)**4, x)
```

Maxima [F]

$$\int (a\sin(e+fx))^m\tan^4(e+fx)\,dx=\int (a\sin(fx+e))^m\tan(fx+e)^4\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^4, x)
```

Giac [F]

$$\int (a\sin(e+fx))^m\tan^4(e+fx)\,dx=\int (a\sin(fx+e))^m\tan(fx+e)^4\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m\tan^4(e+fx)\,dx=\int \tan(e+f\,x)^4\,(a\,\sin\left(e+f\,x\right))^m\,dx$$

```
[In] int(tan(e + f*x)^4*(a*sin(e + f*x))^m,x)
[Out] int(tan(e + f*x)^4*(a*sin(e + f*x))^m, x)
```

3.167 $\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx = \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{3+m}}{a^3f(3+m)}$$

```
[Out] hypergeom([3/2, 3/2+1/2*m],[5/2+1/2*m],sin(f*x+e)^2)*sec(f*x+e)*(a*sin(f*x+
e))^(3+m)*(cos(f*x+e)^2)^(1/2)/a^3/f/(3+m)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2680, 2657}

$$\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx = \frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)(a\sin(e+fx))^{m+3}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+3}{2},\frac{m+5}{2},\sin^2(e+fx)\right)}{a^3f(m+3)}$$

```
[In] Int[(a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[e +
f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2680

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Dist[1/a^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /;
 FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\int \sec^2(e+fx)(a\sin(e+fx))^{2+m}\,dx}{a^2}$$
$$= \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{3+m}}{a^3f(3+m)}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx$$
$$= \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(e+fx)\right)\sin^2(e+fx)(a\sin(e+fx))^m\tan(e+fx)}{f(3+m)}$$

```
[In] Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[e +
f*x]^2]*Sin[e + f*x]^2*(a*Sin[e + f*x])^m*Tan[e + f*x])/(f*(3 + m))
```

Maple [F]

$$\int (\sin(fx+e)\,a)^m\left(\tan^2(fx+e)\right)dx$$

```
[In] int((sin(f*x+e)*a)^m*tan(f*x+e)^2,x)
[Out] int((sin(f*x+e)*a)^m*tan(f*x+e)^2,x)
```

Fricas [F]

$$\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx=\int (a\sin(fx+e))^m\tan(fx+e)^2\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e))^m*tan(f*x + e)^2, x)
```

Sympy [F]

$$\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx=\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx$$

```
[In] integrate((a*sin(f*x+e))**m*tan(f*x+e)**2,x)
[Out] Integral((a*sin(e + f*x))**m*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx=\int (a\sin(fx+e))^m\tan(fx+e)^2\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^2, x)
```

Giac [F]

$$\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx=\int (a\sin(fx+e))^m\tan(fx+e)^2\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx=\int \tan(e+f\,x)^2\,(a\,\sin\left(e+f\,x\right))^m\,dx$$

```
[In] int(tan(e + f*x)^2*(a*sin(e + f*x))^m,x)
[Out] int(tan(e + f*x)^2*(a*sin(e + f*x))^m, x)
```

3.168 $\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1+m}{2},\sin^2(e+fx)\right)(a\sin(e+fx))^{-1+m}}{f(1-m)\sqrt{\cos^2(e+fx)}}$$

```
[Out] -a*cos(f*x+e)*hypergeom([-1/2, -1/2+1/2*m],[1/2+1/2*m],sin(f*x+e)^2)*(a*sin
(f*x+e))^(-1+m)/f/(1-m)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2680, 2657}

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a\cos(e+fx)(a\sin(e+fx))^{m-1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m-1}{2},\frac{m+1}{2},\sin^2(e+fx)\right)}{f(1-m)\sqrt{\cos^2(e+fx)}}$$

```
[In] Int[Cot[e + f*x]^2*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a*Cos[e + f*x]*Hypergeometric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin[e + f*
x]^2]*(a*Sin[e + f*x])^(-1 + m))/(f*(1 - m)*Sqrt[Cos[e + f*x]^2]))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2680

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Dist[1/a^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /;
 FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = a^2 \int \cos^2(e+fx)(a\sin(e+fx))^{-2+m}\,dx$$
$$= -\frac{a\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1+m}{2},\sin^2(e+fx)\right)(a\sin(e+fx))^{-1+m}}{f(1-m)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx$$
$$= \frac{a\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1+m}{2},\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{-1+m}}{f(-1+m)}$$

```
[In] Integrate[Cot[e + f*x]^2*(a*Sin[e + f*x])^m,x]
[Out] (a*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin[
e + f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(-1 + m))/(f*(-1 + m))
```

Maple [F]

$$\int \left(\cot^2(fx+e)\right)(\sin(fx+e)\,a)^m\,dx$$

```
[In] int(cot(f*x+e)^2*(sin(f*x+e)*a)^m,x)
[Out] int(cot(f*x+e)^2*(sin(f*x+e)*a)^m,x)
```

Fricas [F]

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m \cot(fx+e)^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e))^m*cot(f*x + e)^2, x)
```

Sympy [F]

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(e+fx))^m \cot^2(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)**2*(a*sin(f*x+e))**m,x)
[Out] Integral((a*sin(e + f*x))**m*cot(e + f*x)**2, x)
```

Maxima [F]

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m \cot(fx+e)^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^2, x)
```

Giac [F]

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m \cot(fx+e)^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \int \cot(e+f\,x)^2\,(a\,\sin(e+f\,x))^m\,dx$$

```
[In] int(cot(e + f*x)^2*(a*sin(e + f*x))^m,x)
[Out] int(cot(e + f*x)^2*(a*sin(e + f*x))^m, x)
```

3.169 $\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a^3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(e+fx)\right)(a\sin(e+fx))^{-3+m}}{f(3-m)\sqrt{\cos^2(e+fx)}}$$

```
[Out] -a^3*cos(f*x+e)*hypergeom([-3/2, -3/2+1/2*m],[-1/2+1/2*m],sin(f*x+e)^2)*(a*
sin(f*x+e))^(-3+m)/f/(3-m)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2680, 2657}

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a^3\cos(e+fx)(a\sin(e+fx))^{m-3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m-3}{2},\frac{m-1}{2},\sin^2(e+fx)\right)}{f(3-m)\sqrt{\cos^2(e+fx)}}$$

```
[In] Int[Cot[e + f*x]^4*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a^3*Cos[e + f*x]*Hypergeometric2F1[-3/2, (-3 + m)/2, (-1 + m)/2, Sin[e +
 f*x]^2]*(a*Sin[e + f*x])^(-3 + m))/(f*(3 - m)*Sqrt[Cos[e + f*x]^2]))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2680

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Dist[1/a^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /;
 FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = a^4 \int \cos^4(e+fx)(a\sin(e+fx))^{-4+m}\,dx$$
$$= -\frac{a^3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(e+fx)\right)(a\sin(e+fx))^{\text{-}}}{f(3-m)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx$$
$$= \frac{\sqrt{\cos^2(e+fx)}\csc^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(e+fx)\right)\sec(e+fx)(}{f(-3+m)}$$

```
[In] Integrate[Cot[e + f*x]^4*(a*Sin[e + f*x])^m,x]
[Out] (Sqrt[Cos[e + f*x]^2]*Csc[e + f*x]^3*Hypergeometric2F1[-3/2, (-3 + m)/2, (-
1 + m)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^m)/(f*(-3 + m))
```

Maple [F]

$$\int \left(\cot^4(fx+e)\right)(\sin(fx+e)\,a)^m\,dx$$

```
[In] int(cot(f*x+e)^4*(sin(f*x+e)*a)^m,x)
[Out] int(cot(f*x+e)^4*(sin(f*x+e)*a)^m,x)
```

Fricas [F]

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m\cot(fx+e)^4\,dx$$

```
[In] integrate(cot(f*x+e)^4*(a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e))^m*cot(f*x + e)^4, x)
```

Sympy [F]

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(e+fx))^m\cot^4(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)**4*(a*sin(f*x+e))**m,x)
[Out] Integral((a*sin(e + f*x))**m*cot(e + f*x)**4, x)
```

Maxima [F]

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m\cot(fx+e)^4\,dx$$

```
[In] integrate(cot(f*x+e)^4*(a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^4, x)
```

Giac [F]

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m\cot(fx+e)^4\,dx$$

```
[In] integrate(cot(f*x+e)^4*(a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = \int \cot(e+f\,x)^4\,(a\,\sin(e+f\,x))^m\,dx$$

```
[In] int(cot(e + f*x)^4*(a*sin(e + f*x))^m,x)
[Out] int(cot(e + f*x)^4*(a*sin(e + f*x))^m, x)
```

3.170 $\int (a\sin(e+fx))^m (b\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int (a\sin(e+fx))^m (b\tan(e+fx))^{3/2}\,dx = \frac{2\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{4}(5+2m),\frac{1}{4}(9+2m),\sin^2(e+fx)\right)(a\sin(e+fx))^m$$
$$bf(5+2m)$$

```
[Out] 2*(cos(f*x+e)^2)^(5/4)*hypergeom([5/4, 5/4+1/2*m],[9/4+1/2*m],sin(f*x+e)^2)
*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(5/2)/b/f/(5+2*m)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2682, 2657}

$$\int (a\sin(e+fx))^m (b\tan(e+fx))^{3/2}\,dx = \frac{2\cos^2(e+fx)^{5/4}(b\tan(e+fx))^{5/2}(a\sin(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{4}(2m+5),\frac{1}{4}(2m\right.$$
$$bf(2m+5)$$

```
[In] Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, (5 + 2*m)/4, (9 + 2*m)/4,
Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(5/2))/(b*f*(5 + 2*m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(a\cos^{\frac{5}{2}}(e+fx)(b\tan(e+fx))^{5/2}\right)\int\frac{(a\sin(e+fx))^{\frac{3}{2}+m}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{b(a\sin(e+fx))^{5/2}}$$

$$= \frac{2\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{4}(5+2m),\frac{1}{4}(9+2m),\sin^2(e+fx)\right)(a\sin(e+fx))^m(b}{bf(5+2m)}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int(a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},\frac{1}{4}(5+2m),\frac{1}{4}(9+2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}(a\sin(e+}{bf(5+2m)}$$

```
[In] Integrate[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2),x]
[Out] (2*Hypergeometric2F1[(2 + m)/2, (5 + 2*m)/4, (9 + 2*m)/4, -Tan[e + f*x]^2]*
(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(5/2))/(b*f*(5 +
 2*m))
```

Maple [F]

$$\int(\sin(fx+e)\,a)^m\,(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] int((sin(f*x+e)*a)^m*(b*tan(f*x+e))^(3/2),x)
[Out] int((sin(f*x+e)*a)^m*(b*tan(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(fx+e))^{\frac{3}{2}}(a\sin(fx+e))^m\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m*b*tan(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**m*(b*tan(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(fx+e))^{\frac{3}{2}}(a\sin(fx+e))^m\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^(3/2)*(a*sin(f*x + e))^m, x)
```

Giac [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(fx+e))^{\frac{3}{2}}(a\sin(fx+e))^m\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(3/2)*(a*sin(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx = \int (a\ \sin(e+f\,x))^m\,(b\tan(e+f\,x))^{3/2}\,dx$$

```
[In] int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(3/2), x)
```

3.171 $\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}(3+2m),\frac{1}{4}(7+2m),\sin^2(e+fx)\right)(a\sin(e+fx))^m(b\tan(e+}{bf(3+2m)}$$

```
[Out] 2*(cos(f*x+e)^2)^(3/4)*hypergeom([3/4, 3/4+1/2*m],[7/4+1/2*m],sin(f*x+e)^2)
*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2)/b/f/(3+2*m)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2682, 2657}

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{3/4}(b\tan(e+fx))^{3/2}(a\sin(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}(2m+3),\frac{1}{4}(2m+7),\sin^2(e+}{bf(2m+3)}$$

```
[In] Int[(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, (3 + 2*m)/4, (7 + 2*m)/4,
Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2))/(b*f*(3 + 2*m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(a\cos^{\frac{3}{2}}(e+fx)(b\tan(e+fx))^{3/2}\right)\int \frac{(a\sin(e+fx))^{\frac{1}{2}+m}}{\sqrt{\cos(e+fx)}}\,dx}{b(a\sin(e+fx))^{3/2}}$$

$$= \frac{2\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}(3+2m),\frac{1}{4}(7+2m),\sin^2(e+fx)\right)(a\sin(e+fx))^m(b}{bf(3+2m)}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx$$
$$= \frac{2\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},\frac{1}{4}(3+2m),\frac{1}{4}(7+2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}(a\sin(e+fx))^m(b\,\text{ta}}{bf(3+2m)}$$

```
[In] Integrate[(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]],x]
[Out] (2*Hypergeometric2F1[(2 + m)/2, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*
(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2))/(b*f*(3 +
 2*m))
```

Maple [F]

$$\int (\sin(fx+e)\,a)^m\sqrt{b\tan(fx+e)}dx$$

```
[In] int((sin(f*x+e)*a)^m*(b*tan(f*x+e))^(1/2),x)
[Out] int((sin(f*x+e)*a)^m*(b*tan(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx=\int \sqrt{b\tan\left(fx+e\right)}(a\sin\left(fx+e\right))^m\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m, x)
```

Sympy [F]

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx=\int (a\sin\left(e+fx\right))^m\sqrt{b\tan\left(e+fx\right)}\,dx$$

```
[In] integrate((a*sin(f*x+e))**m*(b*tan(f*x+e))**(1/2),x)
[Out] Integral((a*sin(e + f*x))**m*sqrt(b*tan(e + f*x)), x)
```

Maxima [F]

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx=\int \sqrt{b\tan\left(fx+e\right)}(a\sin\left(fx+e\right))^m\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m, x)
```

Giac [F]

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx=\int \sqrt{b\tan\left(fx+e\right)}(a\sin\left(fx+e\right))^m\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx=\int (a\,\sin\left(e+f\,x\right))^m\,\sqrt{b\tan\left(e+f\,x\right)}\,dx$$

```
[In] int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(1/2),x)
[Out] int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(1/2), x)
```

3.172 $\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4}(1+2m),\frac{1}{4}(5+2m),\sin^2(e+fx)\right)(a\sin(e+fx))^m\sqrt{b\tan(e+}}{bf(1+2m)}$$

```
[Out] 2*(cos(f*x+e)^2)^(1/4)*hypergeom([1/4, 1/4+1/2*m],[5/4+1/2*m],sin(f*x+e)^2)
*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2)/b/f/(1+2*m)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2682, 2657}

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}(a\sin(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4}(2m+1),\frac{1}{4}(2m+5),\sin^2(e+f}{bf(2m+1)}$$

```
[In] Int[(a*Sin[e + f*x])^m/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + 2*m)/4, (5 + 2*m)/4,
Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]])/(b*f*(1 + 2*m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int\sqrt{\cos(e+fx)}(a\sin(e+fx))^{-\frac{1}{2}+m}\,dx}{b\sqrt{a\sin(e+fx)}}$$

$$= \frac{2\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4}(1+2m),\frac{1}{4}(5+2m),\sin^2(e+fx)\right)(a\sin(e+fx))^m\sqrt{\cdots}}{bf(1+2m)}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int\frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx$$
$$= \frac{2\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},\frac{1}{4}(1+2m),\frac{1}{4}(5+2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}(a\sin(e+fx))^m\sqrt{b\cdots}}{bf(1+2m)}$$

```
[In] Integrate[(a*Sin[e + f*x])^m/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[(2 + m)/2, (1 + 2*m)/4, (5 + 2*m)/4, -Tan[e + f*x]^2]*
(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]])/(b*f*(1 + 2
*m))
```

Maple [F]

$$\int\frac{(\sin(fx+e)\,a)^m}{\sqrt{b\tan(fx+e)}}dx$$

```
[In] int((sin(f*x+e)*a)^m/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] int((sin(f*x+e)*a)^m/(b*tan(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^m}{\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m/(b*tan(f*x + e)), x)
```

Sympy [F]

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx$$

```
[In] integrate((a*sin(f*x+e))**m/(b*tan(f*x+e))**(1/2),x)
[Out] Integral((a*sin(e + f*x))**m/sqrt(b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^m}{\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^m/sqrt(b*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^m}{\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m/sqrt(b*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\,\sin\left(e+f\,x\right))^m}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

```
[In] int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(1/2),x)
[Out] int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(1/2), x)
```

3.173 $\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4}(-1+2m),\frac{1}{4}(3+2m),\sin^2(e+fx)\right)(a\sin(e+fx))^m}{bf(1-2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] -2*hypergeom([-1/4, -1/4+1/2*m],[3/4+1/2*m],sin(f*x+e)^2)*(a*sin(f*x+e))^m/
b/f/(1-2*m)/(cos(f*x+e)^2)^(1/4)/(b*tan(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2682, 2657}

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2(a\sin(e+fx))^m\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4}(2m-1),\frac{1}{4}(2m+3),\sin^2(e+fx)\right)}{bf(1-2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Int[(a*Sin[e + f*x])^m/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, (-1 + 2*m)/4, (3 + 2*m)/4, Sin[e + f*x]^2]*(a*S
in[e + f*x])^m)/(b*f*(1 - 2*m)*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{a\sin(e+fx)}\right)\int \cos^{\frac{3}{2}}(e+fx)(a\sin(e+fx))^{-\frac{3}{2}+m}\,dx}{b\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= -\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4}(-1+2m),\frac{1}{4}(3+2m),\sin^2(e+fx)\right)(a\sin(e+fx))^m}{bf(1-2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},\frac{1}{4}(-1+2m),\frac{1}{4}(3+2m),-\tan^2(e+fx)\right)\sec^2(e+f}{bf(-1+2m)\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[(a*Sin[e + f*x])^m/(b*Tan[e + f*x])^(3/2),x]
[Out] (2*Hypergeometric2F1[(2 + m)/2, (-1 + 2*m)/4, (3 + 2*m)/4, -Tan[e + f*x]^2]
*(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m)/(b*f*(-1 + 2*m)*Sqrt[b*Tan[e +
f*x]])
```

Maple [F]

$$\int \frac{(\sin(fx+e)\,a)^m}{(b\tan(fx+e))^{\frac{3}{2}}}dx$$

```
[In] int((sin(f*x+e)*a)^m/(b*tan(f*x+e))^(3/2),x)
[Out] int((sin(f*x+e)*a)^m/(b*tan(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^m}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m/(b^2*tan(f*x + e)^2), x)
```

Sympy [F]

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))**m/(b*tan(f*x+e))**(3/2),x)
[Out] Integral((a*sin(e + f*x))**m/(b*tan(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^m}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^m/(b*tan(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^m}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m/(b*tan(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx=\int \frac{{\left(a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(b\,\tan\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

```
[In] int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(3/2), x)
```

3.174 $\int (a\sin(e+fx))^m (b\tan(e+fx))^n \, dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int (a\sin(e+fx))^m (b\tan(e+fx))^n \, dx = \frac{\cos^2(e+fx)^{\frac{1+n}{2}} \text{Hypergeometric2F1}\left(\frac{1+n}{2}, \frac{1}{2}(1+m+n), \frac{1}{2}(3+m+n), \sin^2(e+fx)\right) (a\sin(e+fx))^m (b\,\text{t}}{bf(1+m+n)}$$

```
[Out] (cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1/2+1/2*n, 1/2+1/2*m+1/2*n],[3/2+1/2*
m+1/2*n],sin(f*x+e)^2)*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(1+n)/b/f/(1+m+n)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2682, 2657}

$$\int (a\sin(e+fx))^m (b\tan(e+fx))^n \, dx = \frac{\cos^2(e+fx)^{\frac{n+1}{2}} (a\sin(e+fx))^m (b\tan(e+fx))^{n+1} \text{Hypergeometric2F1}\left(\frac{n+1}{2}, \frac{1}{2}(m+n+1), \frac{1}{2}(m+n+3)\right.}{bf(m+n+1)}$$

```
[In] Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + m + n)/2, (
3 + m + n)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 + m + n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{(a\cos^{1+n}(e+fx)(a\sin(e+fx))^{-1-n}(b\tan(e+fx))^{1+n})\int \cos^{-n}(e+fx)(a\sin(e+fx))^{m+n}\,dx}{b}$$

$$= \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\sin^2(e+fx)\right)(a\sin(e+\ldots}{bf(1+m+n)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.82 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.13

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx$$

$$= \frac{(3+m+\ldots}{f(1+m+n)\left((3+m+n)\,\text{AppellF1}\left(\frac{1}{2}(1+m+n),n,1+m,\frac{1}{2}(3+m+n),\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\ldots\right.\right.\right.}$$

```
[In] Integrate[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((3 + m + n)*AppellF1[(1 + m + n)/2, n, 1 + m, (3 + m + n)/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x]
)^n)/(f*(1 + m + n)*((3 + m + n)*AppellF1[(1 + m + n)/2, n, 1 + m, (3 + m +
 n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((1 + m)*AppellF1[(3 +
m + n)/2, n, 2 + m, (5 + m + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 - n*AppellF1[(3 + m + n)/2, 1 + n, 1 + m, (5 + m + n)/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
```

Maple [F]

$$\int \left(\sin \left(fx+e\right) a\right)^{m} \left(b \tan \left(fx+e\right)\right)^{n}dx$$

```
[In] int((sin(f*x+e)*a)^m*(b*tan(f*x+e))^n,x)
[Out] int((sin(f*x+e)*a)^m*(b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx = \int \left(a \sin \left(fx+e\right)\right)^{m} \left(b \tan \left(fx+e\right)\right)^{n}\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx = \int \left(a \sin \left(e+fx\right)\right)^{m} \left(b \tan \left(e+fx\right)\right)^{n}\,dx$$

```
[In] integrate((a*sin(f*x+e))**m*(b*tan(f*x+e))**n,x)
[Out] Integral((a*sin(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Maxima [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx = \int \left(a \sin \left(fx+e\right)\right)^{m} \left(b \tan \left(fx+e\right)\right)^{n}\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Giac [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\sin\left(fx+e\right))^m\left(b\tan\left(fx+e\right)\right)^n\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\,\sin\left(e+f\,x\right))^m\,(b\tan(e+f\,x))^n\,dx$$

```
[In] int((a*sin(e + f*x))^m*(b*tan(e + f*x))^n,x)
[Out] int((a*sin(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

3.175 $\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(3,\frac{5+n}{2},\frac{7+n}{2},-\tan^2(e+fx)\right)(b\tan(e+fx))^{5+n}}{b^5f(5+n)}$$

```
[Out] hypergeom([3, 5/2+1/2*n],[7/2+1/2*n],-tan(f*x+e)^2)*(b*tan(f*x+e))^(5+n)/b^
5/f/(5+n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2671, 371}

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{(b\tan(e+fx))^{n+5}\text{Hypergeometric2F1}\left(3,\frac{n+5}{2},\frac{n+7}{2},-\tan^2(e+fx)\right)}{b^5f(n+5)}$$

```
[In] Int[Sin[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[3, (5 + n)/2, (7 + n)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*x
])^(5 + n))/(b^5*f*(5 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\text{Subst}\left(\int \frac{x^{4+n}}{(b^2+x^2)^3}\,dx, x, b\tan(e+fx)\right)}{f}\\ &= \frac{\text{Hypergeometric2F1}\left(3, \frac{5+n}{2}, \frac{7+n}{2}, -\tan^2(e+fx)\right)(b\tan(e+fx))^{5+n}}{b^5 f(5+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(3, \frac{5+n}{2}, \frac{7+n}{2}, -\tan^2(e+fx)\right)\tan^5(e+fx)(b\tan(e+fx))^n}{f(5+n)}$$

```
[In] Integrate[Sin[e + f*x]^4*(b*Tan[e + f*x])^n,x]
[Out] (Hypergeometric2F1[3, (5 + n)/2, (7 + n)/2, -Tan[e + f*x]^2]*Tan[e + f*x]^5
*(b*Tan[e + f*x])^n)/(f*(5 + n))
```

Maple [F]

$$\int \left(\sin^4(fx+e)\right)(b\tan(fx+e))^n\,dx$$

```
[In] int(sin(f*x+e)^4*(b*tan(f*x+e))^n,x)
[Out] int(sin(f*x+e)^4*(b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \sin(fx+e)^4\,dx$$

```
[In] integrate(sin(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*tan(f*x + e))^n, x)
```

Sympy [F]

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(e+fx))^n \sin^4(e+fx)\,dx$$

```
[In] integrate(sin(f*x+e)**4*(b*tan(f*x+e))**n,x)
[Out] Integral((b*tan(e + f*x))**n*sin(e + f*x)**4, x)
```

Maxima [F]

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \sin(fx+e)^4\,dx$$

```
[In] integrate(sin(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^4, x)
```

Giac [F]

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \sin(fx+e)^4\,dx$$

```
[In] integrate(sin(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \int \sin\left(e+f\,x\right)^4\,\left(b\tan(e+f\,x)\right)^n\,dx$$

```
[In] int(sin(e + f*x)^4*(b*tan(e + f*x))^n,x)
[Out] int(sin(e + f*x)^4*(b*tan(e + f*x))^n, x)
```

3.176 $\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{3+n}{2},\frac{5+n}{2},-\tan^2(e+fx)\right)(b\tan(e+fx))^{3+n}}{b^3f(3+n)}$$

```
[Out] hypergeom([2, 3/2+1/2*n],[5/2+1/2*n],-tan(f*x+e)^2)*(b*tan(f*x+e))^(3+n)/b^
3/f/(3+n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2671, 371}

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{(b\tan(e+fx))^{n+3}\,\text{Hypergeometric2F1}\left(2,\frac{n+3}{2},\frac{n+5}{2},-\tan^2(e+fx)\right)}{b^3f(n+3)}$$

```
[In] Int[Sin[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*x
])^(3 + n))/(b^3*f*(3 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\text{Subst}\left(\int \frac{x^{2+n}}{(b^2+x^2)^2}\,dx, x, b\tan(e+fx)\right)}{f}\\ &= \frac{\text{Hypergeometric2F1}\left(2, \frac{3+n}{2}, \frac{5+n}{2}, -\tan^2(e+fx)\right)(b\tan(e+fx))^{3+n}}{b^3f(3+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(2, \frac{3+n}{2}, \frac{5+n}{2}, -\tan^2(e+fx)\right)\tan^3(e+fx)(b\tan(e+fx))^n}{f(3+n)}$$

```
[In] Integrate[Sin[e + f*x]^2*(b*Tan[e + f*x])^n,x]
[Out] (Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, -Tan[e + f*x]^2]*Tan[e + f*x]^3
*(b*Tan[e + f*x])^n)/(f*(3 + n))
```

Maple [F]

$$\int \left(\sin^2(fx+e)\right)(b\tan(fx+e))^n\,dx$$

```
[In] int(sin(f*x+e)^2*(b*tan(f*x+e))^n,x)
[Out] int(sin(f*x+e)^2*(b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\sin(fx+e)^2\,dx$$

```
[In] integrate(sin(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e))^n, x)
```

Sympy [F]

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(e+fx))^n\sin^2(e+fx)\,dx$$

```
[In] integrate(sin(f*x+e)**2*(b*tan(f*x+e))**n,x)
[Out] Integral((b*tan(e + f*x))**n*sin(e + f*x)**2, x)
```

Maxima [F]

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\sin(fx+e)^2\,dx$$

```
[In] integrate(sin(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^2, x)
```

Giac [F]

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\sin(fx+e)^2\,dx$$

```
[In] integrate(sin(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \int \sin\left(e+f\,x\right)^2\,\left(b\tan(e+f\,x)\right)^n\,dx$$

```
[In] int(sin(e + f*x)^2*(b*tan(e + f*x))^n,x)
[Out] int(sin(e + f*x)^2*(b*tan(e + f*x))^n, x)
```

3.177 $\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = -\frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}$$

```
[Out] -b*(b*tan(f*x+e))^(-1+n)/f/(1-n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2671, 30}

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = -\frac{b(b\tan(e+fx))^{n-1}}{f(1-n)}$$

```
[In] Int[Csc[e + f*x]^2*(b*Tan[e + f*x])^n,x]
[Out] -((b*(b*Tan[e + f*x])^(-1 + n))/(f*(1 - n)))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int x^{-2+n}\,dx, x, b\tan(e+fx)\right)}{f}$$
$$= -\frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{b(b\tan(e+fx))^{-1+n}}{f(-1+n)}$$

```
[In] Integrate[Csc[e + f*x]^2*(b*Tan[e + f*x])^n,x]
[Out] (b*(b*Tan[e + f*x])^(-1 + n))/(f*(-1 + n))
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{\mathrm{e}^{n\ln(b\tan(fx+e))}}{f(-1+n)\tan(fx+e)}$	30
default	$\frac{\mathrm{e}^{n\ln(b\tan(fx+e))}}{f(-1+n)\tan(fx+e)}$	30
risch	Expression too large to display	1750

```
[In] int(csc(f*x+e)^2*(b*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
[Out] 1/f/(-1+n)*exp(n*ln(b*tan(f*x+e)))/tan(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^n\cos(fx+e)}{(fn-f)\sin(fx+e)}$$

```
[In] integrate(csc(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] (b*sin(f*x + e)/cos(f*x + e))^n*cos(f*x + e)/((f*n - f)*sin(f*x + e))
```

Sympy [F]

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan{(e+fx)})^n\csc^2{(e+fx)}\,dx$$

```
[In] integrate(csc(f*x+e)**2*(b*tan(f*x+e))**n,x)
[Out] Integral((b*tan(e + f*x))**n*csc(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx=\frac{b^n\tan{(fx+e)}^n}{f(n-1)\tan{(fx+e)}}$$

```
[In] integrate(csc(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] b^n*tan(f*x + e)^n/(f*(n - 1)*tan(f*x + e))
```

Giac [F]

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan{(fx+e)})^n\csc{(fx+e)}^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx=-\frac{\sin{(2\,e+2\,f\,x)}\,{\left(\frac{b\,\sin(2\,e+2\,f\,x)}{2\,\cos(e+f\,x)^2}\right)}^n}{2\,f\,\left(\cos{(e+f\,x)}^2-1\right)\,(n-1)}$$

```
[In] int((b*tan(e + f*x))^n/sin(e + f*x)^2,x)
[Out] -(sin(2*e + 2*f*x)*((b*sin(2*e + 2*f*x))/(2*cos(e + f*x)^2))^n)/(2*f*(cos(e
 + f*x)^2 - 1)*(n - 1))
```

3.178 $\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx=-\frac{b^3(b\tan(e+fx))^{-3+n}}{f(3-n)}-\frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}$$

```
[Out] -b^3*(b*tan(f*x+e))^(-3+n)/f/(3-n)-b*(b*tan(f*x+e))^(-1+n)/f/(1-n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2671, 14}

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx=-\frac{b^3(b\tan(e+fx))^{n-3}}{f(3-n)}-\frac{b(b\tan(e+fx))^{n-1}}{f(1-n)}$$

```
[In] Int[Csc[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((b^3*(b*Tan[e + f*x])^(-3 + n))/(f*(3 - n))) - (b*(b*Tan[e + f*x])^(-1 +
n))/(f*(1 - n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
```

```
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\text{Subst}\left(\int x^{-4+n}(b^2+x^2)\,dx, x, b\tan(e+fx)\right)}{f}\\ &= \frac{b\text{Subst}\left(\int (b^2x^{-4+n}+x^{-2+n})\,dx, x, b\tan(e+fx)\right)}{f}\\ &= -\frac{b^3(b\tan(e+fx))^{-3+n}}{f(3-n)} - \frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{b(-2+n+\cos(2(e+fx)))\csc^2(e+fx)(b\tan(e+fx))^{-1+n}}{f(-3+n)(-1+n)}$$

```
[In] Integrate[Csc[e + f*x]^4*(b*Tan[e + f*x])^n,x]
[Out] (b*(-2 + n + Cos[2*(e + f*x)])*Csc[e + f*x]^2*(b*Tan[e + f*x])^(-1 + n))/(f
*(-3 + n)*(-1 + n))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 20.03 (sec) , antiderivative size = 5281, normalized size of antiderivative = 99.64

method	result	size
risch	Expression too large to display	5281

```
[In] int(csc(f*x+e)^4*(b*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.62

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{\left(2\cos\left(fx+e\right)^3+(n-3)\cos\left(fx+e\right)\right)\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^n}{\left(fn^2-\left(fn^2-4\,fn+3\,f\right)\cos\left(fx+e\right)^2-4\,fn+3\,f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(csc(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] (2*cos(f*x + e)^3 + (n - 3)*cos(f*x + e))*(b*sin(f*x + e)/cos(f*x + e))^n/(
(f*n^2 - (f*n^2 - 4*f*n + 3*f)*cos(f*x + e)^2 - 4*f*n + 3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = \int \left(b\tan\left(e+fx\right)\right)^n\csc^4\left(e+fx\right)\,dx$$

```
[In] integrate(csc(f*x+e)**4*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((b*tan(e + f*x))**n*csc(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{\frac{b^n\tan(fx+e)^n}{(n-1)\tan(fx+e)}+\frac{b^n\tan(fx+e)^n}{(n-3)\tan(fx+e)^3}}{f}$$

```
[In] integrate(csc(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] (b^n*tan(f*x + e)^n/((n - 1)*tan(f*x + e)) + b^n*tan(f*x + e)^n/((n - 3)*ta
n(f*x + e)^3))/f
```

Giac [F]

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)^4\,dx$$

[In] integrate(csc(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="giac")

[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^4, x)

Mupad [B] (verification not implemented)

Time = 4.74 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.60

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx =$$
$$-\frac{2\left(-\frac{b\sin(2e+2fx)}{2\sin(e+fx)^2-2}\right)^n(9\sin(2e+2fx)-6\sin(4e+4fx)+\sin(6e+6fx)-4n\sin(2e+2fx)+2}{f\left(30\sin(e+fx)^2-12\sin(2e+2fx)^2+2\sin(3e+3fx)^2\right)(n^2-4n+3)}$$

[In] int((b*tan(e + f*x))^n/sin(e + f*x)^4,x)

[Out] -(2*(-(b*sin(2*e + 2*f*x))/(2*sin(e + f*x)^2 - 2))^n*(9*sin(2*e + 2*f*x) - 6*sin(4*e + 4*f*x) + sin(6*e + 6*f*x) - 4*n*sin(2*e + 2*f*x) + 2*n*sin(4*e + 4*f*x)))/(f*(2*sin(3*e + 3*f*x)^2 - 12*sin(2*e + 2*f*x)^2 + 30*sin(e + f*x)^2)*(n^2 - 4*n + 3))

3.179 $\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = -\frac{b^5(b\tan(e+fx))^{-5+n}}{f(5-n)} - \frac{2b^3(b\tan(e+fx))^{-3+n}}{f(3-n)} - \frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}$$

```
[Out] -b^5*(b*tan(f*x+e))^(-5+n)/f/(5-n)-2*b^3*(b*tan(f*x+e))^(-3+n)/f/(3-n)-b*(b
*tan(f*x+e))^(-1+n)/f/(1-n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2671, 276}

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = -\frac{b^5(b\tan(e+fx))^{n-5}}{f(5-n)} - \frac{2b^3(b\tan(e+fx))^{n-3}}{f(3-n)} - \frac{b(b\tan(e+fx))^{n-1}}{f(1-n)}$$

```
[In] Int[Csc[e + f*x]^6*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((b^5*(b*Tan[e + f*x])^(-5 + n))/(f*(5 - n))) - (2*b^3*(b*Tan[e + f*x])^(-
3 + n))/(f*(3 - n)) - (b*(b*Tan[e + f*x])^(-1 + n))/(f*(1 - n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int x^{-6+n}\left(b^2+x^2\right)^2\,dx,x,b\tan(e+fx)\right)}{f}\\
&= \frac{b\text{Subst}\left(\int \left(b^4x^{-6+n}+2b^2x^{-4+n}+x^{-2+n}\right)\,dx,x,b\tan(e+fx)\right)}{f}\\
&= -\frac{b^5(b\tan(e+fx))^{-5+n}}{f(5-n)}-\frac{2b^3(b\tan(e+fx))^{-3+n}}{f(3-n)}-\frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \frac{b\left(8-6n+n^2+2(-3+n)\cos(2(e+fx))+\cos(4(e+fx))\right)\csc^4(e+fx)(b\tan(e+fx))^{-1+n}}{f(-5+n)(-3+n)(-1+n)}$$

```
[In] Integrate[Csc[e + f*x]^6*(b*Tan[e + f*x])^n,x]
[Out] (b*(8 - 6*n + n^2 + 2*(-3 + n)*Cos[2*(e + f*x)] + Cos[4*(e + f*x)])*Csc[e +
 f*x]^4*(b*Tan[e + f*x])^(-1 + n))/(f*(-5 + n)*(-3 + n)*(-1 + n))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 151.82 (sec) , antiderivative size = 10580, normalized size of antiderivative = 132.25

method	result	size
risch	Expression too large to display	10580

```
[In] int(csc(f*x+e)^6*(b*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

[Out] result too large to display

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.80

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \frac{\left(8\cos(fx+e)^5+4(n-5)\cos(fx+e)^3+(n^2-8n+15)\cos(fx+e)\right)\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^n}{\left((fn^3-9fn^2+23fn-15f)\cos(fx+e)^4+fn^3-9fn^2-2(fn^3-9fn^2+23fn-15f)\cos(fx+e)^2+23fn-15f\right)\sin(fx+e)}$$

```
[In] integrate(csc(f*x+e)^6*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] (8*cos(f*x + e)^5 + 4*(n - 5)*cos(f*x + e)^3 + (n^2 - 8*n + 15)*cos(f*x + e
))*(b*sin(f*x + e)/cos(f*x + e))^n/(((f*n^3 - 9*f*n^2 + 23*f*n - 15*f)*cos(
f*x + e)^4 + f*n^3 - 9*f*n^2 - 2*(f*n^3 - 9*f*n^2 + 23*f*n - 15*f)*cos(f*x
+ e)^2 + 23*f*n - 15*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**6*(b*tan(f*x+e))**n,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \frac{\frac{b^n\tan(fx+e)^n}{(n-1)\tan(fx+e)}+\frac{2\,b^n\tan(fx+e)^n}{(n-3)\tan(fx+e)^3}+\frac{b^n\tan(fx+e)^n}{(n-5)\tan(fx+e)^5}}{f}$$

```
[In] integrate(csc(f*x+e)^6*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] (b^n*tan(f*x + e)^n/((n - 1)*tan(f*x + e)) + 2*b^n*tan(f*x + e)^n/((n - 3)*
tan(f*x + e)^3) + b^n*tan(f*x + e)^n/((n - 5)*tan(f*x + e)^5))/f
```

Giac [F]

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)^6\,dx$$

```
[In] integrate(csc(f*x+e)^6*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \int \frac{(b\tan(e+f\,x))^n}{\sin(e+f\,x)^6}\,dx$$

```
[In] int((b*tan(e + f*x))^n/sin(e + f*x)^6,x)
[Out] int((b*tan(e + f*x))^n/sin(e + f*x)^6, x)
```

3.180 $\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{4+n}{2},\frac{6+n}{2},\sin^2(e+fx)\right)\sin^3(e+fx)(b\tan(e+fx))^{1+n}}{bf(4+n)}$$

```
[Out] (cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([2+1/2*n, 1/2+1/2*n],[3+1/2*n],sin(f*x
+e)^2)*sin(f*x+e)^3*(b*tan(f*x+e))^(1+n)/b/f/(4+n)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2682, 2657}

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \frac{\sin^3(e+fx)\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+4}{2},\frac{n+6}{2},\sin^2(e+fx)\right)}{bf(n+4)}$$

```
[In] Int[Sin[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (4 + n)/2, (6 +
n)/2, Sin[e + f*x]^2]*Sin[e + f*x]^3*(b*Tan[e + f*x])^(1 + n))/(b*f*(4 + n)
)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

integral
$$= \frac{\left(\cos^{1+n}(e+fx)\sin^{-1-n}(e+fx)(b\tan(e+fx))^{1+n}\right)\int\cos^{-n}(e+fx)\sin^{3+n}(e+fx)\,dx}{b}$$
$$= \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{4+n}{2},\frac{6+n}{2},\sin^2(e+fx)\right)\sin^3(e+fx)(b\tan(e+fx))^{1+n}}{bf(4+n)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.30 (sec) , antiderivative size = 456, normalized size of antiderivative = 5.85

$$\int\sin^3(e+fx)(b\tan(e+fx))^n\,dx$$
$$= \frac{}{f(2+n)\left(-2(4+n)\,\text{AppellF1}\left(1+\frac{n}{2},n,4,2+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx)\right)\right.}$$

```
[In] Integrate[Sin[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

```
[Out] (4*(4 + n)*(AppellF1[1 + n/2, n, 3, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] - AppellF1[1 + n/2, n, 4, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2])*Cos[(e + f*x)/2]^3*Sin[(e + f*x)/2]*Sin[e + f*x]^3*(b*Tan[e +
f*x])^n)/(f*(2 + n)*(-2*(4 + n)*AppellF1[1 + n/2, n, 4, 2 + n/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(3*AppellF1[2 + n/2,
n, 4, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[2 + n/
2, n, 5, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*(-AppellF1[2
 + n/2, 1 + n, 3, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + Appel
lF1[2 + n/2, 1 + n, 4, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*
(-1 + Cos[e + f*x]) + (4 + n)*AppellF1[1 + n/2, n, 3, 2 + n/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])))
```

Maple [F]

$$\int \left(\sin^3\left(fx+e\right)\right)\left(b\tan\left(fx+e\right)\right)^n dx$$

```
[In] int(sin(f*x+e)^3*(b*tan(f*x+e))^n,x)
[Out] int(sin(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx=\int \left(b\tan\left(fx+e\right)\right)^n\sin\left(fx+e\right)^3\,dx$$

```
[In] integrate(sin(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e))^n*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx=\text{Timed out}$$

```
[In] integrate(sin(f*x+e)**3*(b*tan(f*x+e))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx=\int \left(b\tan\left(fx+e\right)\right)^n\sin\left(fx+e\right)^3\,dx$$

```
[In] integrate(sin(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^3, x)
```

Giac [F]

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \sin(fx+e)^3\,dx$$

```
[In] integrate(sin(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \int \sin(e+f\,x)^3\,(b\tan(e+f\,x))^n\,dx$$

```
[In] int(sin(e + f*x)^3*(b*tan(e + f*x))^n,x)
[Out] int(sin(e + f*x)^3*(b*tan(e + f*x))^n, x)
```

3.181 $\int \sin(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx = \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)\sin(e+fx)(b\tan(e+fx))^{1+n}}{bf(2+n)}$$

```
[Out] (cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[2+1/2*n],sin(f*x
+e)^2)*sin(f*x+e)*(b*tan(f*x+e))^(1+n)/b/f/(2+n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2682, 2657}

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx = \frac{\sin(e+fx)\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+4}{2},\sin^2(e+fx)\right)}{bf(n+2)}$$

```
[In] Int[Sin[e + f*x]*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n)/2, (4 +
n)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*Tan[e + f*x])^(1 + n))/(b*f*(2 + n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{\left(\cos^{1+n}(e+fx)\sin^{-1-n}(e+fx)(b\tan(e+fx))^{1+n}\right)\int\cos^{-n}(e+fx)\sin^{1+n}(e+fx)\,dx}{b}$$

$$= \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)\sin(e+fx)(b\tan(e+fx))^{1+n}}{bf(2+n)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.58 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.32

$$\int\sin(e+fx)(b\tan(e+fx))^n\,dx$$

$$= \frac{8(4+n)\,\text{AppellF1}\left(1+\frac{n}{2},n,2,\right.}{f(2+n)\left(2\left(2\,\text{AppellF1}\left(2+\frac{n}{2},n,3,3+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-n\,\text{AppellF1}\left(2+\frac{n}{2},1\right.\right.\right.}$$

```
[In] Integrate[Sin[e + f*x]*(b*Tan[e + f*x])^n,x]
```

```
[Out] (8*(4 + n)*AppellF1[1 + n/2, n, 2, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Cos[(e + f*x)/2]^4*Sin[(e + f*x)/2]^2*(b*Tan[e + f*x])^n)/(f*(2 +
 n)*(2*(2*AppellF1[2 + n/2, n, 3, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] - n*AppellF1[2 + n/2, 1 + n, 2, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2])*(-1 + Cos[e + f*x]) + (4 + n)*AppellF1[1 + n/2, n, 2, 2 +
n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])))
```

Maple [F]

$$\int \sin(fx+e)\left(b\tan(fx+e)\right)^n\,dx$$

```
[In] int(sin(f*x+e)*(b*tan(f*x+e))^n,x)
[Out] int(sin(f*x+e)*(b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx = \int \left(b\tan(fx+e)\right)^n\sin(fx+e)\,dx$$

```
[In] integrate(sin(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e))^n*sin(f*x + e), x)
```

Sympy [F]

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx = \int \left(b\tan(e+fx)\right)^n\sin(e+fx)\,dx$$

```
[In] integrate(sin(f*x+e)*(b*tan(f*x+e))**n,x)
[Out] Integral((b*tan(e + f*x))**n*sin(e + f*x), x)
```

Maxima [F]

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx = \int \left(b\tan(fx+e)\right)^n\sin(fx+e)\,dx$$

```
[In] integrate(sin(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e), x)
```

Giac [F]

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \sin(fx+e)\,dx$$

```
[In] integrate(sin(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx = \int \sin(e+f\,x)\,\left(b\tan(e+f\,x)\right)^n\,dx$$

```
[In] int(sin(e + f*x)*(b*tan(e + f*x))^n,x)
[Out] int(sin(e + f*x)*(b*tan(e + f*x))^n, x)
```

3.182 $\int \csc(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{2-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

```
[Out] -cos(f*x+e)*hypergeom([1-1/2*n, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*tan
(f*x+e))^n/f/(1-n)/((sin(f*x+e)^2)^(1/2*n))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2681, 2656}

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = -\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{2-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

```
[In] Int[Csc[e + f*x]*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (2 - n)/2, (3 - n)/2, Cos[e +
f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\text{integral} = \left(\cos^n(e+fx)\sin^{-n}(e+fx)(b\tan(e+fx))^n\right)\int \cos^{-n}(e+fx)\sin^{-1+n}(e+fx)\,dx$$
$$= -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{2-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{n}{2},n,1+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^n(b\tan(e+fx))^n}{fn}$$

```
[In] Integrate[Csc[e + f*x]*(b*Tan[e + f*x])^n,x]
[Out] (Hypergeometric2F1[n/2, n, 1 + n/2, Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(
e + f*x)/2]^2)^n*(b*Tan[e + f*x])^n)/(f*n)
```

Maple [F]

$$\int \csc(fx+e)\,(b\tan(fx+e))^n\,dx$$

```
[In] int(csc(f*x+e)*(b*tan(f*x+e))^n,x)
[Out] int(csc(f*x+e)*(b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(fx+e))^n\csc(fx+e)\;dx$$

```
[In] integrate(csc(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e))^n*csc(f*x + e), x)
```

Sympy [F]

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(e+fx))^n\csc(e+fx)\,dx$$

```
[In] integrate(csc(f*x+e)*(b*tan(f*x+e))**n,x)
[Out] Integral((b*tan(e + f*x))**n*csc(e + f*x), x)
```

Maxima [F]

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(fx+e))^n\csc(fx+e)\;dx$$

```
[In] integrate(csc(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e), x)
```

Giac [F]

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(fx+e))^n\csc(fx+e)\;dx$$

```
[In] integrate(csc(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = \int \frac{(b\tan(e+f\,x))^n}{\sin\left(e+f\,x\right)}\,dx$$

```
[In] int((b*tan(e + f*x))^n/sin(e + f*x),x)
[Out] int((b*tan(e + f*x))^n/sin(e + f*x), x)
```

3.183 $\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{4-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

```
[Out] -cos(f*x+e)*hypergeom([2-1/2*n, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*tan
(f*x+e))^n/f/(1-n)/((sin(f*x+e)^2)^(1/2*n))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2681, 2656}

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = -\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{4-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

```
[In] Int[Csc[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (4 - n)/2, (3 - n)/2, Cos[e +
f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\text{integral} = \left(\cos^n(e+fx)\sin^{-n}(e+fx)(b\tan(e+fx))^n\right)\int \cos^{-n}(e+fx)\sin^{-3+n}(e+fx)\,dx$$

$$= -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{4-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 13.22 (sec) , antiderivative size = 743, normalized size of antiderivative = 9.53

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx$$

$$= \frac{\cot^2\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(-1+\frac{n}{2},n,\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^n(b\,\mathrm{t}}{f(-8+4n)}$$
$$+\frac{(4+n)\,\text{AppellF1}}{4f(2+n)\left(2\left(\text{AppellF1}\left(2+\frac{n}{2},n,2,3+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-n\,\text{AppellF1}\left(2+\frac{n}{2},\right.\right.\right.}$$
$$+\frac{\text{Hypergeometric2F1}\left(1+\frac{n}{2},n,2+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^n\tan^2\left(\frac{1}{2}(e+fx)\right.}{f(8+4n)}$$
$$+\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\left((2+n)\,\text{Hypergeometric2F1}\left(\frac{n}{2},n,1+\frac{n}{2},\mathrm{t}\right.\right.}{fn(2+n)\left(-8\,\text{AppellF1}\left(1+\frac{n}{2},n,1,2+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\tan\left(\frac{1}{2}(e+fx)\right)+\frac{8}{}\right.}$$

```
[In] Integrate[Csc[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Cot[(e + f*x)/2]^2*Hypergeometric2F1[-1 + n/2, n, n/2, Tan[(e + f*x)/2]^2]
*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(b*Tan[e + f*x])^n)/(f*(-8 + 4*n)) + (
(4 + n)*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sin[e + f*x]^2*(b*Tan[e + f*x])^n)/(4*f*(2 + n)*(2*(AppellF1[2 + n/2
```

```
, n, 2, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[2 +
n/2, 1 + n, 1, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos
[e + f*x]) + (4 + n)*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]))) + (Hypergeometric2F1[1 + n/2, n, 2
 + n/2, Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*
x)/2]^2*(b*Tan[e + f*x])^n)/(f*(8 + 4*n)) + (4*Cos[(e + f*x)/2]^2*Cot[(e +
f*x)/2]*((2 + n)*Hypergeometric2F1[n/2, n, 1 + n/2, Tan[(e + f*x)/2]^2] - n
*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
Tan[(e + f*x)/2]^2)*(b*Tan[e + f*x])^n)/(f*n*(2 + n)*(-8*AppellF1[1 + n/2,
n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2] +
(8*(2*AppellF1[2 + n/2, n, 2, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] - 2*n*AppellF1[2 + n/2, 1 + n, 1, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + ((4 + n)*Cot[(e + f*x)/2]^4)/(Cos[e + f*x]*Sec[(e + f*x)/2]
^2)^n)*Tan[(e + f*x)/2]^3)/(4 + n)))
```

Maple [F]

$$\int \left(\csc^3(fx+e)\right)(b\tan(fx+e))^n\,dx$$

```
[In] int(csc(f*x+e)^3*(b*tan(f*x+e))^n,x)
[Out] int(csc(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(fx+e))^n\csc(fx+e)^3\,dx$$

```
[In] integrate(csc(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e))^n*csc(f*x + e)^3, x)
```

Sympy [F]

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(e+fx))^n\csc^3(e+fx)\,dx$$

```
[In] integrate(csc(f*x+e)**3*(b*tan(f*x+e))**n,x)
[Out] Integral((b*tan(e + f*x))**n*csc(e + f*x)**3, x)
```

Maxima [F]

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\csc(fx+e)^3\,dx$$

```
[In] integrate(csc(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^3, x)
```

Giac [F]

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\csc(fx+e)^3\,dx$$

```
[In] integrate(csc(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = \int \frac{(b\tan(e+f\,x))^n}{\sin(e+f\,x)^3}\,dx$$

```
[In] int((b*tan(e + f*x))^n/sin(e + f*x)^3,x)
[Out] int((b*tan(e + f*x))^n/sin(e + f*x)^3, x)
```

3.184 $\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{6-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

```
[Out] -cos(f*x+e)*hypergeom([3-1/2*n, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*tan
(f*x+e))^n/f/(1-n)/((sin(f*x+e)^2)^(1/2*n))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2681, 2656}

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = -\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{6-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

```
[In] Int[Csc[e + f*x]^5*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (6 - n)/2, (3 - n)/2, Cos[e +
f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\text{integral} = \left(\cos^n(e+fx)\sin^{-n}(e+fx)(b\tan(e+fx))^n\right)\int\cos^{-n}(e+fx)\sin^{-5+n}(e+fx)\,dx$$
$$= -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{6-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 14.17 (sec) , antiderivative size = 1017, normalized size of antiderivative = 13.04

$$\int\csc^5(e+fx)(b\tan(e+fx))^n\,dx$$
$$= \frac{3\cot^2\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(-1+\frac{n}{2},n,\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^n(b}{16f(-2+n)}$$
$$+\frac{\cot^2\left(\frac{1}{2}(e+fx)\right)\left((-2+n)\cot^2\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(-2+\frac{n}{2},n,-1+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right.}{16f(}$$
$$+\frac{3(4+n)\,\text{AppellF1}}{16f(2+n)\left(2\left(\text{AppellF1}\left(2+\frac{n}{2},n,2,3+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-n\,\text{AppellF1}\left(2+\frac{n}{2}\right.\right.\right.}$$
$$+\frac{3\,\text{Hypergeometric2F1}\left(1+\frac{n}{2},n,2+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^n\tan^2\left(\frac{1}{2}(e+f\right.}{16f(2+n)}$$
$$+\frac{\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^n\tan^2\left(\frac{1}{2}(e+fx)\right)\left((4+n)\,\text{Hypergeometric2F1}\left(1+\frac{n}{2},n,2+\frac{n}{2},\tan^2\left(\frac{1}{2}\right.\right.\right.}{16f(2}$$
$$+\frac{3\cos^2\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\left((2+n)\,\text{Hypergeometric2F1}\left(\frac{n}{2},n,1+\frac{n}{2},\right.\right.}{fn(2+n)\left(-8\,\text{AppellF1}\left(1+\frac{n}{2},n,1,2+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\tan\left(\frac{1}{2}(e+fx)\right)+\frac{8}{}\right.}$$

```
[In] Integrate[Csc[e + f*x]^5*(b*Tan[e + f*x])^n,x]
```

[Out] (3*Cot[(e + f*x)/2]^2*Hypergeometric2F1[-1 + n/2, n, n/2, Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(b*Tan[e + f*x])^n)/(16*f*(-2 + n)) + (Cot[(e + f*x)/2]^2*((-2 + n)*Cot[(e + f*x)/2]^2*Hypergeometric2F1[-2 + n/2, n, -1 + n/2, Tan[(e + f*x)/2]^2] + (-4 + n)*Hypergeometric2F1[-1 + n/2, n, n/2, Tan[(e + f*x)/2]^2])*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(b*Tan[e + f*x])^n)/(16*f*(-4 + n)*(-2 + n)) + (3*(4 + n)*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]^2*(b*Tan[e + f*x])^n)/(16*f*(2 + n)*(2*(AppellF1[2 + n/2, n, 2, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[2 + n/2, 1 + n, 1, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x]) + (4 + n)*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]))) + (3*Hypergeometric2F1[1 + n/2, n, 2 + n/2, Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^2*(b*Tan[e + f*x])^n)/(16*f*(2 + n)) + ((Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^2*((4 + n)*Hypergeometric2F1[1 + n/2, n, 2 + n/2, Tan[(e + f*x)/2]^2] + (2 + n)*Hypergeometric2F1[2 + n/2, n, 3 + n/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)*(b*Tan[e + f*x])^n)/(16*f*(2 + n)*(4 + n)) + (3*Cos[(e + f*x)/2]^2*Cot[(e + f*x)/2]*((2 + n)*Hypergeometric2F1[n/2, n, 1 + n/2, Tan[(e + f*x)/2]^2] - n*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)*(b*Tan[e + f*x])^n)/(f*n*(2 + n)*(-8*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2] + (8*(2*AppellF1[2 + n/2, n, 2, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*n*AppellF1[2 + n/2, 1 + n, 1, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + ((4 + n)*Cot[(e + f*x)/2]^4)/(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n)*Tan[(e + f*x)/2]^3)/(4 + n)))

Maple [F]

$$\int \left(\csc^5(fx+e)\right)(b\tan(fx+e))^n\,dx$$

[In] int(csc(f*x+e)^5*(b*tan(f*x+e))^n,x)

[Out] int(csc(f*x+e)^5*(b*tan(f*x+e))^n,x)

Fricas [F]

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\csc(fx+e)^5\,dx$$

[In] integrate(csc(f*x+e)^5*(b*tan(f*x+e))^n,x, algorithm="fricas")

[Out] integral((b*tan(f*x + e))^n*csc(f*x + e)^5, x)

Sympy [F]

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(e+fx))^n \csc^5(e+fx)\,dx$$

```
[In] integrate(csc(f*x+e)**5*(b*tan(f*x+e))**n,x)
[Out] Integral((b*tan(e + f*x))**n*csc(e + f*x)**5, x)
```

Maxima [F]

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)^5\,dx$$

```
[In] integrate(csc(f*x+e)^5*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^5, x)
```

Giac [F]

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)^5\,dx$$

```
[In] integrate(csc(f*x+e)^5*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \int \frac{(b\tan(e+fx))^n}{\sin(e+fx)^5}\,dx$$

```
[In] int((b*tan(e + f*x))^n/sin(e + f*x)^5,x)
[Out] int((b*tan(e + f*x))^n/sin(e + f*x)^5, x)
```

3.185 $\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\sin^2(e+fx\right.}{bf(5+2n)}$$

```
[Out] 2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1/2+1/2*n, 5/4+1/2*n],[9/4+1/2*n],s
in(f*x+e)^2)*(a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1+n)/b/f/(5+2*n)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2682, 2657}

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \frac{2(a\sin(e+fx))^{3/2}\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n}{\right.}{bf(2n+5)}$$

```
[In] Int[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^n,x]
```

```
[Out] (2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (5 + 2*n)/4, (
9 + 2*n)/4, Sin[e + f*x]^2]*(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(1 + n)
)/(b*f*(5 + 2*n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{(a\cos^{1+n}(e+fx)(a\sin(e+fx))^{-1-n}(b\tan(e+fx))^{1+n})\int\cos^{-n}(e+fx)(a\sin(e+fx))^{\frac{3}{2}+n}\,dx}{b}$$

$$= \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\sin^2(e+fx)\right)(a\sin(e+fx))^{3/2}(}{bf(5+2n)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 32.84 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.34

$$\int(a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \frac{8(9+2n)}{f(5+2n)\left(2(9+2n)\text{AppellF1}\left(\frac{5}{4}+\frac{n}{2},n,\frac{5}{2},\frac{9}{4}+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}\right.\right.\right.}$$

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^n,x]
```

```
[Out] (8*(9 + 2*n)*AppellF1[5/4 + n/2, n, 5/2, 9/4 + n/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Sin[(e + f*x)/2]*(a*Sin[e + f*x])^(3/2
)*(b*Tan[e + f*x])^n)/(f*(5 + 2*n)*(2*(9 + 2*n)*AppellF1[5/4 + n/2, n, 5/2,
 9/4 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2
*(5*AppellF1[9/4 + n/2, n, 7/2, 13/4 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] - 2*n*AppellF1[9/4 + n/2, 1 + n, 5/2, 13/4 + n/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x])))
```

Maple [F]

$$\int \left(\sin \left(f x+e\right) a\right)^{\frac{3}{2}} \left(b \tan \left(f x+e\right)\right)^{n}d x$$

```
[In] int((sin(f*x+e)*a)^(3/2)*(b*tan(f*x+e))^n,x)
[Out] int((sin(f*x+e)*a)^(3/2)*(b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \int \left(a \sin \left(f x + e\right)\right)^{\frac{3}{2}} \left(b \tan \left(f x + e\right)\right)^{n}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n*a*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(3/2)*(b*tan(f*x+e))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \int \left(a \sin \left(f x + e\right)\right)^{\frac{3}{2}} \left(b \tan \left(f x + e\right)\right)^{n}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^n, x)
```

Giac [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \int (a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^n\,dx$$

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \int (a\,\sin(e+f\,x))^{3/2}\,(b\tan(e+f\,x))^n\,dx$$

```
[In] int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^n,x)
[Out] int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^n, x)
```

3.186 $\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\operatorname{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\sin^2(e+fx)\right)\sqrt{a\sin(e+fx)}(b\tan(e+$$
$$bf(3+2n)$$

```
[Out] 2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1/2+1/2*n, 3/4+1/2*n],[7/4+1/2*n],s
in(f*x+e)^2)*(a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1+n)/b/f/(3+2*n)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2682, 2657}

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \frac{2\sqrt{a\sin(e+fx)}\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\operatorname{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{4}(2n+3),\frac{1}{4}(2n+7),\sin^2(\right.}{bf(2n+3)}$$

```
[In] Int[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^n,x]
```

```
[Out] (2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (3 + 2*n)/4, (
7 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(1 + n))/
(b*f*(3 + 2*n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{\left(a \cos^{1+n}(e+fx)(a\sin(e+fx))^{-1-n}(b\tan(e+fx))^{1+n}\right)\int \cos^{-n}(e+fx)(a\sin(e+fx))^{\frac{1}{2}+n}\,dx}{b}$$

$$= \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\sin^2(e+fx)\right)\sqrt{a\sin(e+fx)}(b\tan$$
$$bf(3+2n)$$

Mathematica [A] (verified)

Time = 11.65 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(-1+n)}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\sin^2(e+fx)\right)\sqrt{a\sin(e+fx)}\sin(2(e+$$
$$f(3+2n)$$

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^n,x]
[Out] ((Cos[e + f*x]^2)^((-1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (3 + 2*n)/4, (7
 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*Sin[2*(e + f*x)]*(b*Tan[e +
 f*x])^n)/(f*(3 + 2*n))
```

Maple [F]

$$\int \sqrt{\sin(fx+e)\,a}\,(b\tan(fx+e))^n\,dx$$

```
[In] int((sin(f*x+e)*a)^(1/2)*(b*tan(f*x+e))^n,x)
[Out] int((sin(f*x+e)*a)^(1/2)*(b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \int \sqrt{a\sin\left(fx+e\right)}(b\tan\left(fx+e\right))^n\,dx$$

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n, x)
```

Sympy [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \int \sqrt{a\sin\left(e+fx\right)}(b\tan\left(e+fx\right))^n\,dx$$

```
[In] integrate((a*sin(f*x+e))**(1/2)*(b*tan(f*x+e))**n,x)
[Out] Integral(sqrt(a*sin(e + f*x))*(b*tan(e + f*x))**n, x)
```

Maxima [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \int \sqrt{a\sin\left(fx+e\right)}(b\tan\left(fx+e\right))^n\,dx$$

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n, x)
```

Giac [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \int \sqrt{a\sin\left(fx+e\right)}(b\tan\left(fx+e\right))^n\,dx$$

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \int \sqrt{a\,\sin\left(e+f\,x\right)}\,(b\tan(e+f\,x))^n\,dx$$

```
[In] int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^n,x)
[Out] int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^n, x)
```

3.187 $\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\sin^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1+2n)\sqrt{a\sin(e+fx)}}$$

```
[Out] 2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1/2+1/2*n, 1/4+1/2*n],[5/4+1/2*n],s
in(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1+2*n)/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2682, 2657}

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{4}(2n+1),\frac{1}{4}(2n+5),\sin^2(e+fx)\right)}{bf(2n+1)\sqrt{a\sin(e+fx)}}$$

```
[In] Int[(b*Tan[e + f*x])^n/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + 2*n)/4, (
5 + 2*n)/4, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 + 2*n)*Sqrt[a
*Sin[e + f*x]])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{\left(a\cos^{1+n}(e+fx)(a\sin(e+fx))^{-1-n}(b\tan(e+fx))^{1+n}\right)\int\cos^{-n}(e+fx)(a\sin(e+fx))^{-\frac{1}{2}+n}\,dx}{b}$$

$$= \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\sin^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1+2n)\sqrt{a\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 11.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int\frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(-1+n)}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\sin^2(e+fx)\right)\sin(2(e+fx))(b\tan(e+}{(f+2fn)\sqrt{a\sin(e+fx)}}$$

```
[In] Integrate[(b*Tan[e + f*x])^n/Sqrt[a*Sin[e + f*x]],x]
[Out] ((Cos[e + f*x]^2)^((-1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + 2*n)/4, (5
 + 2*n)/4, Sin[e + f*x]^2]*Sin[2*(e + f*x)]*(b*Tan[e + f*x])^n)/((f + 2*f*n
)*Sqrt[a*Sin[e + f*x]])
```

Maple [F]

$$\int \frac{(b\tan(fx+e))^n}{\sqrt{\sin(fx+e)\,a}}dx$$

```
[In] int((b*tan(f*x+e))^n/(sin(f*x+e)*a)^(1/2),x)
[Out] int((b*tan(f*x+e))^n/(sin(f*x+e)*a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^n}{\sqrt{a\sin(fx+e)}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n/(a*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx$$

```
[In] integrate((b*tan(f*x+e))**n/(a*sin(f*x+e))**(1/2),x)
[Out] Integral((b*tan(e + f*x))**n/sqrt(a*sin(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^n}{\sqrt{a\sin(fx+e)}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^n/sqrt(a*sin(f*x + e)), x)
```

Giac [F]

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^n}{\sqrt{a\sin(fx+e)}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n/sqrt(a*sin(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(e+f\,x))^n}{\sqrt{a\,\sin(e+f\,x)}}\,dx$$

```
[In] int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(1/2),x)
[Out] int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(1/2), x)
```

3.188 $\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\sin^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1-2n)(a\sin(e+fx))^{3/2}}$$

```
[Out] -2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([-1/4+1/2*n, 1/2+1/2*n],[3/4+1/2*n]
,sin(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1-2*n)/(a*sin(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2682, 2657}

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{2\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{4}(2n-1),\frac{1}{4}(2n+3),\sin^2(e+fx)\right)}{bf(1-2n)(a\sin(e+fx))^{3/2}}$$

```
[In] Int[(b*Tan[e + f*x])^n/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (-1 + 2*n)/4,
 (3 + 2*n)/4, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - 2*n)*(a*S
in[e + f*x])^(3/2))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{(a\cos^{1+n}(e+fx)(a\sin(e+fx))^{-1-n}(b\tan(e+fx))^{1+n})\int\cos^{-n}(e+fx)(a\sin(e+fx))^{-\frac{3}{2}+n}\,dx}{b}$$

$$= -\frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\sin^2(e+fx)\right)(b\tan(e+fx)}{bf(1-2n)(a\sin(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 9.65 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int\frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \frac{2b\cos^2(e+fx)^{\frac{1}{2}(-1+n)}\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\sin^2(e+\right.}{a^2f(-1+2n)}$$

```
[In] Integrate[(b*Tan[e + f*x])^n/(a*Sin[e + f*x])^(3/2),x]
[Out] (2*b*(Cos[e + f*x]^2)^((-1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (-1 + 2*n)/
4, (3 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(-1 +
 n))/(a^2*f*(-1 + 2*n))
```

Maple [F]

$$\int\frac{(b\tan(fx+e))^n}{(\sin(fx+e)\,a)^{\frac{3}{2}}}dx$$

```
[In] int((b*tan(f*x+e))^n/(sin(f*x+e)*a)^(3/2),x)
[Out] int((b*tan(f*x+e))^n/(sin(f*x+e)*a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e))^n}{(a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(-sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n/(a^2*cos(f*x + e)^2 - a^2
), x)
```

Sympy [F]

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))**n/(a*sin(f*x+e))**(3/2),x)
[Out] Integral((b*tan(e + f*x))**n/(a*sin(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e))^n}{(a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^n/(a*sin(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e))^n}{(a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n/(a*sin(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(e+f\,x))^n}{(a\,\sin(e+f\,x))^{3/2}}\,dx$$

```
[In] int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(3/2),x)
[Out] int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(3/2), x)
```

3.189 $\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 86

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{(a\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(1-m+n)} \text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{2}(1-m+n),\frac{3+n}{2},\sin^2(e+fx)\right)(b\tan}{bf(1+n)}$$

```
[Out] (a*cos(f*x+e))^m*(cos(f*x+e)^2)^(1/2-1/2*m+1/2*n)*hypergeom([1/2+1/2*n, 1/2
-1/2*m+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1+n)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2683, 2697}

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{(a\cos(e+fx))^m(b\tan(e+fx))^{n+1} \cos^2(e+fx)^{\frac{1}{2}(-m+n+1)} \text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{2}(-m+n+1),\frac{n+3}{2}\right.}{bf(n+1)}$$

```
[In] Int[(a*Cos[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((a*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + n)/2)*Hypergeometric2F1[(1 +
 n)/2, (1 - m + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 + n))
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cos[e + f*x])^FracPart[m]*(Sec[e + f*x]/a)^FracPar
```

```
t[m], Int[(b*Tan[e + f*x])^n/(Sec[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \left((a\cos(e+fx))^m\left(\frac{\sec(e+fx)}{a}\right)^m\right)\int\left(\frac{\sec(e+fx)}{a}\right)^{-m}(b\tan(e+fx))^n\,dx$$

$$= \frac{(a\cos(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(1-m+n)}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{2}(1-m+n),\frac{3+n}{2},\sin^2(e+fx)\right)}{bf(1+n)}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int(a\cos(e+fx))^m(b\tan(e+fx))^n\,dx$$
$$= \frac{(a\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}\tan(e+fx)(b\tan(e}{f(1+n)}$$

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Tan[e + f*x])^n,x]
[Out] ((a*Cos[e + f*x])^m*Hypergeometric2F1[(2 + m)/2, (1 + n)/2, (3 + n)/2, -Tan
[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(b*Tan[e + f*x])^n)/(f*(1
+ n))
```

Maple [F]

$$\int(a\cos(fx+e))^m(b\tan(fx+e))^n\,dx$$

```
[In] int((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x)
[Out] int((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (a\cos(e+fx))^m (b\tan(e+fx))^n \, dx = \int (a\cos(fx+e))^m (b\tan(fx+e))^n \, dx$$

```
[In] integrate((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*cos(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F]

$$\int (a\cos(e+fx))^m (b\tan(e+fx))^n \, dx = \int (a\cos(e+fx))^m (b\tan(e+fx))^n \, dx$$

```
[In] integrate((a*cos(f*x+e))**m*(b*tan(f*x+e))**n,x)
[Out] Integral((a*cos(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Maxima [F]

$$\int (a\cos(e+fx))^m (b\tan(e+fx))^n \, dx = \int (a\cos(fx+e))^m (b\tan(fx+e))^n \, dx$$

```
[In] integrate((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*cos(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Giac [F]

$$\int (a\cos(e+fx))^m (b\tan(e+fx))^n \, dx = \int (a\cos(fx+e))^m (b\tan(fx+e))^n \, dx$$

```
[In] integrate((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*cos(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\,\cos\left(e+f\,x\right))^m\,(b\tan(e+f\,x))^n\,dx$$

```
[In] int((a*cos(e + f*x))^m*(b*tan(e + f*x))^n,x)
[Out] int((a*cos(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

3.190 $\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),-\tan^2(e+fx)\right)(a\tan(e+fx))^{1+m}(b\tan(e+fx))^n}{af(1+m+n)}$$

```
[Out] hypergeom([1, 1/2+1/2*m+1/2*n],[3/2+1/2*m+1/2*n],-tan(f*x+e)^2)*(a*tan(f*x+
e))^(1+m)*(b*tan(f*x+e))^n/a/f/(1+m+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {20, 3557, 371}

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{(a\tan(e+fx))^{m+1}(b\tan(e+fx))^n\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),-\tan^2(e+fx)\right)}{af(m+n+1)}$$

```
[In] Int[(a*Tan[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[e + f*x]^2]*(a*Tan
[e + f*x])^(1 + m)*(b*Tan[e + f*x])^n)/(a*f*(1 + m + n))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
```

```
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left((a\tan(e+fx))^{-n}(b\tan(e+fx))^n\right)\int (a\tan(e+fx))^{m+n}\,dx$$

$$= \frac{(a(a\tan(e+fx))^{-n}(b\tan(e+fx))^n)\,\text{Subst}\left(\int \frac{x^{m+n}}{a^2+x^2}\,dx, x, a\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+m+n), \frac{1}{2}(3+m+n), -\tan^2(e+fx)\right)(a\tan(e+fx))^{1+m}(b\tan(e+}{af(1+m+n)}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+m+n), \frac{1}{2}(3+m+n), -\tan^2(e+fx)\right)\tan(e+fx)(a\tan(e+fx))^m(b\tan(e}{f(1+m+n)}$$

```
[In] Integrate[(a*Tan[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[e + f*x]^2]*Tan[e
+ f*x]*(a*Tan[e + f*x])^m*(b*Tan[e + f*x])^n)/(f*(1 + m + n))
```

Maple [F]

$$\int (a \tan (fx+e))^m (b \tan (fx+e))^n \, dx$$

```
[In] int((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x)
[Out] int((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n \, dx = \int (a \tan (fx+e))^m (b \tan (fx+e))^n \, dx$$

```
[In] integrate((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*tan(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F]

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n \, dx = \int (a \tan (e+fx))^m (b \tan (e+fx))^n \, dx$$

```
[In] integrate((a*tan(f*x+e))**m*(b*tan(f*x+e))**n,x)
[Out] Integral((a*tan(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Maxima [F]

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n \, dx = \int (a \tan (fx+e))^m (b \tan (fx+e))^n \, dx$$

```
[In] integrate((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*tan(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Giac [F]

$$\int (a\tan(e+fx))^m (b\tan(e+fx))^n\,dx = \int (a\tan(fx+e))^m\,(b\tan(fx+e))^n\,dx$$

```
[In] integrate((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*tan(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\tan(e+fx))^m (b\tan(e+fx))^n\,dx = \int (a\tan(e+f\,x))^m\,(b\tan(e+f\,x))^n\,dx$$

```
[In] int((a*tan(e + f*x))^m*(b*tan(e + f*x))^n,x)
[Out] int((a*tan(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

3.191 $\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 232

$$\begin{aligned}\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx &= \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} \\ &\quad - \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} \\ &\quad + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\end{aligned}$$

```
[Out] 2/5*d^3/f/(d*cot(f*x+e))^(5/2)+1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^
(1/2))*d^(1/2)/f*2^(1/2)-1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))
*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))
^(1/2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(
f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)-2*d/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules

used = {16, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx$$
$$= \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} + \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$

```
[In] Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^4,x]
```

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*d^3)/(5*f*(d*Cot[e + f*x])^(5/2)) - (2*d)/(f*Sqrt[d*Cot[e + f*x]]) - (Sq
rt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(
2*Sqrt[2]*f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d
*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = d^4 \int \frac{1}{(d\cot(e+fx))^{7/2}}\,dx$$

$$= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - d^2\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$$

$$= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} + \int \sqrt{d\cot(e+fx)}\,dx$$

$$= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{d\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{(2d)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{d\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} - \frac{d\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}$$

$$= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}$$

$$= \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}$$
$$+ \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}}$$
$$- \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$
$$+ \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.47

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx =$$
$$-\frac{\sqrt{d\cot(e+fx)}\left(-2+10\cot^2(e+fx)+5\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}\cot^{\frac{9}{4}}(e+fx)\right.}{5f}$$

```
[In] Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^4,x]
```

```
[Out] -1/5*(Sqrt[d*Cot[e + f*x]]*(-2 + 10*Cot[e + f*x]^2 + 5*ArcTan[(-Cot[e + f*x
]^2)^(1/4)]*(-Cot[e + f*x])^(1/4)*Cot[e + f*x]^(9/4) + 5*ArcTanh[(-Cot[e +
f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(5/4))*Tan[e + f*x]^3)/f
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. 2(179) = 358.

Time = 17.60 (sec) , antiderivative size = 707, normalized size of antiderivative = 3.05

method	result
default	$-\frac{\csc(fx+e)\sqrt{-\frac{d\left(\csc(fx+e)(1-\cos(fx+e))^2-\sin(fx+e)\right)}{1-\cos(fx+e)}}\,(1-\cos(fx+e))\left(-40\left(\csc^7(fx+e)\right)(1-\cos(fx+e))^7+5\ln\left(\frac{\csc(fx+e)(1-\cos(}{}\right.\right.}{}$

```
[In] int((cot(f*x+e)*d)^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20/f*csc(f*x+e)*(-d/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2-sin(f*x+
e)))^(1/2)*(1-cos(f*x+e))*(-40*csc(f*x+e)^7*(1-cos(f*x+e))^7+5*ln(1/(1-cos(
f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+
e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)+2-2*cos(f*x+e)-sin(f*x+e)))*(csc(f*x+e)^
```

```
3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5/2)-10*arctan(1/(1-cos(f*x+e))*
(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)+1-c
os(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5/2)-5*l
n(-1/(1-cos(f*x+e))*(-csc(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(csc(f*x+e)^
3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-2+2*cos(f*x+e)+sin(f*x+e)))
*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5/2)-10*arctan(1/(1
-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+
e))^(1/2)-1+cos(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+
e))^(5/2)+112*csc(f*x+e)^5*(1-cos(f*x+e))^5-40*csc(f*x+e)^3*(1-cos(f*x+e))^
3)/(csc(f*x+e)*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(1-cos(f*x+e)))^(1/2)/(csc
(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.91

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx = -\frac{5\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d}{\tan(fx+e)}}\right)-5i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(i\,f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d}{\tan(fx+e)}}\right)+5i\,f\left(-\right.}{}$$

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] -1/10*(5*f*(-d^2/f^4)^(1/4)*log(f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/tan(f*x + e
))) - 5*I*f*(-d^2/f^4)^(1/4)*log(I*f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/tan(f*x
+ e))) + 5*I*f*(-d^2/f^4)^(1/4)*log(-I*f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/tan(
f*x + e))) - 5*f*(-d^2/f^4)^(1/4)*log(-f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/tan(
f*x + e))) - 4*(tan(f*x + e)^3 - 5*tan(f*x + e))*sqrt(d/tan(f*x + e)))/f
```

Sympy [F]

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{d\cot{(e+fx)}}\tan^4{(e+fx)}\,dx$$

```
[In] integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*tan(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.89

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx =$$

$$-\frac{d^5\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\cdots\right)}{d^4}\cdots\right)}{20\,f}$$

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
[Out] -1/20*d^5*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*
sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^2 - 5
*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

Giac [F]

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{d\cot(fx+e)}\tan(fx+e)^4\,dx$$

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate(sqrt(d*cot(f*x + e))*tan(f*x + e)^4, x)
```

Mupad [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.42

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx = \frac{\frac{2d^3}{5}-\frac{2d^3}{\tan(e+fx)^2}}{f\left(\frac{d}{\tan(e+fx)}\right)^{5/2}}-\frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{f}+\frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{f}$$

```
[In] int(tan(e + f*x)^4*(d*cot(e + f*x))^(1/2),x)

[Out] ((2*d^3)/5 - (2*d^3)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1/
4)*d^(1/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f + ((-1)^(1/
4)*d^(1/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.192 $\int \sqrt{d \cot(e+fx)} \tan^3(e+fx) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 214

$$\begin{aligned}\int \sqrt{d \cot(e+fx)} \tan^3(e+fx) \, dx &= -\frac{\sqrt{d} \arctan\left(1-\frac{\sqrt{2}\sqrt{d \cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d} \arctan\left(1+\frac{\sqrt{2}\sqrt{d \cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} \\ &\quad + \frac{2d^2}{3f(d \cot(e+fx))^{3/2}} - \frac{\sqrt{d} \log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d \cot(e+fx)}\right)}{2\sqrt{2}f} \\ &\quad + \frac{\sqrt{d} \log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d \cot(e+fx)}\right)}{2\sqrt{2}f}\end{aligned}$$

```
[Out] 2/3*d^2/f/(d*cot(f*x+e))^(3/2)-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^
(1/2))*d^(1/2)/f*2^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))
*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))
^(1/2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(
f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules

used = {16, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx$$
$$= -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f}$$
$$+ \frac{2d^2}{3f(d\cot(e+fx))^{3/2}} - \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$
$$+ \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$

```
[In] Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^3,x]
```

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d^2)/(3*f*(d*Cot[e + f*x])^(3/2)) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Co
t[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (Sqrt[d]*Log[Sq
rt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^3 \int \frac{1}{(d\cot(e+fx))^{5/2}}\,dx \\
&= \frac{2d^2}{3f(d\cot(e+fx))^{3/2}} - d\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx
\end{aligned}
$$

$$=\frac{2d^2}{3f(d\cot(e+fx))^{3/2}}+\frac{d^2\text{Subst}\left(\int\frac{1}{\sqrt{x}(d^2+x^2)}\,dx,x,d\cot(e+fx)\right)}{f}$$

$$=\frac{2d^2}{3f(d\cot(e+fx))^{3/2}}+\frac{(2d^2)\,\text{Subst}\left(\int\frac{1}{d^2+x^4}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{f}$$

$$=\frac{2d^2}{3f(d\cot(e+fx))^{3/2}}+\frac{d\text{Subst}\left(\int\frac{d-x^2}{d^2+x^4}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{f}+\frac{d\text{Subst}\left(\int\frac{d+x^2}{d^2+x^4}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{f}$$

$$=\frac{2d^2}{3f(d\cot(e+fx))^{3/2}}-\frac{\sqrt{d}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}-\frac{\sqrt{d}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}+\frac{d\text{Subst}\left(\int\frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2f}+\frac{d\text{Subst}\left(\int\frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2f}$$

$$=\frac{2d^2}{3f(d\cot(e+fx))^{3/2}}-\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}+\frac{\sqrt{d}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{\sqrt{d}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}$$

$$=-\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{2d^2}{3f(d\cot(e+fx))^{3/2}}-\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.43

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx =$$
$$-\frac{\sqrt{d\cot(e+fx)}\left(-2+3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+fx))^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx}\right.\right.}{3f}$$

```
[In] Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^3,x]
```

```
[Out] -1/3*(Sqrt[d*Cot[e + f*x]]*(-2 + 3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e
+ f*x]^2)^(3/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3/4
))*Tan[e + f*x]^2)/f
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. 2(163) = 326.

Time = 17.70 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.65

method	result
default	$-\frac{(\sec^2(fx+e))(\cos(fx+e)+1)\left(-6\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\cos(fx+e)\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)+\cos(fx+e)-1}{\cos(fx+e)-1}\right)\right.}{}$

```
[In] int((cot(f*x+e)*d)^(1/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/f*sec(f*x+e)^2*(cos(f*x+e)+1)*(-6*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)*cos(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))-6*(-sin(f*x+e)*cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))+3*(-s
in(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*ln(-(cot(f*x+e)*cos
(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-
3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f
*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))-3*(-sin(f*x+e)*cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*
sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+
e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*
x+e)+2)/(cos(f*x+e)-1))+4*2^(1/2)*cos(f*x+e)-4*2^(1/2))*(cot(f*x+e)*d)^(1/2
)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = \frac{4\sqrt{\frac{d}{\tan(fx+e)}}\tan(fx+e)^2 + 3f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}} + \sqrt{\frac{d}{\tan(fx+e)}}\right) + 3if\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(if\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}} + \cdots}{6\cdots}$$

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
[Out] 1/6*(4*sqrt(d/tan(f*x + e))*tan(f*x + e)^2 + 3*f*(-d^2/f^4)^(1/4)*log(f*(-d
^2/f^4)^(1/4) + sqrt(d/tan(f*x + e))) + 3*I*f*(-d^2/f^4)^(1/4)*log(I*f*(-d^
2/f^4)^(1/4) + sqrt(d/tan(f*x + e))) - 3*I*f*(-d^2/f^4)^(1/4)*log(-I*f*(-d^
2/f^4)^(1/4) + sqrt(d/tan(f*x + e))) - 3*f*(-d^2/f^4)^(1/4)*log(-f*(-d^2/f^
4)^(1/4) + sqrt(d/tan(f*x + e))))/f
```

Sympy [F]

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = \int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx$$

```
[In] integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e)**3,x)
[Out] Integral(sqrt(d*cot(e + f*x))*tan(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.89

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = \frac{d^4\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}} - \frac{\sqrt{2}\log\left(\cdots\right.}{\cdots}\cdots}{d^2}\cdots\right.}{12f}$$

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/12*d^4*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2*(d/t
an(f*x + e))^(3/2)))/f
```

Giac [F]

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = \int \sqrt{d\cot(fx+e)}\tan(fx+e)^3\,dx$$

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*tan(f*x + e)^3, x)
```

Mupad [B] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.39

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = \frac{2\,d^2}{3\,f\left(\frac{d}{\tan(e+f\,x)}\right)^{3/2}} - \frac{(-1)^{1/4}\,\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} - \frac{(-1)^{1/4}\,\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

```
[In] int(tan(e + f*x)^3*(d*cot(e + f*x))^(1/2),x)
```

```
[Out] (2*d^2)/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)
*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(
1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

3.193 $\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 210

$$\begin{aligned}\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx &= -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}\\&\quad+\frac{2d}{f\sqrt{d\cot(e+fx)}}+\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\\&\quad-\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\end{aligned}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1
/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)-1/4*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/
2)+2*d/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules

used = {16, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$$
$$= -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} + \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$

```
[In] Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^2,x]
```

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d)/(f*Sqrt[d*Cot[e + f*x]]) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e +
f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d]
+ Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^2 \int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx \\
&= \frac{2d}{f\sqrt{d\cot(e+fx)}} - \int \sqrt{d\cot(e+fx)}\,dx
\end{aligned}
$$

$$= \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{d\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{(2d)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{d\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} + \frac{d\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}$$

$$= \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}$$

$$= -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.38

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$$
$$= \frac{d\left(2+\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}-\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}\right)}{f\sqrt{d\cot(e+fx)}}$$

```
[In] Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^2,x]
```

```
[Out] (d*(2 + ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) - ArcTanh[(
-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4)))/(f*Sqrt[d*Cot[e + f*x]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 655 vs. 2(161) = 322.

Time = 18.90 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.12

method	result
default	$\csc(fx+e)\sqrt{-\frac{d\left(\csc(fx+e)(1-\cos(fx+e))^2-\sin(fx+e)\right)}{1-\cos(fx+e)}}\,(1-\cos(fx+e))\left(\ln\left(\frac{\csc(fx+e)(1-\cos(fx+e))^2+2\sin(fx+e)\sqrt{\left(\csc^3(fx+e)\right)(1-\cos(f\cdots}}{1-\cos(fx\cdots}\right.\right.$

```
[In] int((cot(f*x+e)*d)^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*csc(f*x+e)*(-d/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2-sin(f*x+e)
))^(1/2)*(1-cos(f*x+e))*(ln(1/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2+2
*sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)+2-2
*cos(f*x+e)-sin(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+
e))^(1/2)-2*arctan(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e)
)^3-csc(f*x+e)+cot(f*x+e))^(1/2)+1-cos(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e)
)^3-csc(f*x+e)+cot(f*x+e))^(1/2)-ln(-1/(1-cos(f*x+e))*(-csc(f*x+e)*(1-cos(f
*x+e))^2+2*sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))
^(1/2)-2+2*cos(f*x+e)+sin(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e
)+cot(f*x+e))^(1/2)-2*arctan(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-
cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-1+cos(f*x+e)))*(csc(f*x+e)^3*(1-
cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-8*csc(f*x+e)+8*cot(f*x+e))/(csc(
f*x+e)*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(1-cos(f*x+e)))^(1/2)/(csc(f*x+e)^
3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.95

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$$
$$= \frac{f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d}{\tan(fx+e)}}\right)-i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(i\,f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d}{\tan(fx+e)}}\right)+i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}}{}$$

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
[Out] 1/2*(f*(-d^2/f^4)^(1/4)*log(f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/tan(f*x + e)))
- I*f*(-d^2/f^4)^(1/4)*log(I*f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/tan(f*x + e)))
 + I*f*(-d^2/f^4)^(1/4)*log(-I*f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/tan(f*x + e)
)) - f*(-d^2/f^4)^(1/4)*log(-f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/tan(f*x + e)))
 + 4*sqrt(d/tan(f*x + e))*tan(f*x + e))/f
```

Sympy [F]

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$$

```
[In] integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e)**2,x)
[Out] Integral(sqrt(d*cot(e + f*x))*tan(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$$
$$= \frac{d^3\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\right.}{}}{d^2}\right.}{4f}$$

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/4*d^3*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
+ e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt
(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sqrt(d/
tan(f*x + e))))/f
```

Giac [F]

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{d\cot\left(fx+e\right)}\tan\left(fx+e\right)^2\,dx$$

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*tan(f*x + e)^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.38

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx = \frac{2\,d}{f\,\sqrt{\frac{d}{\tan(e+f\,x)}}} + \frac{(-1)^{1/4}\,\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{f} - \frac{(-1)^{1/4}\,\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{f}$$

```
[In] int(tan(e + f*x)^2*(d*cot(e + f*x))^(1/2),x)
```

```
[Out] (2*d)/(f*(d/tan(e + f*x))^(1/2)) + ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d/
tan(e + f*x))^(1/2))/d^(1/2)))/f - ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4)*(d
/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.194 $\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 192

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)-1/2*ar
ctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1/
2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)-1/4*l
n(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2
)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {16, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$

```
[In] Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x],x]
```

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = d\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$

$$= -\frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{d\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} - \frac{d\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}$$

$$= \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}$$

$$= \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.69

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = \frac{d\sqrt{\cot(e+fx)}\left(2\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-2\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right.\right.}{2\sqrt{2}f\sqrt{d\cot(e+fx)}}$$

```
[In] Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x],x]
```

```
[Out] (d*Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTan[
1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[
e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt[2]
*f*Sqrt[d*Cot[e + f*x]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(145) = 290$.

Time = 14.92 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.33

method	result
default	$-\frac{\sqrt{-\frac{d\left(\csc(fx+e)(1-\cos(fx+e))^2-\sin(fx+e)\right)}{1-\cos(fx+e)}}\left(\ln\left(\frac{\csc(fx+e)(1-\cos(fx+e))^2+2\sin(fx+e)\sqrt{\left(\csc^3(fx+e)\right)(1-\cos(fx+e))^3-\csc(fx+e)+\cot}}{1-\cos(fx+e)}\right.\right.}{}$

```
[In] int((cot(f*x+e)*d)^(1/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*(-d/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2-sin(f*x+e)))^(1/2)*(
ln(1/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(csc(f*x+e)^3
*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)+2-2*cos(f*x+e)-sin(f*x+e)))+
2*arctan(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*
x+e)+cot(f*x+e))^(1/2)+1-cos(f*x+e)))-ln(-1/(1-cos(f*x+e))*(-csc(f*x+e)*(1-
cos(f*x+e))^2+2*sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*
x+e))^(1/2)-2+2*cos(f*x+e)+sin(f*x+e)))+2*arctan(1/(1-cos(f*x+e))*(sin(f*x+
e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-1+cos(f*x+e)
)))/(csc(f*x+e)*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(1-cos(f*x+e)))^(1/2)*(cs
c(f*x+e)-cot(f*x+e))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.81

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = -\frac{1}{2}\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right) - \frac{1}{2}i\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right) + \frac{1}{2}i\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(-i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right) + \frac{1}{2}\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(-f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)$$

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="fricas")
[Out] -1/2*(-d^2/f^4)^(1/4)*log(f*(-d^2/f^4)^(1/4) + sqrt(d/tan(f*x + e))) - 1/2*
I*(-d^2/f^4)^(1/4)*log(I*f*(-d^2/f^4)^(1/4) + sqrt(d/tan(f*x + e))) + 1/2*I
*(-d^2/f^4)^(1/4)*log(-I*f*(-d^2/f^4)^(1/4) + sqrt(d/tan(f*x + e))) + 1/2*(
-d^2/f^4)^(1/4)*log(-f*(-d^2/f^4)^(1/4) + sqrt(d/tan(f*x + e)))
```

Sympy [F]

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = \int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx$$

```
[In] integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e),x)
[Out] Integral(sqrt(d*cot(e + f*x))*tan(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = -\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right.}{d^{\frac{3}{2}}}\right.}{4f}$$

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/4*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
+ e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt
(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

Giac [F]

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = \int \sqrt{d\cot(fx+e)}\tan(fx+e)\,dx$$

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*tan(f*x + e), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.32

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} + \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

```
[In] int(tan(e + f*x)*(d*cot(e + f*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f
 + ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1
i)/f
```

3.195 $\int \sqrt{d \cot(e+fx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 192

$$\int \sqrt{d \cot(e+fx)}\,dx = \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)-1/2*ar
ctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1/
2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)+1/4*l
n(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2
)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used

= {3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \sqrt{d\cot(e+fx)}\,dx = \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$

```
[In] Int[Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

<u>Rule 303</u>

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

<u>Rule 335</u>

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

<u>Rule 631</u>

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f} \\
&= -\frac{(2d)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} \\
&= \frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{d\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}
\end{aligned}$$

$$= -\frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} - \frac{d\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}$$

$$= -\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}$$

$$= \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.37

$$\int \sqrt{d\cot(e+fx)}\,dx = \frac{\left(-\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)+\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\right)\sqrt[4]{-\cot(e+fx)}\sqrt{d\cot(e+fx)}}{f\cot^{\frac{3}{4}}(e+fx)}$$

```
[In] Integrate[Sqrt[d*Cot[e + f*x]],x]
[Out] ((-ArcTan[(-Cot[e + f*x]^2)^(1/4)] + ArcTanh[(-Cot[e + f*x]^2)^(1/4)])*(-Co
t[e + f*x])^(1/4)*Sqrt[d*Cot[e + f*x]])/(f*Cot[e + f*x]^(3/4))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{d\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}{\cot(fx+e)d+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right.}{4f\left(d^{2}\right)^{\frac{1}{4}}}$
default	$-\frac{d\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}{\cot(fx+e)d+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right.}{4f\left(d^{2}\right)^{\frac{1}{4}}}$

```
[In] int((cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*d/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(
1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^
(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*
arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \sqrt{d\cot(e+fx)}\,dx = &-\frac{1}{2}\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d\cos\left(2\,fx+2\,e\right)+d}{\sin\left(2\,fx+2\,e\right)}}\right)\\ &+\frac{1}{2}i\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(i\,f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d\cos\left(2\,fx+2\,e\right)+d}{\sin\left(2\,fx+2\,e\right)}}\right)\\ &-\frac{1}{2}i\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(-i\,f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d\cos\left(2\,fx+2\,e\right)+d}{\sin\left(2\,fx+2\,e\right)}}\right)\\ &+\frac{1}{2}\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(-f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d\cos\left(2\,fx+2\,e\right)+d}{\sin\left(2\,fx+2\,e\right)}}\right)\end{aligned}$$

```
[In] integrate((d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(-d^2/f^4)^(1/4)*log(f^3*(-d^2/f^4)^(3/4) + d*sqrt((d*cos(2*f*x + 2*e)
 + d)/sin(2*f*x + 2*e))) + 1/2*I*(-d^2/f^4)^(1/4)*log(I*f^3*(-d^2/f^4)^(3/4
) + d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - 1/2*I*(-d^2/f^4)^(
1/4)*log(-I*f^3*(-d^2/f^4)^(3/4) + d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*
x + 2*e))) + 1/2*(-d^2/f^4)^(1/4)*log(-f^3*(-d^2/f^4)^(3/4) + d*sqrt((d*cos
(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))
```

Sympy [F]

$$\int \sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot\left(e+fx\right)}\,dx$$

```
[In] integrate((d*cot(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*cot(e + f*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.86

$$\int \sqrt{d\cot(e+fx)}\,dx = -\frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e}}{\sqrt{d}}}{4f}$$

```
[In] integrate((d*cot(f*x+e))^(1/2),x, algorithm="maxima")
[Out] -1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x +
e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*
sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(d
/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(d)
*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/f
```

Giac [F]

$$\int \sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot\left(fx+e\right)}\,dx$$

```
[In] integrate((d*cot(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*cot(f*x + e)), x)
```

Mupad [B] (verification not implemented)

Time = 3.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.26

$$\int \sqrt{d\cot(e+fx)}\,dx = -\frac{(-1)^{1/4}\sqrt{d}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\right)}{f}$$

```
[In] int((d*cot(e + f*x))^(1/2),x)
```

```
[Out] -((-1)^(1/4)*d^(1/2)*(atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)) - a
tanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))))/f
```

3.196 $\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 209

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1
/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)+1/4*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/
2)-2*(d*cot(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} - \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$

```
[In] Int[Cot[e + f*x]*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*Sqrt[d*Cot[e + f*x]])/f - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x]
 - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (Sqrt[d]*Log[Sqrt[d] + Sq
rt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (d\cot(e+fx))^{3/2}\,dx}{d}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} - d\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} + \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} + \frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&\quad + \frac{d\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\\
&\quad - \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\\
&\quad + \frac{d\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}\\
&\quad + \frac{d\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\\
&\quad + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\\
&\quad + \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}\\
&\quad - \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}
\end{aligned}
$$

$$= -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.78

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = -\frac{(d\cot(e+fx))^{3/2}\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}} + 2\sqrt{\cot(e+fx)} + \frac{\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right.}{2\sqrt{2}}\right.}{df\cot^{\frac{3}{2}}(e+fx)}$$

```
[In] Integrate[Cot[e + f*x]*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -(((d*Cot[e + f*x])^(3/2)*(ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] -
 ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] + 2*Sqrt[Cot[e + f*x]] + Lo
g[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2]) - Log[1 + Sqrt
[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2])))/(d*f*Cot[e + f*x]^(3/2
)))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{-2\sqrt{\cot(fx+e)d}+\frac{\left(d^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{}}{}\right.\right.}{4}}{f}$
default	$\frac{-2\sqrt{\cot(fx+e)d}+\frac{\left(d^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{}}{}\right.\right.}{4}}{f}$

```
[In] int(cot(f*x+e)*(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*(cot(f*x+e)*d)^(1/2)+1/4*(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d+(d^2
)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/4)
```

```
*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(c
ot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)
))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.21

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right)+i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right)-i\,f\left(-\right.}{}$$

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(f*(-d^2/f^4)^(1/4)*log(f*(-d^2/f^4)^(1/4) + sqrt((d*cos(2*f*x + 2*e) +
 d)/sin(2*f*x + 2*e))) + I*f*(-d^2/f^4)^(1/4)*log(I*f*(-d^2/f^4)^(1/4) + sq
rt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - I*f*(-d^2/f^4)^(1/4)*log(-
I*f*(-d^2/f^4)^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - f
*(-d^2/f^4)^(1/4)*log(-f*(-d^2/f^4)^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/s
in(2*f*x + 2*e))) - 4*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))/f
```

Sympy [F]

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot(e+fx)}\cot(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*cot(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.85

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+\sqrt{2}\sqrt{d}\log\left(\sqrt{2}\sqrt{d}\right.}{4f}$$

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x + e)))/
f
```

Giac [F]

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot\left(fx+e\right)}\cot\left(fx+e\right)\,dx$$

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*cot(f*x + e), x)
```

Mupad [B] (verification not implemented)

Time = 2.96 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = -\frac{2\sqrt{d\cot\left(e+f\,x\right)}}{f} - \frac{(-1)^{1/4}\,\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} - \frac{(-1)^{1/4}\,\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

```
[In] int(cot(e + f*x)*(d*cot(e + f*x))^(1/2),x)
```

```
[Out] - (2*(d*cot(e + f*x))^(1/2))/f - ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*co
t(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4)*(
d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

3.197 $\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 214

$$\begin{aligned}\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = &-\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}\\ &+\frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{2(d\cot(e+fx))^{3/2}}{3df}\\ &+\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\\ &-\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\end{aligned}$$

```
[Out] -2/3*(d*cot(f*x+e))^(3/2)/d/f-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(
1/2))*d^(1/2)/f*2^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*
d^(1/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^
(1/2))*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f
*x+e))^(1/2))*d^(1/2)/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} - \frac{2(d\cot(e+fx))^{3/2}}{3df} + \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$

```
[In] Int[Cot[e + f*x]^2*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*(d*Cot[e + f*x])^(3/2))/(3*d*f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[
e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt
[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (d\cot(e+fx))^{5/2}\,dx}{d^2}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3df} - \int \sqrt{d\cot(e+fx)}\,dx\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3df} + \frac{d\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3df} + \frac{(2d)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3df} - \frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&\quad + \frac{d\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3df} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\\
&\quad + \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\\
&\quad + \frac{d\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}\\
&\quad + \frac{d\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3df} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\\
&\quad - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\\
&\quad + \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}\\
&\quad - \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}
\end{aligned}
$$

$$= -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{2(d\cot(e+fx))^{3/2}}{3df} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.47

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = -\frac{\sqrt{d\cot(e+fx)}\left(-3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\right.}{3f\cot^{\frac{3}{4}}(e+fx)}$$

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -1/3*(Sqrt[d*Cot[e + f*x]]*(-3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*
x])^(1/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4) + 2*Co
t[e + f*x]^(7/4)))/(f*Cot[e + f*x]^(3/4))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.73

method	result
derivativedivides	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{}}{}\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{fd}$
default	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{}}{}\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{fd}$

```
[In] int(cot(f*x+e)^2*(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/d*(1/3*(cot(f*x+e)*d)^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e
)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d+(d^
2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^
(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^
(1/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.53

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$$
$$= \frac{3\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)\sin\left(2\,fx+2\,e\right)-3i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(i\,f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\cdots}\right.}{}$$

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*f*(-d^2/f^4)^(1/4)*log(f^3*(-d^2/f^4)^(3/4) + d*sqrt((d*cos(2*f*x +
2*e) + d)/sin(2*f*x + 2*e)))*sin(2*f*x + 2*e) - 3*I*f*(-d^2/f^4)^(1/4)*log(
I*f^3*(-d^2/f^4)^(3/4) + d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))
*sin(2*f*x + 2*e) + 3*I*f*(-d^2/f^4)^(1/4)*log(-I*f^3*(-d^2/f^4)^(3/4) + d*
sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))*sin(2*f*x + 2*e) - 3*f*(-d
^2/f^4)^(1/4)*log(-f^3*(-d^2/f^4)^(3/4) + d*sqrt((d*cos(2*f*x + 2*e) + d)/s
in(2*f*x + 2*e)))*sin(2*f*x + 2*e) - 4*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*
f*x + 2*e))*(cos(2*f*x + 2*e) + 1))/(f*sin(2*f*x + 2*e))
```

Sympy [F]

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot\left(e+fx\right)}\cot^2\left(e+fx\right)dx$$

```
[In] integrate(cot(f*x+e)**2*(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*cot(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{3\,d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}\right.}{12\,df}$$

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f*x +
e))^(3/2))/(d*f)
```

Giac [F]

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot(fx+e)}\cot(fx+e)^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*cot(f*x + e)^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{f} - \frac{2\,(d\cot(e+f\,x))^{3/2}}{3\,d\,f} - \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{f}$$

```
[In] int(cot(e + f*x)^2*(d*cot(e + f*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/f -
(2*(d*cot(e + f*x))^(3/2))/(3*d*f) - ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4)*
(d*cot(e + f*x))^(1/2))/d^(1/2)))/f
```

3.198 $\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 231

$$\begin{aligned}\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx = {} & \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} \\ & -\frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} \\ & +\frac{2\sqrt{d\cot(e+fx)}}{f}-\frac{2(d\cot(e+fx))^{5/2}}{5d^2f} \\ & +\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} \\ & -\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\end{aligned}$$

```
[Out] -2/5*(d*cot(f*x+e))^(5/2)/d^2/f+1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d
^(1/2))*d^(1/2)/f*2^(1/2)-1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2)
)*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e)
)^(1/2))*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot
(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)+2*(d*cot(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} + \frac{2\sqrt{d\cot(e+fx)}}{f} + \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$

```
[In] Int[Cot[e + f*x]^3*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*Sqrt[d*Cot[e + f*x]])/f - (2*(d*Cot[e + f*x])^(5/2))/(5*d^2*f) + (Sqrt[d
]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sq
rt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot
[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

`IntegerQ[n]`

Rubi steps

$$\text{integral} = \frac{\int (d\cot(e+fx))^{7/2}\,dx}{d^3}$$

$$= -\frac{2(d\cot(e+fx))^{5/2}}{5d^2f} - \frac{\int (d\cot(e+fx))^{3/2}\,dx}{d}$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} + d\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} - \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} - \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} - \frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{d\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} - \frac{d\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}$$

$$= \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.74

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$$

$$= \frac{\sqrt{d\cot(e+fx)}\left(10\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-10\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+40\sqrt{\cot(}\right.}{}$$

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (Sqrt[d*Cot[e + f*x]]*(10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] -
10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 40*Sqrt[Cot[e + f*x]] -
 8*Cot[e + f*x]^(5/2) + 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[
e + f*x]] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/
(20*f*Sqrt[Cot[e + f*x]])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{5}{2}}}{5}-d^2\sqrt{\cot(fx+e)d}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(f}}{(d^2)}\right.\right.}{8}\right.}{f\,d^2}$
default	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{5}{2}}}{5}-d^2\sqrt{\cot(fx+e)d}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(f}}{(d^2)}\right.\right.}{8}\right.}{f\,d^2}$

```
[In] int(cot(f*x+e)^3*(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/d^2*(1/5*(cot(f*x+e)*d)^(5/2)-d^2*(cot(f*x+e)*d)^(1/2)+1/8*d^2*(d^2)^(
1/4)*2^(1/2)*(ln((cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^
2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2
)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/
(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.47

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx =$$
$$-\frac{5\left(f\cos\left(2\,fx+2\,e\right)-f\right)\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)+5\left(i\,f\cos\left(2\,fx+2\,e\right)-i\,f\right)\left(-\right.}{}$$

```
[In] integrate(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/10*(5*(f*cos(2*f*x + 2*e) - f)*(-d^2/f^4)^(1/4)*log(f*(-d^2/f^4)^(1/4) +
 sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 5*(I*f*cos(2*f*x + 2*e)
 - I*f)*(-d^2/f^4)^(1/4)*log(I*f*(-d^2/f^4)^(1/4) + sqrt((d*cos(2*f*x + 2*e
) + d)/sin(2*f*x + 2*e))) + 5*(-I*f*cos(2*f*x + 2*e) + I*f)*(-d^2/f^4)^(1/4
)*log(-I*f*(-d^2/f^4)^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e
))) - 5*(f*cos(2*f*x + 2*e) - f)*(-d^2/f^4)^(1/4)*log(-f*(-d^2/f^4)^(1/4) +
 sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - 8*sqrt((d*cos(2*f*x + 2
*e) + d)/sin(2*f*x + 2*e))*(3*cos(2*f*x + 2*e) - 2))/(f*cos(2*f*x + 2*e) -
f)
```

Sympy [F]

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot(e+fx)}\cot^3(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)**3*(d*cot(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*cot(e + f*x))*cot(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.86

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx =$$

$$-\frac{10\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+10\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+5\sqrt{2}d^{\frac{5}{2}}\log\left(\sqrt{2}\right.}{}$$

```
[In] integrate(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
[Out] -1/20*(10*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/ta
n(f*x + e)))/sqrt(d)) + 10*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + 5*sqrt(2)*d^(5/2)*log(sqrt(2)*sqr
t(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 5*sqrt(2)*d^(5/2)*log(-sq
rt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 40*d^2*sqrt(d/ta
n(f*x + e)) + 8*(d/tan(f*x + e))^(5/2))/(d^2*f)
```

Giac [F]

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot(fx+e)}\cot(fx+e)^3\,dx$$

```
[In] integrate(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*cot(f*x + e))*cot(f*x + e)^3, x)
```

Mupad [B] (verification not implemented)

Time = 3.78 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.39

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{2\sqrt{d\cot(e+f\,x)}}{f} - \frac{2\,(d\cot(e+f\,x))^{5/2}}{5\,d^2\,f} + \frac{(-1)^{1/4}\,\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} + \frac{(-1)^{1/4}\,\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}\,1\mathrm{i}}{\sqrt{d}}\right)}{f}$$

```
[In] int(cot(e + f*x)^3*(d*cot(e + f*x))^(1/2),x)
```

```
[Out] (2*(d*cot(e + f*x))^(1/2))/f - (2*(d*cot(e + f*x))^(5/2))/(5*d^2*f) + ((-1)
^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f + ((
-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2)*1i)/d^(1/2)))/f
```

3.199 $\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 234

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

```
[Out] 2/5*d^4/f/(d*cot(f*x+e))^(5/2)+1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^
(1/2)/d^(1/2))/f*2^(1/2)-1/2*d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/
d^(1/2))/f*2^(1/2)-1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot
(f*x+e))^(1/2))/f*2^(1/2)+1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)
*(d*cot(f*x+e))^(1/2))/f*2^(1/2)-2*d^2/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} + \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}}$$

```
[In] Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*d^4)/(5*f*(d*Cot[e + f*x])^(5/2)) - (2*d^2)/(f*Sqrt[d*Cot[e + f*x]]) - (
d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])
/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt
[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

`IntegerQ[n]`

Rubi steps

$$\begin{aligned}
\text{integral} &= d^5 \int \frac{1}{(d\cot(e+fx))^{7/2}}\,dx\\
&= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - d^3\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx\\
&= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + d\int \sqrt{d\cot(e+fx)}\,dx\\
&= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - \frac{d^2\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}\\
&= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - \frac{(2d^2)\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{d^2\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&\quad - \frac{d^2\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}}\\
&\quad - \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\\
&\quad - \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\\
&\quad - \frac{d^2\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}\\
&\quad - \frac{d^2\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} \\
&\quad - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} \\
&\quad + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} \\
&\quad - \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} \\
&\quad + \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} \\
&= \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} \\
&\quad + \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} \\
&\quad - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} \\
&\quad + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.47

$$
\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = -\frac{(d\cot(e+fx))^{3/2}\left(-2+10\cot^2(e+fx)+5\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}\cot^{\frac{9}{4}}(e+fx)+\right.}{5f}
$$

```
[In] Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] -1/5*((d*Cot[e + f*x])^(3/2)*(-2 + 10*Cot[e + f*x]^2 + 5*ArcTan[(-Cot[e + f
*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4)*Cot[e + f*x]^(9/4) + 5*ArcTanh[(-Cot[e
+ f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(5/4))*Tan[e + f*x]^4)/f
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(181) = 362$.

Time = 3.41 (sec) , antiderivative size = 632, normalized size of antiderivative = 2.70

method	result
default	$\frac{(\sec^3(fx+e))\csc(fx+e)\left(5(\cos^2(fx+e))\sin(fx+e)\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)+2\sin(fx+e)\sqrt{-(\cot^3(fx+e))+3(\cot^2(fx+e)}}{}\right.\right.}{}$

```
[In] int((cot(f*x+e)*d)^(3/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/20/f*sec(f*x+e)^3*csc(f*x+e)*(5*cos(f*x+e)^2*sin(f*x+e)*ln(-(cot(f*x+e)*c
os(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e
)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos
(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-5*cos(f*x+e)^2*sin(f*x+e)*ln(-(cot(f*x+e)*cos(f*x+e)-
2*cot(f*x+e)-2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*
x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-si
n(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)+10*cos(f*x+e)^2*sin(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-sin(
f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-10*cos(f*x+e)^2*sin(f*x+e)*arctan
((-2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f
*x+e)-1)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+24
*2^(1/2)*cos(f*x+e)^3-24*2^(1/2)*cos(f*x+e)^2-4*2^(1/2)*cos(f*x+e)+4*2^(1/2
))*(cos(f*x+e)+1)*d*(cot(f*x+e)*d)^(1/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.95

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx =$$

$$-\frac{5\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\log\left(d^4\sqrt{\frac{d}{\tan(fx+e)}}+\left(-\frac{d^6}{f^4}\right)^{\frac{3}{4}} f^3\right)-5i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\log\left(d^4\sqrt{\frac{d}{\tan(fx+e)}}+i\left(-\frac{d^6}{f^4}\right)^{\frac{3}{4}} f^3\right)+5i\left(-\right.}{}$$

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/10*(5*(-d^6/f^4)^(1/4)*f*log(d^4*sqrt(d/tan(f*x + e)) + (-d^6/f^4)^(3/4)
*f^3) - 5*I*(-d^6/f^4)^(1/4)*f*log(d^4*sqrt(d/tan(f*x + e)) + I*(-d^6/f^4)^
(3/4)*f^3) + 5*I*(-d^6/f^4)^(1/4)*f*log(d^4*sqrt(d/tan(f*x + e)) - I*(-d^6/
```

```
f^4)^(3/4)*f^3) - 5*(-d^6/f^4)^(1/4)*f*log(d^4*sqrt(d/tan(f*x + e)) - (-d^6
/f^4)^(3/4)*f^3) - 4*(d*tan(f*x + e)^3 - 5*d*tan(f*x + e))*sqrt(d/tan(f*x +
 e)))/f
```

Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\tan^5(e+fx)\,dx$$

```
[In] integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**5,x)
[Out] Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.88

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx =$$

$$-\frac{d^6\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\cdots\right.}{\cdots}\right.}{d^4}\cdots\right.}{20\,f}$$

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x, algorithm="maxima")
[Out] -1/20*d^6*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*
sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^2 - 5
*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

Giac [F]

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\tan(fx+e)^5\,dx$$

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
[Out] integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^5, x)
```

Mupad [B] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.41

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = \frac{\frac{2d^4}{5} - \frac{2d^4}{\tan(e+fx)^2}}{f\left(\frac{d}{\tan(e+fx)}\right)^{5/2}} - \frac{(-1)^{1/4}d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{f} + \frac{(-1)^{1/4}d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{f}$$

```
[In] int(tan(e + f*x)^5*(d*cot(e + f*x))^(3/2),x)
[Out] ((2*d^4)/5 - (2*d^4)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1/
4)*d^(3/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f + ((-1)^(1/
4)*d^(3/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.200 $\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 214

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d^3}{3f(d\cot(e+fx))^{3/2}} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

```
[Out] 2/3*d^3/f/(d*cot(f*x+e))^(3/2)-1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^
(1/2)/d^(1/2))/f*2^(1/2)+1/2*d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/
d^(1/2))/f*2^(1/2)-1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot
(f*x+e))^(1/2))/f*2^(1/2)+1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)
*(d*cot(f*x+e))^(1/2))/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules

used = {16, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{d^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{2d^3}{3f(d\cot(e+fx))^{3/2}}$$

```
[In] Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d^3)/(3*f*(d*Cot[e + f*x])^(3/2)) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Co
t[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sq
rt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^4 \int \frac{1}{(d\cot(e+fx))^{5/2}}\,dx \\
&= \frac{2d^3}{3f(d\cot(e+fx))^{3/2}} - d^2 \int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx
\end{aligned}
$$

$$=\frac{2d^3}{3f(d\cot(e+fx))^{3/2}}+\frac{d^3\text{Subst}\left(\int\frac{1}{\sqrt{x}(d^2+x^2)}\,dx,x,d\cot(e+fx)\right)}{f}$$

$$=\frac{2d^3}{3f(d\cot(e+fx))^{3/2}}+\frac{(2d^3)\,\text{Subst}\left(\int\frac{1}{d^2+x^4}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{f}$$

$$=\frac{2d^3}{3f(d\cot(e+fx))^{3/2}}+\frac{d^2\text{Subst}\left(\int\frac{d-x^2}{d^2+x^4}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{f}+\frac{d^2\text{Subst}\left(\int\frac{d+x^2}{d^2+x^4}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{f}$$

$$=\frac{2d^3}{3f(d\cot(e+fx))^{3/2}}-\frac{d^{3/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}+\frac{d^2\text{Subst}\left(\int\frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2f}+\frac{d^2\text{Subst}\left(\int\frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2f}$$

$$=\frac{2d^3}{3f(d\cot(e+fx))^{3/2}}-\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{3/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}$$

$$=-\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{2d^3}{3f(d\cot(e+fx))^{3/2}}-\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.43

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx =$$

$$-\frac{(d\cot(e+fx))^{3/2}\left(-2+3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+fx))^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right.\right.}{3f}$$

[In] Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^4,x]

[Out] -1/3*((d*Cot[e + f*x])^(3/2)*(-2 + 3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3/4))*Tan[e + f*x]^3)/f

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. 2(163) = 326.

Time = 2.09 (sec) , antiderivative size = 569, normalized size of antiderivative = 2.66

method	result
default	$\frac{(\sec^2(fx+e))\left(6\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\cos(fx+e)\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)+\cos(fx+e)-1}{\cos(fx+e)-1}\right)+6\sqrt{-\frac{\sin(fx+e)\cos(f}{(\cos(fx+e)+}}\right.}{}$

[In] int((cot(f*x+e)*d)^(3/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)

[Out] 1/12/f*sec(f*x+e)^2*(6*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))+6*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))-3*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))+3*ln((2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-cot(f*x+e)*cos(f*x+e)+sin(f*x+e)+2*cos(f*x+e)+2*cot(f*x+e)-csc(f*x+e)-2)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-4*2^(1/2)*cos(f*x+e)+4*2^(1/2))*(cos(f*x+e)+1)*d*(cot(f*x+e)*d)^(1/2)*2^(1/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.92

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \frac{4\,d\sqrt{\frac{d}{\tan(fx+e)}}\tan(fx+e)^2 + 3\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\log\left(d\sqrt{\frac{d}{\tan(fx+e)}} + \left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\right) + 3i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\log\left(d\right.}{}$$

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
[Out] 1/6*(4*d*sqrt(d/tan(f*x + e))*tan(f*x + e)^2 + 3*(-d^6/f^4)^(1/4)*f*log(d*s
qrt(d/tan(f*x + e)) + (-d^6/f^4)^(1/4)*f) + 3*I*(-d^6/f^4)^(1/4)*f*log(d*sq
rt(d/tan(f*x + e)) + I*(-d^6/f^4)^(1/4)*f) - 3*I*(-d^6/f^4)^(1/4)*f*log(d*s
qrt(d/tan(f*x + e)) - I*(-d^6/f^4)^(1/4)*f) - 3*(-d^6/f^4)^(1/4)*f*log(d*sq
rt(d/tan(f*x + e)) - (-d^6/f^4)^(1/4)*f))/f
```

Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\tan^4(e+fx)\,dx$$

```
[In] integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**4,x)
[Out] Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.89

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \frac{d^5\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+}\right.}{d^{\frac{3}{2}}}\right.}{d^2}\right.}{12\,f}$$

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 1/12*d^5*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2*(d/t
an(f*x + e))^(3/2)))/f
```

Giac [F]

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\tan(fx+e)^4\,dx$$

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^4, x)
```

Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.39

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \frac{2\,d^3}{3\,f\,\left(\frac{d}{\tan(e+f\,x)}\right)^{3/2}} - \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} - \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

```
[In] int(tan(e + f*x)^4*(d*cot(e + f*x))^(3/2),x)
```

```
[Out] (2*d^3)/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)
*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(
1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

3.201 $\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 212

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

```
[Out] -1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+1/2*d
^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+1/4*d^(3/2)
*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)-1/4*
d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/
2)+2*d^2/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{d^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{2d^2}{f\sqrt{d\cot(e+fx)}}$$

```
[In] Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^3,x]
```

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d^2)/(f*Sqrt[d*Cot[e + f*x]]) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e
+ f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (d^(3/2)*Log[Sqrt[d
] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^3 \int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx \\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - d\int \sqrt{d\cot(e+fx)}\,dx \\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{d^2\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f} \\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} \\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - \frac{d^2\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} \\
&\quad + \frac{d^2\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} \\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} \\
&\quad + \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} \\
&\quad + \frac{d^2\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} \\
&\quad + \frac{d^2\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} \\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} \\
&\quad - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} \\
&\quad + \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} \\
&\quad - \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}
\end{aligned}
$$

$$= -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.39

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{d^2\left(2+\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}-\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e}\right.}{f\sqrt{d\cot(e+fx)}}$$

```
[In] Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^3,x]
[Out] (d^2*(2 + ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) - ArcTanh
[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4)))/(f*Sqrt[d*Cot[e + f*x]]
)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(163) = 326.

Time = 3.09 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.70

method	result
default	$-\frac{\sec(fx+e)\csc(fx+e)\left(\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)+2\sin(fx+e)\sqrt{-\left(\cot^3(fx+e)\right)+3\left(co\right.}}{}\right.\right.}{}$

```
[In] int((cot(f*x+e)*d)^(3/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
[Out] -1/4/f*sec(f*x+e)*csc(f*x+e)*((-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*sin(f*x+e)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*
x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot
(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)
-1))+2*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)*arctan((2
^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e
```

```
)-1)/(cos(f*x+e)-1))+2*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(
f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(
f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))-sin(f*x+e)*(-sin(f*x+e)*cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*sin(f*x+e)*(
-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e
)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos
(f*x+e)-1))+4*2^(1/2)*cos(f*x+e)-4*2^(1/2))*(cos(f*x+e)+1)*d*(cot(f*x+e)*d)
^(1/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.99

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{4\,d\sqrt{\frac{d}{\tan(fx+e)}}\tan(fx+e) + \left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\log\left(d^4\sqrt{\frac{d}{\tan(fx+e)}} + \left(-\frac{d^6}{f^4}\right)^{\frac{3}{4}} f^3\right) - i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\log\left(d^4\sqrt{\right.}\right.}{}$$

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*d*sqrt(d/tan(f*x + e))*tan(f*x + e) + (-d^6/f^4)^(1/4)*f*log(d^4*sqr
t(d/tan(f*x + e)) + (-d^6/f^4)^(3/4)*f^3) - I*(-d^6/f^4)^(1/4)*f*log(d^4*sq
rt(d/tan(f*x + e)) + I*(-d^6/f^4)^(3/4)*f^3) + I*(-d^6/f^4)^(1/4)*f*log(d^4
*sqrt(d/tan(f*x + e)) - I*(-d^6/f^4)^(3/4)*f^3) - (-d^6/f^4)^(1/4)*f*log(d^
4*sqrt(d/tan(f*x + e)) - (-d^6/f^4)^(3/4)*f^3))/f
```

Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\tan^3(e+fx)\,dx$$

```
[In] integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.89

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{d^4\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}}{d^2}\right.}{4f}$$

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x, algorithm="maxima")
[Out] 1/4*d^4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
+ e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt
(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sqrt(d/
tan(f*x + e))))/f
```

Giac [F]

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\tan(fx+e)^3\,dx$$

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
[Out] integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^3, x)
```

Mupad [B] (verification not implemented)

Time = 3.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.39

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{2\,d^2}{f\,\sqrt{\frac{d}{\tan(e+f\,x)}}} + \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{f} - \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{f}$$

```
[In] int(tan(e + f*x)^3*(d*cot(e + f*x))^(3/2),x)
```

```
[Out] (2*d^2)/(f*(d/tan(e + f*x))^(1/2)) + ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(
d/tan(e + f*x))^(1/2))/d^(1/2)))/f - ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)*
(d/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.202 $\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 192

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

```
[Out] 1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/2*d^
(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+1/4*d^(3/2)*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)-1/4*d
^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2
)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

$= \{16, 3557, 335, 217, 1179, 642, 1176, 631, 210\}$

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$

```
[In] Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d^2 \int \frac{1}{\sqrt{d \cot(e+fx)}}\,dx \\
&= -\frac{d^3 \text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f} \\
&= -\frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}
\end{aligned}$$

$$= -\frac{d^2\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{d^2\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^2\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} - \frac{d^2\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}$$

$$= \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}$$

$$= \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.70

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx=\frac{d^2\sqrt{\cot(e+fx)}\left(2\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-2\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+\log\left(1-\dots\right.\right.}{2\sqrt{2}f\sqrt{d\cot(e+\dots}}$$

```
[In] Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (d^2*Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTa
n[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Co
t[e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt[
2]*f*Sqrt[d*Cot[e + f*x]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. 2(145) = 290.

Time = 2.05 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.33

method	result
default	$\frac{\left(\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)+2\sin(fx+e)\sqrt{-\left(\cot^3(fx+e)\right)+3\left(\cot^2(fx+e)\right)\csc(fx+e)-3\left(\csc^2(fx+e)\right)\cot(fx+e)+\csc^3(fx+e)+\dots}}{\cos(fx+e)-1}\right.\right.}{\dots}$

```
[In] int((cot(f*x+e)*d)^(3/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*(ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)^3+
3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)
-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))-2*
arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-
cos(f*x+e)+1)/(cos(f*x+e)-1))-ln((2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^
2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^
(1/2)-cot(f*x+e)*cos(f*x+e)+sin(f*x+e)+2*cos(f*x+e)+2*cot(f*x+e)-csc(f*x+e)
-2)/(cos(f*x+e)-1))-2*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1)))*d*(cot(f*x+e)*d)^(1/2)
/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)-csc(f*x+e))*2^
(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = -\frac{1}{2}\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d\sqrt{\frac{d}{\tan(fx+e)}}+\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\right) - \frac{1}{2}i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d\sqrt{\frac{d}{\tan(fx+e)}}+i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\right) + \frac{1}{2}i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d\sqrt{\frac{d}{\tan(fx+e)}}-i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\right) + \frac{1}{2}\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d\sqrt{\frac{d}{\tan(fx+e)}}-\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\right)$$

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
[Out] -1/2*(-d^6/f^4)^(1/4)*log(d*sqrt(d/tan(f*x + e)) + (-d^6/f^4)^(1/4)*f) - 1/
2*I*(-d^6/f^4)^(1/4)*log(d*sqrt(d/tan(f*x + e)) + I*(-d^6/f^4)^(1/4)*f) + 1
/2*I*(-d^6/f^4)^(1/4)*log(d*sqrt(d/tan(f*x + e)) - I*(-d^6/f^4)^(1/4)*f) +
1/2*(-d^6/f^4)^(1/4)*log(d*sqrt(d/tan(f*x + e)) - (-d^6/f^4)^(1/4)*f)
```

Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\tan^2(e+fx)\,dx$$

```
[In] integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**2,x)
[Out] Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = -\frac{d^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}\right.}{4f}$$

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] -1/4*d^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
+ e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt
(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

Giac [F]

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\tan(fx+e)^2\,dx$$

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.32

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} + \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

```
[In] int(tan(e + f*x)^2*(d*cot(e + f*x))^(3/2),x)
[Out] ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f
 + ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1
i)/f
```

3.203 $\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 192

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

```
[Out] 1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/2*d^
(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/4*d^(3/2)*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)+1/4*d
^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2
)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used

= {16, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$

```
[In] Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x],x]
```

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = d\int \sqrt{d\cot(e+fx)}\,dx$$

$$= -\frac{d^2\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{(2d^2)\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$=\frac{d^2\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{f}-\frac{d^2\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{f}$$

$$=-\frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}-\frac{d^2\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2f}-\frac{d^2\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2f}$$

$$=-\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}$$

$$=\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.38

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = \frac{d\left(-\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)+\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\right)\sqrt[4]{-\cot(e+fx)}\sqrt{d\cot(e+fx)}}{f\cot^{\frac{3}{4}}(e+fx)}$$

[In] Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x],x]

[Out] (d*(-ArcTan[(-Cot[e + f*x]^2)^(1/4)] + ArcTanh[(-Cot[e + f*x]^2)^(1/4)])*(-Cot[e + f*x])^(1/4)*Sqrt[d*Cot[e + f*x]])/(f*Cot[e + f*x]^(3/4))

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)\right)}{4f\left(d^2\right)^{\frac{1}{4}}}$	138

[In] int((cot(f*x+e)*d)^(3/2)*tan(f*x+e),x,method=_RETURNVERBOSE)

[Out] -1/4*d^2/f/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.94

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx =$$
$$-\frac{1}{2}\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d^4\sqrt{\frac{d}{\tan(fx+e)}}+\left(-\frac{d^6}{f^4}\right)^{\frac{3}{4}}f^3\right)$$
$$+\frac{1}{2}i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d^4\sqrt{\frac{d}{\tan(fx+e)}}+i\left(-\frac{d^6}{f^4}\right)^{\frac{3}{4}}f^3\right)$$
$$-\frac{1}{2}i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d^4\sqrt{\frac{d}{\tan(fx+e)}}-i\left(-\frac{d^6}{f^4}\right)^{\frac{3}{4}}f^3\right)$$
$$+\frac{1}{2}\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d^4\sqrt{\frac{d}{\tan(fx+e)}}-\left(-\frac{d^6}{f^4}\right)^{\frac{3}{4}}f^3\right)$$

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e),x, algorithm="fricas")
[Out] -1/2*(-d^6/f^4)^(1/4)*log(d^4*sqrt(d/tan(f*x + e)) + (-d^6/f^4)^(3/4)*f^3)
+ 1/2*I*(-d^6/f^4)^(1/4)*log(d^4*sqrt(d/tan(f*x + e)) + I*(-d^6/f^4)^(3/4)*
f^3) - 1/2*I*(-d^6/f^4)^(1/4)*log(d^4*sqrt(d/tan(f*x + e)) - I*(-d^6/f^4)^(
3/4)*f^3) + 1/2*(-d^6/f^4)^(1/4)*log(d^4*sqrt(d/tan(f*x + e)) - (-d^6/f^4)^
(3/4)*f^3)
```

Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\tan(e+fx)\,dx$$

```
[In] integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e),x)
[Out] Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx =$$

$$-\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}} -\right.}{4f}$$

[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e),x, algorithm="maxima")

[Out] -1/4*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/f

Giac [F]

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\tan(fx+e)\,dx$$

[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e),x, algorithm="giac")

[Out] integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e), x)

Mupad [B] (verification not implemented)

Time = 2.86 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.28

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx =$$

$$-\frac{(-1)^{1/4}\,d^{3/2}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\right)}{f}$$

[In] int(tan(e + f*x)*(d*cot(e + f*x))^(3/2),x)

[Out] -((-1)^(1/4)*d^(3/2)*(atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)) - atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))))/f

3.204 $\int (d\cot(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 210

$$\int (d\cot(e+fx))^{3/2}\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

```
[Out] -1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+1/2*d
^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/4*d^(3/2)
*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)+1/4*
d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/
2)-2*d*(d*cot(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used

= {3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int (d\cot(e+fx))^{3/2}\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} - \frac{2d\sqrt{d\cot(e+fx)}}{f}$$

```
[In] Int[(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*d*Sqrt[d*Cot[e + f*x]])/f - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*
x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sqrt[d] +
Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d\sqrt{d\cot(e+fx)}}{f} - d^2\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx \\
&= -\frac{2d\sqrt{d\cot(e+fx)}}{f} + \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f} \\
&= -\frac{2d\sqrt{d\cot(e+fx)}}{f} + \frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}
\end{aligned}$$

$$=-\frac{2d\sqrt{d\cot(e+fx)}}{f}+\frac{d^2\text{Subst}\left(\int\frac{d-x^2}{d^2+x^4}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{f}+\frac{d^2\text{Subst}\left(\int\frac{d+x^2}{d^2+x^4}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{f}$$

$$=-\frac{2d\sqrt{d\cot(e+fx)}}{f}-\frac{d^{3/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}+\frac{d^2\text{Subst}\left(\int\frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2f}+\frac{d^2\text{Subst}\left(\int\frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2f}$$

$$=-\frac{2d\sqrt{d\cot(e+fx)}}{f}-\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{3/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}$$

$$=-\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}-\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.77

$$\int (d\cot(e+fx))^{3/2}\,dx =$$

$$-\frac{(d\cot(e+fx))^{3/2}\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}+2\sqrt{\cot(e+fx)}+\frac{\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}+c\right.}{2\sqrt{2}}\right.}{f\cot^{\frac{3}{2}}(e+fx)}$$

```
[In] Integrate[(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -(((d*Cot[e + f*x])^(3/2)*(ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] -
 ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] + 2*Sqrt[Cot[e + f*x]] + Lo
g[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2]) - Log[1 + Sqrt
[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2])))/(f*Cot[e + f*x]^(3/2))
)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2d\left(\sqrt{\cot(fx+e)d}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\right.\right.}{8}\right.}{f}$
default	$-\frac{2d\left(\sqrt{\cot(fx+e)d}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\right.\right.}{8}\right.}{f}$

```
[In] int((cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*d*((cot(f*x+e)*d)^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d+(d^2
)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/4)
*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(c
ot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)
))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.25

$$\int (d\cot(e+fx))^{3/2}\,dx = \frac{\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\log\left(d\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}+\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\right)+i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\log\left(d\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}+i\left(-\frac{d^6}{f^4}\right)\right.}{}$$

```
[In] integrate((d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((-d^6/f^4)^(1/4)*f*log(d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e
)) + (-d^6/f^4)^(1/4)*f) + I*(-d^6/f^4)^(1/4)*f*log(d*sqrt((d*cos(2*f*x + 2
*e) + d)/sin(2*f*x + 2*e)) + I*(-d^6/f^4)^(1/4)*f) - I*(-d^6/f^4)^(1/4)*f*l
og(d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)) - I*(-d^6/f^4)^(1/4)*f
) - (-d^6/f^4)^(1/4)*f*log(d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)
) - (-d^6/f^4)^(1/4)*f) - 4*d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e
)))/f
```

Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*cot(e + f*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.85

$$\int (d\cot(e+fx))^{3/2}\,dx = \frac{\left(2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+\sqrt{2}\sqrt{\right.}{}$$

```
[In] integrate((d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x + e)))*
d/f
```

Giac [F]

$$\int (d\cot(e+fx))^{3/2}\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.36

$$\int (d\cot(e+fx))^{3/2}\,dx = -\frac{2\,d\,\sqrt{d\cot(e+f\,x)}}{f} - \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} - \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

```
[In] int((d*cot(e + f*x))^(3/2),x)
```

```
[Out] - (2*d*(d*cot(e + f*x))^(1/2))/f - ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*
cot(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)
*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

3.205 $\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 211

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{2(d\cot(e+fx))^{3/2}}{3f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

```
[Out] -2/3*(d*cot(f*x+e))^(3/2)/f-1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/
2)/d^(1/2))/f*2^(1/2)+1/2*d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(
1/2))/f*2^(1/2)+1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*
x+e))^(1/2))/f*2^(1/2)-1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d
*cot(f*x+e))^(1/2))/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules

used = {16, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{d^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} - \frac{2(d\cot(e+fx))^{3/2}}{3f}$$

```
[In] Int[Cot[e + f*x]*(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*(d*Cot[e + f*x])^(3/2))/(3*f) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e
+ f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (d^(3/2)*Log[Sqrt[d
] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\int (d\cot(e+fx))^{5/2}\,dx}{d}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3f} - d\int \sqrt{d\cot(e+fx)}\,dx$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3f} + \frac{d^2\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3f} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3f} - \frac{d^2\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{d^2\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3f} + \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^2\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} + \frac{d^2\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}$$

$$= -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{2(d\cot(e+fx))^{3/2}}{3f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.47

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = -\frac{(d\cot(e+fx))^{3/2}\left(-3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}+3\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-c\right.}{3f\cot^{\frac{7}{4}}(e+fx)}$$

[In] Integrate[Cot[e + f*x]*(d*Cot[e + f*x])^(3/2),x]

[Out] -1/3*((d*Cot[e + f*x])^(3/2)*(-3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4) + 2*Cot[e + f*x]^(7/4)))/(f*Cot[e + f*x]^(7/4))

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{-\frac{2(\cot(fx+e)d)^{\frac{3}{2}}}{3}+\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot}}{\left(d^2\right.}\right.\right.}{4\left(d^2\right)^{\frac{1}{4}}}}{f}$
default	$\frac{-\frac{2(\cot(fx+e)d)^{\frac{3}{2}}}{3}+\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot}}{\left(d^2\right.}\right.\right.}{4\left(d^2\right)^{\frac{1}{4}}}}{f}$

[In] int(cot(f*x+e)*(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/f*(-2/3*(cot(f*x+e)*d)^(3/2)+1/4*d^2/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.60

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{3\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\log\left(d^4\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}+\left(-\frac{d^6}{f^4}\right)^{\frac{3}{4}} f^3\right)\sin(2\,fx+2\,e) - 3i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\log\left(d^4\sqrt{\right.}$$

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(-d^6/f^4)^(1/4)*f*log(d^4*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x +
 2*e)) + (-d^6/f^4)^(3/4)*f^3)*sin(2*f*x + 2*e) - 3*I*(-d^6/f^4)^(1/4)*f*lo
g(d^4*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)) + I*(-d^6/f^4)^(3/4)*
f^3)*sin(2*f*x + 2*e) + 3*I*(-d^6/f^4)^(1/4)*f*log(d^4*sqrt((d*cos(2*f*x +
2*e) + d)/sin(2*f*x + 2*e)) - I*(-d^6/f^4)^(3/4)*f^3)*sin(2*f*x + 2*e) - 3*
(-d^6/f^4)^(1/4)*f*log(d^4*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))
- (-d^6/f^4)^(3/4)*f^3)*sin(2*f*x + 2*e) - 4*(d*cos(2*f*x + 2*e) + d)*sqrt(
(d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))/(f*sin(2*f*x + 2*e))
```

Sympy [F]

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\cot(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*cot(e + f*x))**(3/2)*cot(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.87

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{3\,d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+\right.}}}{\sqrt{d}}\right.}{12\,f}$$

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f*x +
e))^(3/2))/f
```

Giac [F]

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\cot(fx+e)\,dx$$

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2)*cot(f*x + e), x)
```

Mupad [B] (verification not implemented)

Time = 3.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{f} - \frac{2\,(d\cot(e+f\,x))^{3/2}}{3\,f} - \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{f}$$

```
[In] int(cot(e + f*x)*(d*cot(e + f*x))^(3/2),x)
```

```
[Out] ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/f -
(2*(d*cot(e + f*x))^(3/2))/(3*f) - ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)*(d
*cot(e + f*x))^(1/2))/d^(1/2)))/f
```

3.206 $\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 232

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

```
[Out] -2/5*(d*cot(f*x+e))^(5/2)/d/f+1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(
1/2)/d^(1/2))/f*2^(1/2)-1/2*d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d
^(1/2))/f*2^(1/2)+1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(
f*x+e))^(1/2))/f*2^(1/2)-1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*
(d*cot(f*x+e))^(1/2))/f*2^(1/2)+2*d*(d*cot(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules

used = {16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} + \frac{2d\sqrt{d\cot(e+fx)}}{f}$$

```
[In] Int[Cot[e + f*x]^2*(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*d*Sqrt[d*Cot[e + f*x]])/f - (2*(d*Cot[e + f*x])^(5/2))/(5*d*f) + (d^(3/2
)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sq
rt[2]*f) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot
[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\int (d\cot(e+fx))^{7/2}\,dx}{d^2}$$

$$= -\frac{2(d\cot(e+fx))^{5/2}}{5df} - \int (d\cot(e+fx))^{3/2}\,dx$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} + d^2\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} - \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} - \frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} - \frac{d^2\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{d^2\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} + \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^2\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} - \frac{d^2\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}$$

$$= \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.74

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = -\frac{(d\cot(e+fx))^{3/2}\left(-10\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)+10\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)-40\sqrt{}\right.}{}$$

```
[In] Integrate[Cot[e + f*x]^2*(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -1/20*((d*Cot[e + f*x])^(3/2)*(-10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e +
f*x]]] + 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] - 40*Sqrt[Cot[e
+ f*x]] + 8*Cot[e + f*x]^(5/2) - 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x
]] + Cot[e + f*x]] + 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e +
 f*x]]))/(f*Cot[e + f*x]^(3/2))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{5}{2}}}{5}-d^2\sqrt{\cot(fx+e)d}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+}}{(d^2)^{\frac{1}{4}}}\right)\right)}{8}\right)}{fd}$
default	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{5}{2}}}{5}-d^2\sqrt{\cot(fx+e)d}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+}}{(d^2)^{\frac{1}{4}}}\right)\right)}{8}\right)}{fd}$

```
[In] int(cot(f*x+e)^2*(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/d*(1/5*(cot(f*x+e)*d)^(5/2)-d^2*(cot(f*x+e)*d)^(1/2)+1/8*d^2*(d^2)^(1/
4)*2^(1/2)*(ln((cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)
^(1/2))/(cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))
)+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d
^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.51

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx =$$

$$-\frac{5\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}(f\cos(2\,fx+2\,e)-f)\log\left(d\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}+\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\right)+5\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}(i\,f\cos(2\,fx+2\,e)-}{}$$

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/10*(5*(-d^6/f^4)^(1/4)*(f*cos(2*f*x + 2*e) - f)*log(d*sqrt((d*cos(2*f*x
+ 2*e) + d)/sin(2*f*x + 2*e)) + (-d^6/f^4)^(1/4)*f) + 5*(-d^6/f^4)^(1/4)*(I
*f*cos(2*f*x + 2*e) - I*f)*log(d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x +
2*e)) + I*(-d^6/f^4)^(1/4)*f) + 5*(-d^6/f^4)^(1/4)*(-I*f*cos(2*f*x + 2*e) +
 I*f)*log(d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)) - I*(-d^6/f^4)^
(1/4)*f) - 5*(-d^6/f^4)^(1/4)*(f*cos(2*f*x + 2*e) - f)*log(d*sqrt((d*cos(2*
f*x + 2*e) + d)/sin(2*f*x + 2*e)) - (-d^6/f^4)^(1/4)*f) - 8*(3*d*cos(2*f*x
+ 2*e) - 2*d)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))/(f*cos(2*f*x
 + 2*e) - f)
```

Sympy [F]

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\cot^2(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)**2*(d*cot(f*x+e))**(3/2),x)
[Out] Integral((d*cot(e + f*x))**(3/2)*cot(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.86

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = -\frac{10\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+10\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+5\sqrt{2}d^{\frac{5}{2}}\log\left(\sqrt{2}\sqrt{}\right.}{}$$

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
[Out] -1/20*(10*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/ta
n(f*x + e)))/sqrt(d)) + 10*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + 5*sqrt(2)*d^(5/2)*log(sqrt(2)*sqr
t(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 5*sqrt(2)*d^(5/2)*log(-sq
rt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 40*d^2*sqrt(d/ta
n(f*x + e)) + 8*(d/tan(f*x + e))^(5/2))/(d*f)
```

Giac [F]

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\cot(fx+e)^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*cot(f*x + e))^(3/2)*cot(f*x + e)^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.65 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.39

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{2\,d\,\sqrt{d\cot\left(e+f\,x\right)}}{f} - \frac{2\,{\left(d\cot(e+f\,x)\right)}^{5/2}}{5\,d\,f} + \frac{{\left(-1\right)}^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1{}\mathrm{i}}{f} + \frac{{\left(-1\right)}^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{d\cot(e+f\,x)}\,1{}\mathrm{i}}{\sqrt{d}}\right)}{f}$$

```
[In] int(cot(e + f*x)^2*(d*cot(e + f*x))^(3/2),x)
```

```
[Out] (2*d*(d*cot(e + f*x))^(1/2))/f - (2*(d*cot(e + f*x))^(5/2))/(5*d*f) + ((-1)
^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f + ((
-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2)*1i)/d^(1/2)))/f
```

3.207 $\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 231

$$\begin{aligned}\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx &= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}\\ &\quad+\frac{2d^2}{5f(d\cot(e+fx))^{5/2}}-\frac{2}{f\sqrt{d\cot(e+fx)}}\\ &\quad-\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}\\ &\quad+\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}\end{aligned}$$

```
[Out] 2/5*d^2/f/(d*cot(f*x+e))^(5/2)+1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^
(1/2))/f*2^(1/2)/d^(1/2)-1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))
/f*2^(1/2)/d^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))
^(1/2))/f*2^(1/2)/d^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(
f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)-2/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}f} + \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}$$

```
[In] Int[Tan[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + (2*d
^2)/(5*f*(d*Cot[e + f*x])^(5/2)) - 2/(f*Sqrt[d*Cot[e + f*x]]) - Log[Sqrt[d]
 + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*
f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*
Sqrt[2]*Sqrt[d]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^3\int \frac{1}{(d\cot(e+fx))^{7/2}}\,dx\\
&= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - d\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx\\
&= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d}\\
&= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}\\
&= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&\quad - \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}\\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}\\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}
\end{aligned}
$$

$$= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}$$

$$= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.41

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{-5\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)} + 5\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)} + 2(}{5f\sqrt{d\cot(e+fx)}}$$

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (-5*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) + 5*ArcTanh[(-C
ot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) + 2*(-5 + Tan[e + f*x]^2))/(5
*f*Sqrt[d*Cot[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(178) = 356$.

Time = 3.46 (sec) , antiderivative size = 714, normalized size of antiderivative = 3.09

method	result
default	$\frac{\left(\left(\csc^2(fx+e)\right)(1-\cos(fx+e))^2-1\right)\left(-40\left(\csc^7(fx+e)\right)(1-\cos(fx+e))^7+5\ln\left(\frac{\csc(fx+e)(1-\cos(fx+e))^2+2\sin(fx+e)\sqrt{\left(\csc^3(fx+e)\right)(1-}}{1-c}\right.\right.}{}$

[In] int(tan(f*x+e)^3/(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] 1/20/f*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(-40*csc(f*x+e)^7*(1-cos(f*x+e))^7
+5*ln(1/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(csc(f*x+e
)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)+2-2*cos(f*x+e)-sin(f*x+e)
))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5/2)-10*arctan(1/
(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*
x+e))^(1/2)+1-cos(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*
x+e))^(5/2)-5*ln(-1/(1-cos(f*x+e))*(-csc(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+
e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-2+2*cos(f*x+
e)+sin(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5/2)
-10*arctan(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(
f*x+e)+cot(f*x+e))^(1/2)-1+cos(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(
f*x+e)+cot(f*x+e))^(5/2)+112*csc(f*x+e)^5*(1-cos(f*x+e))^5-40*csc(f*x+e)^3*
(1-cos(f*x+e))^3)/(-d/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2-sin(f*x+e
)))^(1/2)/(csc(f*x+e)*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(1-cos(f*x+e)))^(1/
2)/(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.97

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{5\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)-5i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(i\,d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)}{}$$

[In] integrate(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")

```
[Out] -1/10*(5*d*f*(-1/(d^2*f^4))^(1/4)*log(d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt(d
/tan(f*x + e))) - 5*I*d*f*(-1/(d^2*f^4))^(1/4)*log(I*d^2*f^3*(-1/(d^2*f^4))
```

```
^(3/4) + sqrt(d/tan(f*x + e))) + 5*I*d*f*(-1/(d^2*f^4))^(1/4)*log(-I*d^2*f^
3*(-1/(d^2*f^4))^(3/4) + sqrt(d/tan(f*x + e))) - 5*d*f*(-1/(d^2*f^4))^(1/4)
*log(-d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt(d/tan(f*x + e))) - 4*(tan(f*x + e
)^3 - 5*tan(f*x + e))*sqrt(d/tan(f*x + e)))/(d*f)
```

Sympy [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan^3{\left(e+fx \right)}}{\sqrt{d\cot{\left(e+fx \right)}}}\,dx$$

```
[In] integrate(tan(f*x+e)**3/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/sqrt(d*cot(e + f*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.90

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{d^4\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\cdots}{\cdots}\right)}{d^4}\cdots\right)}{20\,f}$$

```
[In] integrate(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*d^4*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*
sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^2 - 5
*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

Giac [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan(fx+e)^3}{\sqrt{d\cot(fx+e)}}\,dx$$

```
[In] integrate(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^3/sqrt(d*cot(f*x + e)), x)
```

Mupad [B] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.42

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\frac{2d^2}{5}-\frac{2d^2}{\tan(e+fx)^2}}{f\left(\frac{d}{\tan(e+fx)}\right)^{5/2}} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f} + \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

```
[In] int(tan(e + f*x)^3/(d*cot(e + f*x))^(1/2),x)
[Out] ((2*d^2)/5 - (2*d^2)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1/
4)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f) + ((-1)^(
1/4)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
```

3.208 $\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 212

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{2d}{3f(d\cot(e+fx))^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

```
[Out] 2/3*d/f/(d*cot(f*x+e))^(3/2)-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1
/2))/f*2^(1/2)/d^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f
*2^(1/2)/d^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(
1/2))/f*2^(1/2)/d^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*
x+e))^(1/2))/f*2^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}f} + \frac{2d}{3f(d\cot(e+fx))^{3/2}} - \frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}$$

```
[In] Int[Tan[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + (
2*d)/(3*f*(d*Cot[e + f*x])^(3/2)) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sq
rt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) + Log[Sqrt[d] + Sqrt[d]*C
ot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^2 \int \frac{1}{(d\cot(e+fx))^{5/2}}\,dx \\
&= \frac{2d}{3f(d\cot(e+fx))^{3/2}} - \int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx \\
&= \frac{2d}{3f(d\cot(e+fx))^{3/2}} + \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f} \\
&= \frac{2d}{3f(d\cot(e+fx))^{3/2}} + \frac{(2d)\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} \\
&= \frac{2d}{3f(d\cot(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} \\
&\quad + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} \\
&= \frac{2d}{3f(d\cot(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} \\
&= \frac{2d}{3f(d\cot(e+fx))^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} \\
&\quad + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}
\end{aligned}
$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{2d}{3f(d\cot(e+fx))^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.40

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{d\left(-2+3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+fx))^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+\right.}{3f(d\cot(e+fx))^{3/2}}$$

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -1/3*(d*(-2 + 3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3/4) + 3
*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3/4)))/(f*(d*Cot[e + f
*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(161) = 322$.

Time = 2.27 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.28

method	result
default	$-\frac{\sec(fx+e)\csc(fx+e)(\cos(fx+e)+1)\left(-6\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\cos(fx+e)\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)+\cos(fx\right.}{\cos(fx+e)-1}\right.}{}$

```
[In] int(tan(f*x+e)^2/(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/f*sec(f*x+e)*csc(f*x+e)*(cos(f*x+e)+1)*(-6*(-sin(f*x+e)*cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))-6*(-sin(f*x+
e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*arctan((2^(1/2)*(-sin(f*x+
e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-
1))+3*cos(f*x+e)*ln(2*cot(f*x+e)*2^(1/2)*(-csc(f*x+e)^2*cot(f*x+e)*(cos(f*x
```

```
+e)-1)^2)^(1/2)+2*csc(f*x+e)*2^(1/2)*(-csc(f*x+e)^2*cot(f*x+e)*(cos(f*x+e)-
1)^2)^(1/2)-2*cot(f*x+e)+2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-3*cos(f*x+e)*ln(-2*cot(f*x+e)*2^(1/2)*(-csc(f*x+e)^2*cot(f*x+e)*(cos(f*x+e
)-1)^2)^(1/2)-2*csc(f*x+e)*2^(1/2)*(-csc(f*x+e)^2*cot(f*x+e)*(cos(f*x+e)-1)
^2)^(1/2)-2*cot(f*x+e)+2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4
*2^(1/2)*cos(f*x+e)-4*2^(1/2))/(cot(f*x+e)*d)^(1/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$
$$= \frac{3\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)+3i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)-3i\,d}{}$$

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*d*f*(-1/(d^2*f^4))^(1/4)*log(d*f*(-1/(d^2*f^4))^(1/4) + sqrt(d/tan(f
*x + e))) + 3*I*d*f*(-1/(d^2*f^4))^(1/4)*log(I*d*f*(-1/(d^2*f^4))^(1/4) + s
qrt(d/tan(f*x + e))) - 3*I*d*f*(-1/(d^2*f^4))^(1/4)*log(-I*d*f*(-1/(d^2*f^4
))^(1/4) + sqrt(d/tan(f*x + e))) - 3*d*f*(-1/(d^2*f^4))^(1/4)*log(-d*f*(-1/
(d^2*f^4))^(1/4) + sqrt(d/tan(f*x + e))) + 4*sqrt(d/tan(f*x + e))*tan(f*x +
 e)^2)/(d*f)
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan^2\left(e+fx\right)}{\sqrt{d\cot\left(e+fx\right)}}\,dx$$

```
[In] integrate(tan(f*x+e)**2/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/sqrt(d*cot(e + f*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.90

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{d^3\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-\frac{\sqrt{2}\log\ldots}{\ldots}\ldots}{d^2}\ldots\right)}{12f}$$

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*d^3*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2*(d/t
an(f*x + e))^(3/2)))/f
```

Giac [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan(fx+e)^2}{\sqrt{d\cot(fx+e)}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(d*cot(f*x + e)), x)
```

Mupad [B] (verification not implemented)

Time = 2.99 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.38

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{2\,d}{3\,f\,\left(\frac{d}{\tan(e+f\,x)}\right)^{3/2}} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f}$$

```
[In] int(tan(e + f*x)^2/(d*cot(e + f*x))^(1/2),x)
```

```
[Out] (2*d)/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d/tan(e
+ f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d/
tan(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f)
```

3.209 $\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 209

$$
\begin{aligned}
\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = & -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} \\
& + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{2}{f\sqrt{d\cot(e+fx)}} \\
& + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} \\
& - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}
\end{aligned}
$$

```
[Out] -1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)+1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)+1/4*ln(d^(1
/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)-1/4*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/
2)+2/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {16, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}f} + \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}$$

```
[In] Int[Tan[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + 2
/(f*Sqrt[d*Cot[e + f*x]]) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sq
rt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f
*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = d\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$$
$$= \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d}$$
$$= \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$
$$= \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$
$$= \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$
$$= \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} + \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$
$$= \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.38

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{2+\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}-\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}}{f\sqrt{d\cot(e+fx)}}$$

```
[In] Integrate[Tan[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
[Out] (2 + ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) - ArcTanh[(-Co
t[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4))/(f*Sqrt[d*Cot[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(160) = 320.

Time = 3.13 (sec) , antiderivative size = 663, normalized size of antiderivative = 3.17

method	result
default	$-\frac{\left(\left(\csc^2(fx+e)\right)(1-\cos(fx+e))^2-1\right)\left(\ln\left(\frac{\csc(fx+e)(1-\cos(fx+e))^2+2\sin(fx+e)\sqrt{\left(\csc^3(fx+e)\right)(1-\cos(fx+e))^3-\csc(fx+e)+\cot(fx+e)}}{1-\cos(fx+e)}\right.\right.}{}$

```
[In] int(tan(f*x+e)/(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4/f*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(ln(1/(1-cos(f*x+e))*(csc(f*x+e)*(
1-cos(f*x+e))^2+2*sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(
f*x+e))^(1/2)+2-2*cos(f*x+e)-sin(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-cs
c(f*x+e)+cot(f*x+e))^(1/2)-2*arctan(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e
)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)+1-cos(f*x+e)))*(csc(f*x+e
)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-ln(-1/(1-cos(f*x+e))*(-cs
```

```
c(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f
*x+e)+cot(f*x+e))^(1/2)-2+2*cos(f*x+e)+sin(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*
x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-2*arctan(1/(1-cos(f*x+e))*(sin(f*x+e)*
(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-1+cos(f*x+e)))*
(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-8*csc(f*x+e)+8*
cot(f*x+e))/(-d/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2-sin(f*x+e)))^(1
/2)/(csc(f*x+e)*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(1-cos(f*x+e)))^(1/2)/(cs
c(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.01

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$
$$= \frac{df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)-i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(i\,d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)+i\,d}{}$$

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(d*f*(-1/(d^2*f^4))^(1/4)*log(d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt(d/tan
(f*x + e))) - I*d*f*(-1/(d^2*f^4))^(1/4)*log(I*d^2*f^3*(-1/(d^2*f^4))^(3/4)
 + sqrt(d/tan(f*x + e))) + I*d*f*(-1/(d^2*f^4))^(1/4)*log(-I*d^2*f^3*(-1/(d
^2*f^4))^(3/4) + sqrt(d/tan(f*x + e))) - d*f*(-1/(d^2*f^4))^(1/4)*log(-d^2*
f^3*(-1/(d^2*f^4))^(3/4) + sqrt(d/tan(f*x + e))) + 4*sqrt(d/tan(f*x + e))*t
an(f*x + e))/(d*f)
```

Sympy [F]

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)/sqrt(d*cot(e + f*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{d^2\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\cdots\right.}{}}{d^2}\cdots\right.}{4f}$$

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*d^2*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
+ e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt
(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sqrt(d/
tan(f*x + e))))/f
```

Giac [F]

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan(fx+e)}{\sqrt{d\cot(fx+e)}}\,dx$$

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)/sqrt(d*cot(f*x + e)), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.38

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{2}{f\sqrt{\frac{d}{\tan(e+f\,x)}}}+\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f}-\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

```
[In] int(tan(e + f*x)/(d*cot(e + f*x))^(1/2),x)
```

```
[Out] 2/(f*(d/tan(e + f*x))^(1/2)) + ((-1)^(1/4)*atan(((-1)^(1/4)*(d/tan(e + f*x)
)^(1/2))/d^(1/2)))/(d^(1/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d/tan(e + f
*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
```

3.210 $\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 192

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)-1/2*ar
ctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)+1/4*ln(d^(1/
2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)-1/4*l
n(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2
)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used

$= \{3557, 335, 217, 1179, 642, 1176, 631, 210\}$

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}$$

```
[In] Int[1/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*S
qrt[d]*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{(2d)\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2f} - \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

$$= \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}$$

$$= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\sqrt{\cot(e+fx)}\left(2\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-2\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(e\right.\right.}{2\sqrt{2}f\sqrt{d\cot(e+fx)}}$$

```
[In] Integrate[1/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTan[1
+ Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e
+ f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt[2]*f
*Sqrt[d*Cot[e + f*x]])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

method	result
derivativedivides	$-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4fd}$
default	$-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4fd}$

```
[In] int(1/(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/d*(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(
1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^
(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*
arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{1}{2}\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right) - \frac{1}{2}i\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right) + \frac{1}{2}i\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(-i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right) + \frac{1}{2}\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(-df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)$$

```
[In] integrate(1/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(-1/(d^2*f^4))^(1/4)*log(d*f*(-1/(d^2*f^4))^(1/4) + sqrt((d*cos(2*f*x
+ 2*e) + d)/sin(2*f*x + 2*e))) - 1/2*I*(-1/(d^2*f^4))^(1/4)*log(I*d*f*(-1/(
```

```
d^2*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 1/2*I*
(-1/(d^2*f^4))^(1/4)*log(-I*d*f*(-1/(d^2*f^4))^(1/4) + sqrt((d*cos(2*f*x +
2*e) + d)/sin(2*f*x + 2*e))) + 1/2*(-1/(d^2*f^4))^(1/4)*log(-d*f*(-1/(d^2*f
^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))
```

Sympy [F]

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\cot\left(e+fx\right)}}\,dx$$

```
[In] integrate(1/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(d*cot(e + f*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}\right.}{4f}$$

```
[In] integrate(1/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x +
e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*
sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(d
/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sqrt(d)
*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

Giac [F]

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\cot(fx+e)}}\,dx$$

```
[In] integrate(1/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(d*cot(f*x + e)), x)
```

Mupad [B] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.30

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f} + \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f}$$

```
[In] int(1/(d*cot(e + f*x))^(1/2),x)
[Out] ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*
f) + ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^
(1/2)*f)
```

3.211 $\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 192

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)-1/2*ar
ctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)-1/4*ln(d^(1/
2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)+1/4*l
n(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2
)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used

= {16, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}$$

```
[In] Int[Cot[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*S
qrt[d]*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}\\
&= -\frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}
\end{aligned}
$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} - \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

$$= -\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}$$

$$= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.39

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\left(-\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)+\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\right)\sqrt[4]{-\cot(e+fx)}\sqrt{d\cot(e+fx)}}{df\cot^{\frac{3}{4}}(e+fx)}$$

```
[In] Integrate[Cot[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] ((-ArcTan[(-Cot[e + f*x]^2)^(1/4)] + ArcTanh[(-Cot[e + f*x]^2)^(1/4)])*(-Co
t[e + f*x])^(1/4)*Sqrt[d*Cot[e + f*x]])/(d*f*Cot[e + f*x]^(3/4))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)\right)}{4f\left(d^2\right)^{\frac{1}{4}}}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)\right)}{4f\left(d^2\right)^{\frac{1}{4}}}$

```
[In] int(cot(f*x+e)/(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/
2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1
/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*ar
ctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.23

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{1}{2}\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right) + \frac{1}{2}i\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(i\,d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right) - \frac{1}{2}i\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(-i\,d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right) + \frac{1}{2}\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(-d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right)$$

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(-1/(d^2*f^4))^(1/4)*log(d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt((d*cos(2*
f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 1/2*I*(-1/(d^2*f^4))^(1/4)*log(I*d^2*f
^3*(-1/(d^2*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))
- 1/2*I*(-1/(d^2*f^4))^(1/4)*log(-I*d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt((d*
cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 1/2*(-1/(d^2*f^4))^(1/4)*log(-d^
2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)
))
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)/sqrt(d*cot(e + f*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+}{4f}$$

[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/f

Giac [F]

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot(fx+e)}{\sqrt{d\cot(fx+e)}}\,dx$$

[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(cot(f*x + e)/sqrt(d*cot(f*x + e)), x)

Mupad [B] (verification not implemented)

Time = 3.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.30

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}-\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

[In] int(cot(e + f*x)/(d*cot(e + f*x))^(1/2),x)

[Out] ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)

3.212 $\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 212

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{2\sqrt{d\cot(e+fx)}}{df} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)+1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)-1/4*ln(d^(1
/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)+1/4*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/
2)-2*(d*cot(f*x+e))^(1/2)/d/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}f} - \frac{2\sqrt{d\cot(e+fx)}}{df} - \frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}$$

```
[In] Int[Cot[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - (
2*Sqrt[d*Cot[e + f*x]])/(d*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2
]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e
 + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\int (d\cot(e+fx))^{3/2}\,dx}{d^2}$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{df} - \int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{df} + \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{df} + \frac{(2d)\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{df} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{df} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} + \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{df} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{2\sqrt{d\cot(e+fx)}}{df} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.76

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\sqrt{\cot(e+fx)}\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}+2\sqrt{\cot(e+fx)}+\frac{\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}+\text{co}\right.}{2\sqrt{2}}\right.}{f\sqrt{d\cot(e+fx)}}$$

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -((Sqrt[Cot[e + f*x]]*(ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] - Arc
Tan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] + 2*Sqrt[Cot[e + f*x]] + Log[1
- Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*
Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2])))/(f*Sqrt[d*Cot[e + f*x]]))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2\left(\sqrt{\cot(fx+e)d}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\right.\right.}{8}\right)}{fd}$
default	$-\frac{2\left(\sqrt{\cot(fx+e)d}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\right.\right.}{8}\right)}{fd}$

```
[In] int(cot(f*x+e)^2/(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/d*((cot(f*x+e)*d)^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d+(d^2
)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/4)
*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(c
ot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)
))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.25

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right)+i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right)$$

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(d*f*(-1/(d^2*f^4))^(1/4)*log(d*f*(-1/(d^2*f^4))^(1/4) + sqrt((d*cos(2*
f*x + 2*e) + d)/sin(2*f*x + 2*e))) + I*d*f*(-1/(d^2*f^4))^(1/4)*log(I*d*f*(
-1/(d^2*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - I*
d*f*(-1/(d^2*f^4))^(1/4)*log(-I*d*f*(-1/(d^2*f^4))^(1/4) + sqrt((d*cos(2*f*
x + 2*e) + d)/sin(2*f*x + 2*e))) - d*f*(-1/(d^2*f^4))^(1/4)*log(-d*f*(-1/(d
^2*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - 4*sqrt(
(d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))/(d*f)
```

Sympy [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

```
[In] integrate(cot(f*x+e)**2/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(d*cot(e + f*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.85

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+\sqrt{2}\sqrt{d}\log\left(\sqrt{2}\sqrt{d}\right.}{4\,df}$$

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
[Out] 1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x + e)))/
(d*f)
```

Giac [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot(fx+e)^2}{\sqrt{d\cot(fx+e)}}\,dx$$

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^2/sqrt(d*cot(f*x + e)), x)
```

Mupad [B] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.36

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{2\sqrt{d\cot(e+fx)}}{df}-\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f}-\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f}$$

```
[In] int(cot(e + f*x)^2/(d*cot(e + f*x))^(1/2),x)
[Out] - (2*(d*cot(e + f*x))^(1/2))/(d*f) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e
+ f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*
cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f)
```

3.213 $\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 214

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

```
[Out] -2/3*(d*cot(f*x+e))^(3/2)/d^2/f-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d
^(1/2))/f*2^(1/2)/d^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2)
)/f*2^(1/2)/d^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e)
)^(1/2))/f*2^(1/2)/d^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot
(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}f} - \frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}$$

```
[In] Int[Cot[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - (
2*(d*Cot[e + f*x])^(3/2))/(3*d^2*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] -
Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) - Log[Sqrt[d] + Sqrt[d]
*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (d\cot(e+fx))^{5/2}\,dx}{d^3}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} - \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&\quad + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f}\\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}\\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}\\
&\quad - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}
\end{aligned}
$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.47

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\sqrt[4]{\cot(e+fx)}\left(-3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\text{co}\right.}{3f\sqrt{d\cot(e+fx)}}$$

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -1/3*(Cot[e + f*x]^(1/4)*(-3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]
)^(1/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4) + 2*Cot[
e + f*x]^(7/4)))/(f*Sqrt[d*Cot[e + f*x]])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.73

method	result
derivativedivides	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{}}{}\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{f\,d^2}$
default	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{}}{}\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{f\,d^2}$

```
[In] int(cot(f*x+e)^3/(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/d^2*(1/3*(cot(f*x+e)*d)^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x
+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d+(
d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2
)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d
)^(1/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.58

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$
$$= \frac{3\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)\sin(2\,fx+2\,e)-3i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(i\,d^2f^3\left(-\frac{1}{d^2\right.}}{}$$

```
[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*d*f*(-1/(d^2*f^4))^(1/4)*log(d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt((d*
cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))*sin(2*f*x + 2*e) - 3*I*d*f*(-1/(d^
2*f^4))^(1/4)*log(I*d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e)
 + d)/sin(2*f*x + 2*e)))*sin(2*f*x + 2*e) + 3*I*d*f*(-1/(d^2*f^4))^(1/4)*lo
g(-I*d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x
 + 2*e)))*sin(2*f*x + 2*e) - 3*d*f*(-1/(d^2*f^4))^(1/4)*log(-d^2*f^3*(-1/(d
^2*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))*sin(2*f*x
 + 2*e) - 4*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*(cos(2*f*x + 2*
e) + 1))/(d*f*sin(2*f*x + 2*e))
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

```
[In] integrate(cot(f*x+e)**3/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/sqrt(d*cot(e + f*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.72 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$
$$= \frac{3\,d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}\right.}{12\,d^2 f}$$

```
[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f*x +
e))^(3/2))/(d^2*f)
```

Giac [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)^3}{\sqrt{d\cot\left(fx+e\right)}}\,dx$$

```
[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/sqrt(d*cot(f*x + e)), x)
```

Mupad [B] (verification not implemented)

Time = 3.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{\sqrt{d}\,f} - \frac{2\,(d\cot(e+f\,x))^{3/2}}{3\,d^2\,f} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

```
[In] int(cot(e + f*x)^3/(d*cot(e + f*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
- (2*(d*cot(e + f*x))^(3/2))/(3*d^2*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*c
ot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
```

3.214 $\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 232

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

```
[Out] 2/5*d/f/(d*cot(f*x+e))^(5/2)+1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1
/2))/d^(3/2)/f*2^(1/2)-1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d
^(3/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(
1/2))/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*
x+e))^(1/2))/d^(3/2)/f*2^(1/2)-2/d/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f} + \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}}$$

```
[In] Int[Tan[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + (2*d
)/(5*f*(d*Cot[e + f*x])^(5/2)) - 2/(d*f*Sqrt[d*Cot[e + f*x]]) - Log[Sqrt[d]
 + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*
f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*
Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = d^2 \int \frac{1}{(d\cot(e+fx))^{7/2}}\,dx$$

$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$$

$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d^2}$$

$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{df}$$

$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}$$

$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} - \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}$$

$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df} - \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df}$$

$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}$$

$$= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.42

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{-5\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)} + 5\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right.}{5df\sqrt{d\cot(e+fx)}}$$

```
[In] Integrate[Tan[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] (-5*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) + 5*ArcTanh[(-C
ot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) + 2*(-5 + Tan[e + f*x]^2))/(5
*d*f*Sqrt[d*Cot[e + f*x]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(179) = 358$.

Time = 3.28 (sec) , antiderivative size = 734, normalized size of antiderivative = 3.16

method	result
default	$-\frac{\left(\left(\csc^2(fx+e)\right)(1-\cos(fx+e))^2-1\right)^2\left(-40\left(\csc^7(fx+e)\right)(1-\cos(fx+e))^7+5\ln\left(\frac{\csc(fx+e)(1-\cos(fx+e))^2+2\sin(fx+e)\sqrt{\left(\csc^3(fx+e)\right.}}{}\right.\right.}{}$

```
[In] int(tan(f*x+e)^2/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20/f*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)^2*(-40*csc(f*x+e)^7*(1-cos(f*x+e)
)^7+5*ln(1/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(csc(f*
x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)+2-2*cos(f*x+e)-sin(f*x
+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5/2)-10*arctan
(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot
(f*x+e))^(1/2)+1-cos(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot
(f*x+e))^(5/2)-5*ln(-1/(1-cos(f*x+e))*(-csc(f*x+e)*(1-cos(f*x+e))^2+2*sin(f
*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-2+2*cos(f
*x+e)+sin(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5
/2)-10*arctan(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-c
sc(f*x+e)+cot(f*x+e))^(1/2)-1+cos(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-c
sc(f*x+e)+cot(f*x+e))^(5/2)+112*csc(f*x+e)^5*(1-cos(f*x+e))^5-40*csc(f*x+e)
^3*(1-cos(f*x+e))^3)/(-d/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2-sin(f*
x+e)))^(3/2)/(1-cos(f*x+e))*sin(f*x+e)/(csc(f*x+e)*(csc(f*x+e)^2*(1-cos(f*x
+e))^2-1)*(1-cos(f*x+e)))^(1/2)/(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+c
ot(f*x+e))^(5/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{5\,d^2 f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(d^5f^3\left(-\frac{1}{d^6f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)-5i\,d^2 f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(i\,d^5f^3\left(-\frac{1}{d^6f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)}{}$$

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/10*(5*d^2*f*(-1/(d^6*f^4))^(1/4)*log(d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt
(d/tan(f*x + e))) - 5*I*d^2*f*(-1/(d^6*f^4))^(1/4)*log(I*d^5*f^3*(-1/(d^6*f
```

```
^4))^(3/4) + sqrt(d/tan(f*x + e))) + 5*I*d^2*f*(-1/(d^6*f^4))^(1/4)*log(-I*
d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt(d/tan(f*x + e))) - 5*d^2*f*(-1/(d^6*f^4
))^(1/4)*log(-d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt(d/tan(f*x + e))) - 4*(tan
(f*x + e)^3 - 5*tan(f*x + e))*sqrt(d/tan(f*x + e)))/(d^2*f)
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)**2/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(d*cot(e + f*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.89

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =$$

$$-\frac{d^3\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log(\cdots}{\cdots}}{d^4}\cdots\right)}{20\,f}$$

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/20*d^3*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*
sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^2 - 5
*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

Giac [F]

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\tan\left(fx+e\right)^2}{\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(d*cot(f*x + e))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.40

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\frac{2\,d}{5}-\frac{2\,d}{\tan(e+f\,x)^2}}{f\,\left(\frac{d}{\tan(e+f\,x)}\right)^{5/2}} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{d^{3/2}\,f} + \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

```
[In] int(tan(e + f*x)^2/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] ((2*d)/5 - (2*d)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1/4)*a
tan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f) + ((-1)^(1/4)
*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
```

3.215 $\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 211

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{2}{3f(d\cot(e+fx))^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

```
[Out] 2/3/f/(d*cot(f*x+e))^(3/2)-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2
))/d^(3/2)/f*2^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(
3/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/
2))/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+
e))^(1/2))/d^(3/2)/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules

used = {16, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = &-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}\\ &+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}\\ &+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}+\frac{2}{3f(d\cot(e+fx))^{3/2}}\end{aligned}$$

```
[In] Int[Tan[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + 2
/(3*f*(d*Cot[e + f*x])^(3/2)) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2
]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e
 + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = d\int \frac{1}{(d\cot(e+fx))^{5/2}}\,dx$$

$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} - \frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx}{d}$$

$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}$$

$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df} + \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df}$$

$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{2}{3f(d\cot(e+fx))^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.39

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{-2+3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+fx))^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+fx))}{3f(d\cot(e+fx))^{3/2}}$$

```
[In] Integrate[Tan[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
[Out] -1/3*(-2 + 3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3/4) + 3*Ar
cTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3/4))/(f*(d*Cot[e + f*x])
^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. 2(160) = 320.

Time = 2.72 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.71

method	result
default	$\frac{\left(-6\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)+\cos(fx+e)-1}{\cos(fx+e)-1}\right)-6\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(\right.}{}$

```
[In] int(tan(f*x+e)/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/12/f/(cos(f*x+e)-1)/d/(cot(f*x+e)*d)^(1/2)*(-6*(-sin(f*x+e)*cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*sin(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))-6*(-sin(f*x+
e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)*arctan((2^(1/2)*(-sin(f*x+
e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-
1))+3*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)*ln(-(cot(f
*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*cs
c(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2
)-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))-3*sin(f*x+e)*(-sin(
f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(
f*x+e)-2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2
*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+
e)+csc(f*x+e)+2)/(cos(f*x+e)-1))+4*2^(1/2)*sin(f*x+e)-4*tan(f*x+e)*2^(1/2))
*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.01

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{3\,d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)+3i\,d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(i\,d^2f\left(-\right.\right.}{}$$

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*d^2*f*(-1/(d^6*f^4))^(1/4)*log(d^2*f*(-1/(d^6*f^4))^(1/4) + sqrt(d/t
an(f*x + e))) + 3*I*d^2*f*(-1/(d^6*f^4))^(1/4)*log(I*d^2*f*(-1/(d^6*f^4))^(
1/4) + sqrt(d/tan(f*x + e))) - 3*I*d^2*f*(-1/(d^6*f^4))^(1/4)*log(-I*d^2*f*
(-1/(d^6*f^4))^(1/4) + sqrt(d/tan(f*x + e))) - 3*d^2*f*(-1/(d^6*f^4))^(1/4)
*log(-d^2*f*(-1/(d^6*f^4))^(1/4) + sqrt(d/tan(f*x + e))) + 4*sqrt(d/tan(f*x
 + e))*tan(f*x + e)^2)/(d^2*f)
```

Sympy [F]

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+fx)}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)/(d*cot(e + f*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.90

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{d^2\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan}}\right.}{}\right.}{d^2}\right.}{12\,f}$$

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*d^2*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2*(d/t
an(f*x + e))^(3/2)))/f
```

Giac [F]

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)/(d*cot(f*x + e))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.38

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2}{3f\left(\frac{d}{\tan(e+fx)}\right)^{3/2}} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}$$

```
[In] int(tan(e + f*x)/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] 2/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d/tan(e + f*
x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d/tan(
e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f)
```

3.216 $\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 212

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{2}{df\sqrt{d\cot(e+fx)}}+\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)+1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1
/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)-1/4*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/
2)+2/d/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used

= {3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f} + \frac{2}{df\sqrt{d\cot(e+fx)}}$$

```
[In] Int[(d*Cot[e + f*x])^(-3/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + 2
/(d*f*Sqrt[d*Cot[e + f*x]]) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*
Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e +
 f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d^2} \\
&= \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{df} \\
&= \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}
\end{aligned}
$$

$$= \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}$$

$$= \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df} + \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df}$$

$$= \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.39

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2+\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}-\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)}{df\sqrt{d\cot(e+fx)}}$$

[In] Integrate[(d*Cot[e + f*x])^(-3/2),x]

[Out] (2 + ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) - ArcTanh[(-Co t[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4))/(d*f*Sqrt[d*Cot[e + f*x]])

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{2d\left(-\frac{1}{d^2\sqrt{\cot(fx+e)d}}-\frac{\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}}{}\right.\right.}{8d^2(d^2)^{\frac{1}{4}}}\right.}{f}$
default	$-\frac{2d\left(-\frac{1}{d^2\sqrt{\cot(fx+e)d}}-\frac{\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}}{}\right.\right.}{8d^2(d^2)^{\frac{1}{4}}}\right.}{f}$

[In] int(1/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/f*d*(-1/d^2/(cot(f*x+e)*d)^(1/2)-1/8/d^2/(d^2)^(1/4)*2^(1/2)*(ln((cot(f* x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d+ (d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^ 2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)* d)^(1/2)+1)))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.79

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\left(d^2 f\cos\left(2\,fx+2\,e\right)+d^2 f\right)\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(d^5f^3\left(-\frac{1}{d^6f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)+}{}$$

```
[In] integrate(1/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((d^2*f*cos(2*f*x + 2*e) + d^2*f)*(-1/(d^6*f^4))^(1/4)*log(d^5*f^3*(-1/
(d^6*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + (-I*d
^2*f*cos(2*f*x + 2*e) - I*d^2*f)*(-1/(d^6*f^4))^(1/4)*log(I*d^5*f^3*(-1/(d^
6*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + (I*d^2*f
*cos(2*f*x + 2*e) + I*d^2*f)*(-1/(d^6*f^4))^(1/4)*log(-I*d^5*f^3*(-1/(d^6*f
^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - (d^2*f*cos(
2*f*x + 2*e) + d^2*f)*(-1/(d^6*f^4))^(1/4)*log(-d^5*f^3*(-1/(d^6*f^4))^(3/4
) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 4*sqrt((d*cos(2*f*x
+ 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e))/(d^2*f*cos(2*f*x + 2*e) + d
^2*f)
```

Sympy [F]

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*cot(f*x+e))**(3/2),x)
[Out] Integral((d*cot(e + f*x))**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \frac{d\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(f}}\right.}{}}{d^2}\right.}{4\,f}$$

```
[In] integrate(1/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*d*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x +
e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*
sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(d
/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(d)
*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sqrt(d/ta
n(f*x + e))))/f
```

Giac [F]

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2}{d\,f\,\sqrt{d\cot\left(e+f\,x\right)}} + \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

```
[In] int(1/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] 2/(d*f*(d*cot(e + f*x))^(1/2)) + ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*
x))^(1/2))/d^(1/2)))/(d^(3/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e +
 f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
```

3.217 $\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 192

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)-1/2*ar
ctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1/
2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)-1/4*l
n(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2
)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used

$= \{16, 3557, 335, 217, 1179, 642, 1176, 631, 210\}$

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}$$

```
[In] Int[Cot[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d
^(3/2)*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}\\
&= -\frac{2\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} - \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}
\end{aligned}
$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df} - \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df}$$

$$= \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}$$

$$= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.70

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\sqrt{\cot(e+fx)}\left(2\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right) - 2\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx}\right.\right.}{2\sqrt{2}}$$

```
[In] Integrate[Cot[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
[Out] (Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTan[1
+ Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e
+ f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt[2]*d
*f*Sqrt[d*Cot[e + f*x]])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

method	result
derivativedivides	$-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4f\,d^2}$
default	$-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4f\,d^2}$

```
[In] int(cot(f*x+e)/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*(d^2)^(1/4)/d^2*2^(1/2)*(ln((cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)
^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*
2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-
2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.19

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{1}{2}\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right) - \frac{1}{2}i\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(i\,d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right) + \frac{1}{2}i\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(-i\,d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right) + \frac{1}{2}\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(-d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)$$

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(-1/(d^6*f^4))^(1/4)*log(d^2*f*(-1/(d^6*f^4))^(1/4) + sqrt((d*cos(2*f*
x + 2*e) + d)/sin(2*f*x + 2*e))) - 1/2*I*(-1/(d^6*f^4))^(1/4)*log(I*d^2*f*(
-1/(d^6*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 1/
2*I*(-1/(d^6*f^4))^(1/4)*log(-I*d^2*f*(-1/(d^6*f^4))^(1/4) + sqrt((d*cos(2*
f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 1/2*(-1/(d^6*f^4))^(1/4)*log(-d^2*f*(-
1/(d^6*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+fx)}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))**(3/2),x)
[Out] Integral(cot(e + f*x)/(d*cot(e + f*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}} - \sqrt{2}\cdots}{4f}$$

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x + e)
))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sq
rt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(d/t
an(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

Giac [F]

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)/(d*cot(f*x + e))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 2.85 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.30

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f} + \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}$$

```
[In] int(cot(e + f*x)/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*
f) + ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^
(3/2)*f)
```

3.218 $\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 192

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)-1/2*ar
ctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)-1/4*ln(d^(1/
2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)+1/4*l
n(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2
)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

= {16, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}$$

```
[In] Int[Cot[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d
^(3/2)*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d^2}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{df}$$

$$= -\frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}$$

$$= \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} - \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df} - \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df}$$

$$= -\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}$$

$$= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.39

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\left(-\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)+\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\right)\sqrt[4]{-\cot(e+f}}{d^2f\cot^{\frac{3}{4}}(e+fx)}$$

```
[In] Integrate[Cot[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] ((-ArcTan[(-Cot[e + f*x]^2)^(1/4)] + ArcTanh[(-Cot[e + f*x]^2)^(1/4)])*(-Co
t[e + f*x])^(1/4)*Sqrt[d*Cot[e + f*x]])/(d^2*f*Cot[e + f*x]^(3/4))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

method	result
derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}{\cot(fx+e)d+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)\right.}{4fd\left(d^{2}\right)^{\frac{1}{4}}}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}{\cot(fx+e)d+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)\right.}{4fd\left(d^{2}\right)^{\frac{1}{4}}}$

```
[In] int(cot(f*x+e)^2/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/d/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(
1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^
(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*
arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.23

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{1}{2}\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(d^5f^3\left(-\frac{1}{d^6f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right) + \frac{1}{2}i\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(i\,d^5f^3\left(-\frac{1}{d^6f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right) - \frac{1}{2}i\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(-i\,d^5f^3\left(-\frac{1}{d^6f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right) + \frac{1}{2}\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(-d^5f^3\left(-\frac{1}{d^6f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)$$

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(-1/(d^6*f^4))^(1/4)*log(d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt((d*cos(2*
f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 1/2*I*(-1/(d^6*f^4))^(1/4)*log(I*d^5*f
^3*(-1/(d^6*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))
- 1/2*I*(-1/(d^6*f^4))^(1/4)*log(-I*d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt((d*
```

```
cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 1/2*(-1/(d^6*f^4))^(1/4)*log(-d^
5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)
))
```

Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)**2/(d*cot(f*x+e))**(3/2),x)
[Out] Integral(cot(e + f*x)**2/(d*cot(e + f*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}} + \cdots}{4\,df}$$

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x + e)
))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sq
rt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(d/t
an(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/(d*f)
```

Giac [F]

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^2}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^2/(d*cot(f*x + e))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.30

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f} - \frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

```
[In] int(cot(e + f*x)^2/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
 - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*
f)
```

3.219 $\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 212

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)+1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)-1/4*ln(d^(1
/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)+1/4*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/
2)-2*(d*cot(f*x+e))^(1/2)/d^2/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules

used = {16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f} - \frac{2\sqrt{d\cot(e+fx)}}{d^2f}$$

```
[In] Int[Cot[e + f*x]^3/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - (
2*Sqrt[d*Cot[e + f*x]])/(d^2*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt
[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) + Log[Sqrt[d] + Sqrt[d]*Cot
[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int (d\cot(e+fx))^{3/2}\,dx}{d^3}\\ &= -\frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx}{d}\\ &= -\frac{2\sqrt{d\cot(e+fx)}}{d^2f} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}\end{aligned}$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{d^2f} + \frac{2\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{d^2f} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df} + \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df}$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.76

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =$$

$$-\frac{\cot^{\frac{3}{2}}(e+fx)\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}+2\sqrt{\cot(e+fx)}+\frac{\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}+\cot(e+\right.}{2\sqrt{2}}\right.}{f(d\cot(e+fx))^{3/2}}$$

```
[In] Integrate[Cot[e + f*x]^3/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -((Cot[e + f*x]^(3/2)*(ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] - Arc
Tan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] + 2*Sqrt[Cot[e + f*x]] + Log[1
- Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*
Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2])))/(f*(d*Cot[e + f*x])^(3/2))
)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2\left(\sqrt{\cot(fx+e)d}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\right.\right.}{8}\right.}{f\,d^2}$
default	$-\frac{2\left(\sqrt{\cot(fx+e)d}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\right.\right.}{8}\right.}{f\,d^2}$

```
[In] int(cot(f*x+e)^3/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/d^2*((cot(f*x+e)*d)^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d+(d
^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/
4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*
(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+
1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.32

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{d^2 f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(d^2 f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)+i\,d^2 f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(i\,d^2 f\right.}{}$$

[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 1/2*(d^2*f*(-1/(d^6*f^4))^(1/4)*log(d^2*f*(-1/(d^6*f^4))^(1/4) + sqrt((d*co
s(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + I*d^2*f*(-1/(d^6*f^4))^(1/4)*log(I
*d^2*f*(-1/(d^6*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e
))) - I*d^2*f*(-1/(d^6*f^4))^(1/4)*log(-I*d^2*f*(-1/(d^6*f^4))^(1/4) + sqrt
((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - d^2*f*(-1/(d^6*f^4))^(1/4)*l
og(-d^2*f*(-1/(d^6*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x +
2*e))) - 4*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))/(d^2*f)

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot^3\left(e+fx\right)}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(cot(f*x+e)**3/(d*cot(f*x+e))**(3/2),x)

[Out] Integral(cot(e + f*x)**3/(d*cot(e + f*x))**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.85

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right.}{}$$

[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")

[Out] 1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x + e)))/
(d^2*f)

Giac [F]

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^3}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/(d*cot(f*x + e))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.36

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{d\cot(e+f\,x)}}{d^2\,f} - \frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f} - \frac{(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}$$

```
[In] int(cot(e + f*x)^3/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] - (2*(d*cot(e + f*x))^(1/2))/(d^2*f) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(
e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(
d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f)
```

3.220 $\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 214

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{2(d\cot(e+fx))^{3/2}}{3d^3f}+\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

```
[Out] -2/3*(d*cot(f*x+e))^(3/2)/d^3/f-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d
^(1/2))/d^(3/2)/f*2^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2)
)/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e)
)^(1/2))/d^(3/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot
(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules

used = {16, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f} - \frac{2(d\cot(e+fx))^{3/2}}{3d^3f}$$

```
[In] Int[Cot[e + f*x]^4/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - (
2*(d*Cot[e + f*x])^(3/2))/(3*d^3*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] -
Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) - Log[Sqrt[d] + Sqrt[d]
*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (d\cot(e+fx))^{5/2}\,dx}{d^4} \\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} - \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d^2} \\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{df}
\end{aligned}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} + \frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df} + \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{2(d\cot(e+fx))^{3/2}}{3d^3f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.47

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\cot^{\frac{5}{4}}(e+fx)\left(-3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e}\right.}{3f(d\cot(e+fx))^{3/2}}$$

```
[In] Integrate[Cot[e + f*x]^4/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -1/3*(Cot[e + f*x]^(5/4)*(-3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]
)^(1/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4) + 2*Cot[
e + f*x]^(7/4)))/(f*(d*Cot[e + f*x])^(3/2))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.73

method	result
derivativedivides	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{}}{}\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{f\,d^3}$
default	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{}}{}\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{f\,d^3}$

```
[In] int(cot(f*x+e)^4/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/d^3*(1/3*(cot(f*x+e)*d)^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x
+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d+(
d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2
)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d
)^(1/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.62

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{3\,d^2 f\left(-\frac{1}{d^6 f^4}\right)^{\frac{1}{4}}\log\left(d^5 f^3\left(-\frac{1}{d^6 f^4}\right)^{\frac{3}{4}} + \sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)\sin\left(2\,fx+2\,e\right) - 3i\,d^2 f}{}$$

```
[In] integrate(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*d^2*f*(-1/(d^6*f^4))^(1/4)*log(d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt((
d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))*sin(2*f*x + 2*e) - 3*I*d^2*f*(-1
/(d^6*f^4))^(1/4)*log(I*d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt((d*cos(2*f*x +
2*e) + d)/sin(2*f*x + 2*e)))*sin(2*f*x + 2*e) + 3*I*d^2*f*(-1/(d^6*f^4))^(1
/4)*log(-I*d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin
(2*f*x + 2*e)))*sin(2*f*x + 2*e) - 3*d^2*f*(-1/(d^6*f^4))^(1/4)*log(-d^5*f^
3*(-1/(d^6*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))*s
in(2*f*x + 2*e) - 4*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*(cos(2*
f*x + 2*e) + 1))/(d^2*f*sin(2*f*x + 2*e))
```

Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot^4\left(e+fx\right)}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)**4/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(d*cot(e + f*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{3\,d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log}{}\right.}{12}$$

```
[In] integrate(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f*x +
e))^(3/2))/(d^3*f)
```

Giac [F]

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^4}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(d*cot(f*x + e))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f} - \frac{2\,(d\cot(e+fx))^{3/2}}{3\,d^3\,f} - \frac{(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}$$

```
[In] int(cot(e + f*x)^4/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
- (2*(d*cot(e + f*x))^(3/2))/(3*d^3*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*c
ot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
```

3.221 $\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 234

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

```
[Out] -2/5*(d*cot(f*x+e))^(5/2)/d^4/f+1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d
^(1/2))/d^(3/2)/f*2^(1/2)-1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2)
)/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e)
)^(1/2))/d^(3/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot
(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)+2*(d*cot(f*x+e))^(1/2)/d^2/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} + \frac{2\sqrt{d\cot(e+fx)}}{d^2f}$$

```
[In] Int[Cot[e + f*x]^5/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + (2*S
qrt[d*Cot[e + f*x]])/(d^2*f) - (2*(d*Cot[e + f*x])^(5/2))/(5*d^4*f) + Log[S
qrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^
(3/2)*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]
]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (d\cot(e+fx))^{7/2}\,dx}{d^5}\\
&= -\frac{2(d\cot(e+fx))^{5/2}}{5d^4f} - \frac{\int (d\cot(e+fx))^{3/2}\,dx}{d^3}\\
&= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} + \frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx}{d}\\
&= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}\\
&= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} - \frac{2\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}\\
&\quad - \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}\\
&= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f}\\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}\\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2df}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} \\
&\quad + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} \\
&\quad - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} \\
&= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{2\sqrt{d\cot(e+fx)}}{d^2f} \\
&\quad - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} \\
&\quad - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.74

$$
\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =
$$

$$
-\frac{\cot^{\frac{3}{2}}(e+fx)\left(-10\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)+10\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)-40\sqrt{\cot(e+fx)}\right.}{}
$$

```
[In] Integrate[Cot[e + f*x]^5/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -1/20*(Cot[e + f*x]^(3/2)*(-10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]
]] + 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] - 40*Sqrt[Cot[e + f*
x]] + 8*Cot[e + f*x]^(5/2) - 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] +
 Cot[e + f*x]] + 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x
]]))/(f*(d*Cot[e + f*x])^(3/2))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.73

method	result
derivativedivides	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{5}{2}}}{5}-d^2\sqrt{\cot(fx+e)d}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+}}{(d^2)^{\frac{1}{4}}}\right.\right.}{8}\right.}{f\,d^4}$
default	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{5}{2}}}{5}-d^2\sqrt{\cot(fx+e)d}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+}}{(d^2)^{\frac{1}{4}}}\right.\right.}{8}\right.}{f\,d^4}$

```
[In] int(cot(f*x+e)^5/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/d^4*(1/5*(cot(f*x+e)*d)^(5/2)-d^2*(cot(f*x+e)*d)^(1/2)+1/8*d^2*(d^2)^(
1/4)*2^(1/2)*(ln((cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^
2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2
)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/
(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.63

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =$$

$$-\frac{5\left(d^2f\cos(2fx+2e)-d^2f\right)\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right)+5\left(i\,d^2f\cos(2fx+2e)-\right.}{}$$

```
[In] integrate(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/10*(5*(d^2*f*cos(2*f*x + 2*e) - d^2*f)*(-1/(d^6*f^4))^(1/4)*log(d^2*f*(-
1/(d^6*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 5*(
I*d^2*f*cos(2*f*x + 2*e) - I*d^2*f)*(-1/(d^6*f^4))^(1/4)*log(I*d^2*f*(-1/(d
^6*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 5*(-I*d
^2*f*cos(2*f*x + 2*e) + I*d^2*f)*(-1/(d^6*f^4))^(1/4)*log(-I*d^2*f*(-1/(d^6
*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - 5*(d^2*f*
cos(2*f*x + 2*e) - d^2*f)*(-1/(d^6*f^4))^(1/4)*log(-d^2*f*(-1/(d^6*f^4))^(1
/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - 8*sqrt((d*cos(2*f*
```

```
x + 2*e) + d)/sin(2*f*x + 2*e))*(3*cos(2*f*x + 2*e) - 2))/(d^2*f*cos(2*f*x
+ 2*e) - d^2*f)
```

Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)**5/(d*cot(f*x+e))**(3/2),x)
[Out] Integral(cot(e + f*x)**5/(d*cot(e + f*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.85

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{10\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+10\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+5\sqrt{2}d^{\frac{5}{2}}\log\left(\sqrt{2}\sqrt{}\right.}{}$$

```
[In] integrate(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
[Out] -1/20*(10*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/ta
n(f*x + e)))/sqrt(d)) + 10*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + 5*sqrt(2)*d^(5/2)*log(sqrt(2)*sqr
t(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 5*sqrt(2)*d^(5/2)*log(-sq
rt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 40*d^2*sqrt(d/ta
n(f*x + e)) + 8*(d/tan(f*x + e))^(5/2))/(d^4*f)
```

Giac [F]

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^5}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^5/(d*cot(f*x + e))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.40

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2\sqrt{d\cot(e+f\,x)}}{d^2\,f} - \frac{2\,(d\cot(e+f\,x))^{5/2}}{5\,d^4\,f} + \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f} + \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}\,1\mathrm{i}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

```
[In] int(cot(e + f*x)^5/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] (2*(d*cot(e + f*x))^(1/2))/(d^2*f) - (2*(d*cot(e + f*x))^(5/2))/(5*d^4*f) +
 ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)
*f) + ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2)*1i)/d^(1/2)))/(d^
(3/2)*f)
```

3.222 $\int \cot^m(e+fx)\tan^n(e+fx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx = \frac{\cot^m(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}$$

```
[Out] cot(f*x+e)^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x+e)^2
)*tan(f*x+e)^(1+n)/f/(1-m+n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2684, 3557, 371}

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx = \frac{\cot^m(e+fx)\tan^{n+1}(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n+3),-\tan^2(e+fx)\right)}{f(-m+n+1)}$$

```
[In] Int[Cot[e + f*x]^m*Tan[e + f*x]^n,x]
```

```
[Out] (Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e +
 f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2684

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(\cot^m(e+fx)\tan^m(e+fx)\right)\int \tan^{-m+n}(e+fx)\,dx$$

$$= \frac{\left(\cot^m(e+fx)\tan^m(e+fx)\right)\text{Subst}\left(\int \frac{x^{-m+n}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cot^m(e+fx)\,\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1-m+n), \frac{1}{2}(3-m+n), -\tan^2(e+fx)\right)\tan^{1+n}(e+fx}{f(1-m+n)}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx$$
$$= \frac{\cot^m(e+fx)\,\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1-m+n), \frac{1}{2}(3-m+n), -\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}$$

```
[In] Integrate[Cot[e + f*x]^m*Tan[e + f*x]^n,x]
[Out] (Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e +
 f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

Maple [F]

$$\int (\cot^m (fx + e)) (\tan^n (fx + e)) \, dx$$

```
[In] int(cot(f*x+e)^m*tan(f*x+e)^n,x)
[Out] int(cot(f*x+e)^m*tan(f*x+e)^n,x)
```

Fricas [F]

$$\int \cot^m(e + fx) \tan^n(e + fx) \, dx = \int \cot (fx + e)^m \tan (fx + e)^n \, dx$$

```
[In] integrate(cot(f*x+e)^m*tan(f*x+e)^n,x, algorithm="fricas")
[Out] integral(cot(f*x + e)^m*tan(f*x + e)^n, x)
```

Sympy [F]

$$\int \cot^m(e + fx) \tan^n(e + fx) \, dx = \int \tan^n (e + fx) \cot^m (e + fx) \, dx$$

```
[In] integrate(cot(f*x+e)**m*tan(f*x+e)**n,x)
[Out] Integral(tan(e + f*x)**n*cot(e + f*x)**m, x)
```

Maxima [F]

$$\int \cot^m(e + fx) \tan^n(e + fx) \, dx = \int \cot (fx + e)^m \tan (fx + e)^n \, dx$$

```
[In] integrate(cot(f*x+e)^m*tan(f*x+e)^n,x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^m*tan(f*x + e)^n, x)
```

Giac [F]

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx = \int \cot\left(fx+e\right)^m \tan\left(fx+e\right)^n\,dx$$

```
[In] integrate(cot(f*x+e)^m*tan(f*x+e)^n,x, algorithm="giac")
[Out] integrate(cot(f*x + e)^m*tan(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx = \int \cot(e+f\,x)^m \tan(e+f\,x)^n\,dx$$

```
[In] int(cot(e + f*x)^m*tan(e + f*x)^n,x)
[Out] int(cot(e + f*x)^m*tan(e + f*x)^n, x)
```

3.223 $\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \frac{\cot^m(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1-m+n)}$$

```
[Out] cot(f*x+e)^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x+e)^2
)*(b*tan(f*x+e))^(1+n)/b/f/(1-m+n)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2684, 3557, 371}

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \frac{\cot^m(e+fx)(b\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n+3),-\tan^2(e+fx)\right)}{bf(-m+n+1)}$$

```
[In] Int[Cot[e + f*x]^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e +
 f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - m + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2684

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = (\cot^m(e+fx)(b\tan(e+fx))^m)\int(b\tan(e+fx))^{-m+n}\,dx$$

$$=\frac{(b\cot^m(e+fx)(b\tan(e+fx))^m)\,\text{Subst}\left(\int\frac{x^{-m+n}}{b^2+x^2}\,dx,x,b\tan(e+fx)\right)}{f}$$

$$=\frac{\cot^m(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)(b\tan(e+fx)}{bf(1-m+n)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int\cot^m(e+fx)(b\tan(e+fx))^n\,dx$$
$$=\frac{\cot^{-1+m}(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)(b\tan(e+fx))^n}{f(1-m+n)}$$

```
[In] Integrate[Cot[e + f*x]^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Cot[e + f*x]^(-1 + m)*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -
Tan[e + f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - m + n))
```

Maple [F]

$$\int (\cot^m(fx+e))\,(b\tan(fx+e))^n\,dx$$

```
[In] int(cot(f*x+e)^m*(b*tan(f*x+e))^n,x)
[Out] int(cot(f*x+e)^m*(b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \cot(fx+e)^m\,dx$$

```
[In] integrate(cot(f*x+e)^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e))^n*cot(f*x + e)^m, x)
```

Sympy [F]

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(e+fx))^n \cot^m(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)**m*(b*tan(f*x+e))**n,x)
[Out] Integral((b*tan(e + f*x))**n*cot(e + f*x)**m, x)
```

Maxima [F]

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \cot(fx+e)^m\,dx$$

```
[In] integrate(cot(f*x+e)^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^n*cot(f*x + e)^m, x)
```

Giac [F]

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \cot(fx+e)^m\,dx$$

```
[In] integrate(cot(f*x+e)^m*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n*cot(f*x + e)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \int \cot(e+f\,x)^m\,(b\tan(e+f\,x))^n\,dx$$

```
[In] int(cot(e + f*x)^m*(b*tan(e + f*x))^n,x)
[Out] int(cot(e + f*x)^m*(b*tan(e + f*x))^n, x)
```

3.224 $\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 64

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \frac{(a\cot(e+fx))^m \text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}$$

```
[Out] (a*cot(f*x+e))^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x+
e)^2)*tan(f*x+e)^(1+n)/f/(1-m+n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2684, 3557, 371}

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \frac{\tan^{n+1}(e+fx)(a\cot(e+fx))^m \text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n+3),-\tan^2(e+fx)\right)}{f(-m+n+1)}$$

```
[In] Int[(a*Cot[e + f*x])^m*Tan[e + f*x]^n,x]
```

```
[Out] ((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan
[e + f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2684

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left((a\cot(e+fx))^m \tan^m(e+fx)\right)\int \tan^{-m+n}(e+fx)\,dx$$

$$= \frac{\left((a\cot(e+fx))^m \tan^m(e+fx)\right)\text{Subst}\left(\int \frac{x^{-m+n}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(a\cot(e+fx))^m \text{Hypergeometric2F1}\left(1, \frac{1}{2}(1-m+n), \frac{1}{2}(3-m+n), -\tan^2(e+fx)\right)\tan^{1+n}(e+}{f(1-m+n)}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx$$
$$= \frac{(a\cot(e+fx))^m \text{Hypergeometric2F1}\left(1, \frac{1}{2}(1-m+n), \frac{1}{2}(3-m+n), -\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}$$

```
[In] Integrate[(a*Cot[e + f*x])^m*Tan[e + f*x]^n,x]
[Out] ((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan
[e + f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

Maple [F]

$$\int (a\cot(fx+e))^m (\tan^n(fx+e))\,dx$$

```
[In] int((a*cot(f*x+e))^m*tan(f*x+e)^n,x)
[Out] int((a*cot(f*x+e))^m*tan(f*x+e)^n,x)
```

Fricas [F]

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \int (a\cot(fx+e))^m \tan(fx+e)^n \,dx$$

```
[In] integrate((a*cot(f*x+e))^m*tan(f*x+e)^n,x, algorithm="fricas")
[Out] integral((a*cot(f*x + e))^m*tan(f*x + e)^n, x)
```

Sympy [F]

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \int (a\cot(e+fx))^m \tan^n(e+fx)\,dx$$

```
[In] integrate((a*cot(f*x+e))**m*tan(f*x+e)**n,x)
[Out] Integral((a*cot(e + f*x))**m*tan(e + f*x)**n, x)
```

Maxima [F]

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \int (a\cot(fx+e))^m \tan(fx+e)^n \,dx$$

```
[In] integrate((a*cot(f*x+e))^m*tan(f*x+e)^n,x, algorithm="maxima")
[Out] integrate((a*cot(f*x + e))^m*tan(f*x + e)^n, x)
```

Giac [F]

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \int (a\cot(fx+e))^m \tan(fx+e)^n\,dx$$

```
[In] integrate((a*cot(f*x+e))^m*tan(f*x+e)^n,x, algorithm="giac")
[Out] integrate((a*cot(f*x + e))^m*tan(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \int \tan(e+f\,x)^n\,(a\cot(e+f\,x))^m\,dx$$

```
[In] int(tan(e + f*x)^n*(a*cot(e + f*x))^m,x)
[Out] int(tan(e + f*x)^n*(a*cot(e + f*x))^m, x)
```

3.225 $\int (a\cot(e+fx))^m (b\tan(e+fx))^n \, dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int (a\cot(e+fx))^m (b\tan(e+fx))^n \, dx = \frac{(a\cot(e+fx))^m \text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)(b\tan(e+fx))^{1+\cdots}}{bf(1-m+n)}$$

```
[Out] (a*cot(f*x+e))^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x+
e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1-m+n)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2684, 3557, 371}

$$\int (a\cot(e+fx))^m (b\tan(e+fx))^n \, dx = \frac{(a\cot(e+fx))^m (b\tan(e+fx))^{n+1} \text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n+3),-\tan^2(e+\cdots\right)}{bf(-m+n+1)}$$

```
[In] Int[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan
[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - m + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2684

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = ((a\cot(e+fx))^m(b\tan(e+fx))^m)\int (b\tan(e+fx))^{-m+n}\,dx$$

$$= \frac{(b(a\cot(e+fx))^m(b\tan(e+fx))^m)\,\text{Subst}\left(\int \frac{x^{-m+n}}{b^2+x^2}\,dx, x, b\tan(e+fx)\right)}{f}$$

$$= \frac{(a\cot(e+fx))^m\,\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1-m+n), \frac{1}{2}(3-m+n), -\tan^2(e+fx)\right)(b\tan(e+}{bf(1-m+n)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx$$

$$= \frac{a(a\cot(e+fx))^{-1+m}\,\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1-m+n), \frac{1}{2}(3-m+n), -\tan^2(e+fx)\right)(b\tan(e+fx)}{f(1-m+n)}$$

```
[In] Integrate[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (a*(a*Cot[e + f*x])^(-1 + m)*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n
)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - m + n))
```

Maple [F]

$$\int (a\cot(fx+e))^m (b\tan(fx+e))^n \, dx$$

```
[In] int((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x)
[Out] int((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n \, dx = \int (a\cot(fx+e))^m (b\tan(fx+e))^n \, dx$$

```
[In] integrate((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*cot(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F]

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n \, dx = \int (a\cot(e+fx))^m (b\tan(e+fx))^n \, dx$$

```
[In] integrate((a*cot(f*x+e))**m*(b*tan(f*x+e))**n,x)
[Out] Integral((a*cot(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Maxima [F]

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n \, dx = \int (a\cot(fx+e))^m (b\tan(fx+e))^n \, dx$$

```
[In] integrate((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*cot(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Giac [F]

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\cot(fx+e))^m\,(b\tan(fx+e))^n\,dx$$

```
[In] integrate((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*cot(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\cot(e+f\,x))^m\,(b\tan(e+f\,x))^n\,dx$$

```
[In] int((a*cot(e + f*x))^m*(b*tan(e + f*x))^n,x)
[Out] int((a*cot(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

3.226 $\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2(d\tan(e+fx))^{3/2}}{3df} + \frac{4(d\tan(e+fx))^{7/2}}{7d^3f} + \frac{2(d\tan(e+fx))^{11/2}}{11d^5f}$$

```
[Out] 2/3*(d*tan(f*x+e))^(3/2)/d/f+4/7*(d*tan(f*x+e))^(7/2)/d^3/f+2/11*(d*tan(f*x
+e))^(11/2)/d^5/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2687, 276}

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2(d\tan(e+fx))^{11/2}}{11d^5f} + \frac{4(d\tan(e+fx))^{7/2}}{7d^3f} + \frac{2(d\tan(e+fx))^{3/2}}{3df}$$

```
[In] Int[Sec[e + f*x]^6*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(d*Tan[e + f*x])^(3/2))/(3*d*f) + (4*(d*Tan[e + f*x])^(7/2))/(7*d^3*f) +
 (2*(d*Tan[e + f*x])^(11/2))/(11*d^5*f)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{dx}(1+x^2)^2\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(\sqrt{dx}+\frac{2(dx)^{5/2}}{d^2}+\frac{(dx)^{9/2}}{d^4}\right)\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{2(d\tan(e+fx))^{3/2}}{3df}+\frac{4(d\tan(e+fx))^{7/2}}{7d^3f}+\frac{2(d\tan(e+fx))^{11/2}}{11d^5f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2(45+28\cos(2(e+fx))+4\cos(4(e+fx)))\sec^4(e+fx)(d\tan(e+fx))^{3/2}}{231df}$$

```
[In] Integrate[Sec[e + f*x]^6*Sqrt[d*Tan[e + f*x]],x]
[Out] (2*(45 + 28*Cos[2*(e + f*x)] + 4*Cos[4*(e + f*x)])*Sec[e + f*x]^4*(d*Tan[e
+ f*x])^(3/2))/(231*d*f)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{2(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{4d^2(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d^4(d\tan(fx+e))^{\frac{3}{2}}}{3}}{f\,d^5}$	52
default	$\frac{\frac{2(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{4d^2(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d^4(d\tan(fx+e))^{\frac{3}{2}}}{3}}{f\,d^5}$	52

```
[In] int(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d^5*(1/11*(d*tan(f*x+e))^(11/2)+2/7*d^2*(d*tan(f*x+e))^(7/2)+1/3*d^4*(d
*tan(f*x+e))^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\left(32\cos\left(fx+e\right)^4+24\cos\left(fx+e\right)^2+21\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin\left(fx+e\right)}{231\,f\cos\left(fx+e\right)^5}$$

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(32*cos(f*x + e)^4 + 24*cos(f*x + e)^2 + 21)*sqrt(d*sin(f*x + e)/cos(
f*x + e))*sin(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(e+fx\right)}\sec^6\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**6*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\left(21\left(d\tan\left(fx+e\right)\right)^{\frac{11}{2}}+66\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}d^2+77\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}d^4\right)}{231\,d^5f}$$

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/231*(21*(d*tan(f*x + e))^(11/2) + 66*(d*tan(f*x + e))^(7/2)*d^2 + 77*(d*t
an(f*x + e))^(3/2)*d^4)/(d^5*f)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{2\left(21\sqrt{d\tan(fx+e)}d^5\tan(fx+e)^5+66\sqrt{d\tan(fx+e)}d^5\tan(fx+e)^3+77\sqrt{d\tan(fx+e)}d^5\tan(fx+e)\right)}{231\,d^5f}$$

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/231*(21*sqrt(d*tan(f*x + e))*d^5*tan(f*x + e)^5 + 66*sqrt(d*tan(f*x + e))
*d^5*tan(f*x + e)^3 + 77*sqrt(d*tan(f*x + e))*d^5*tan(f*x + e))/(d^5*f)
```

Mupad [B] (verification not implemented)

Time = 8.45 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.99

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,64i}{231\,f} - \frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,64i}{231\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)} - \frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,32i}{77\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^2} + \frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,768i}{77\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^3} - \frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,160i}{11\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^4} + \frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,64i}{11\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^5}$$

```
[In] int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^6,x)
```

```
[Out] ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*768i)/(
77*f*(exp(e*2i + f*x*2i) + 1)^3) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp
(e*2i + f*x*2i) + 1))^(1/2)*64i)/(231*f*(exp(e*2i + f*x*2i) + 1)) - ((-(d*(
exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*32i)/(77*f*(ex
p(e*2i + f*x*2i) + 1)^2) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i +
f*x*2i) + 1))^(1/2)*64i)/(231*f) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp
(e*2i + f*x*2i) + 1))^(1/2)*160i)/(11*f*(exp(e*2i + f*x*2i) + 1)^4) + ((-(d
*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*64i)/(11*f*(
exp(e*2i + f*x*2i) + 1)^5)
```

3.227 $\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2(d\tan(e+fx))^{3/2}}{3df} + \frac{2(d\tan(e+fx))^{7/2}}{7d^3f}$$

```
[Out] 2/3*(d*tan(f*x+e))^(3/2)/d/f+2/7*(d*tan(f*x+e))^(7/2)/d^3/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2687, 14}

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2(d\tan(e+fx))^{7/2}}{7d^3f} + \frac{2(d\tan(e+fx))^{3/2}}{3df}$$

```
[In] Int[Sec[e + f*x]^4*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(d*Tan[e + f*x])^(3/2))/(3*d*f) + (2*(d*Tan[e + f*x])^(7/2))/(7*d^3*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{dx}(1+x^2)\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(\sqrt{dx}+\frac{(dx)^{5/2}}{d^2}\right)\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{2(d\tan(e+fx))^{3/2}}{3df}+\frac{2(d\tan(e+fx))^{7/2}}{7d^3f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2(4+3\sec^2(e+fx))\,(d\tan(e+fx))^{3/2}}{21df}$$

```
[In] Integrate[Sec[e + f*x]^4*Sqrt[d*Tan[e + f*x]],x]
[Out] (2*(4 + 3*Sec[e + f*x]^2)*(d*Tan[e + f*x])^(3/2))/(21*d*f)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{2(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d^2(d\tan(fx+e))^{\frac{3}{2}}}{3}}{f\,d^3}$	37
default	$\frac{\frac{2(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d^2(d\tan(fx+e))^{\frac{3}{2}}}{3}}{f\,d^3}$	37

```
[In] int(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/f/d^3*(1/7*(d*tan(f*x+e))^(7/2)+1/3*d^2*(d*tan(f*x+e))^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\left(4\cos\left(fx+e\right)^2+3\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin\left(fx+e\right)}{21\,f\cos\left(fx+e\right)^3}$$

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] 2/21*(4*cos(f*x + e)^2 + 3)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e)/
(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(e+fx\right)}\sec^4\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**4*(d*tan(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\left(3\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}+7\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}d^2\right)}{21\,d^3f}$$

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] 2/21*(3*(d*tan(f*x + e))^(7/2) + 7*(d*tan(f*x + e))^(3/2)*d^2)/(d^3*f)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$=\frac{2\left(3\sqrt{d\tan(fx+e)}d^3\tan(fx+e)^3+7\sqrt{d\tan(fx+e)}d^3\tan(fx+e)\right)}{21\,d^3f}$$

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*(3*sqrt(d*tan(f*x + e))*d^3*tan(f*x + e)^3 + 7*sqrt(d*tan(f*x + e))*d^
3*tan(f*x + e))/(d^3*f)
```

Mupad [B] (verification not implemented)

Time = 7.26 (sec) , antiderivative size = 218, normalized size of antiderivative = 4.84

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx=-\frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,8i}{21\,f}-\frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,8i}{21\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)}+\frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,24i}{7\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^2}-\frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,16i}{7\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^3}$$

```
[In] int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^4,x)
```

```
[Out] ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*24i)/(7
*f*(exp(e*2i + f*x*2i) + 1)^2) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e
*2i + f*x*2i) + 1))^(1/2)*8i)/(21*f*(exp(e*2i + f*x*2i) + 1)) - ((-(d*(exp(
e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*8i)/(21*f) - ((-(d
*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*16i)/(7*f*(e
xp(e*2i + f*x*2i) + 1)^3)
```

3.228 $\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx=\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

```
[Out] 2/3*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2687, 32}

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx=\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

```
[In] Int[Sec[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
[Out] (2*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \sqrt{dx}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{2(d\tan(e+fx))^{3/2}}{3df}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2(d\tan(e+fx))^{3/2}}{3df}$$

```
[In] Integrate[Sec[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
[Out] (2*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2(d\tan(fx+e))^{\frac{3}{2}}}{3df}$	19
default	$\frac{2(d\tan(fx+e))^{\frac{3}{2}}}{3df}$	19

```
[In] int(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(d*tan(f*x+e))^(3/2)/d/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin(fx+e)}{3f\cos(fx+e)}$$

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(e+fx)}\sec^2(e+fx)\,dx$$

```
[In] integrate(sec(f*x+e)**2*(d*tan(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\,(d\tan(fx+e))^{\frac{3}{2}}}{3\,df}$$

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] 2/3*(d*tan(f*x + e))^(3/2)/(d*f)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\,\sqrt{d\tan(fx+e)}\tan(fx+e)}{3\,f}$$

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(d*tan(f*x + e))*tan(f*x + e)/f
```

Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\,\sin(2\,e+2\,f\,x)\,\sqrt{\frac{d\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{3\,f\,\left(\cos(2\,e+2\,f\,x)+1\right)}$$

```
[In] int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^2,x)
[Out] (2*sin(2*e + 2*f*x)*((d*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(3
*f*(cos(2*e + 2*f*x) + 1))
```

3.229 $\int \sqrt{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 192

$$\int \sqrt{d\tan(e+fx)}\,dx = -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f}-\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/2*a
rctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1
/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/f*2^(1/2)-1/4*
ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/f*2^(1/
2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used

$= \{3557, 335, 303, 1176, 631, 210, 1179, 642\}$

$$\int \sqrt{d\tan(e+fx)}\,dx = -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$

```
[In] Int[Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x
]]])/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*
Sqrt[d*Tan[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(e+fx)\right)}{f}\\
&= \frac{(2d)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}\\
&= -\frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}+\frac{d\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} \\
&\quad + \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} \\
&\quad + \frac{d\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2f} \\
&\quad + \frac{d\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2f} \\
&= \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} \\
&\quad - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} \\
&\quad + \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} \\
&\quad - \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} \\
&= -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} \\
&\quad + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} \\
&\quad - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.37

$$
\int \sqrt{d\tan(e+fx)}\,dx
= \frac{\left(\arctan\left(\sqrt[4]{-\tan^2(e+fx)}\right)-\text{arctanh}\left(\sqrt[4]{-\tan^2(e+fx)}\right)\right)\sqrt[4]{-\tan(e+fx)}\sqrt{d\tan(e+fx)}}{f\tan^{\frac{3}{4}}(e+fx)}
$$

```
[In] Integrate[Sqrt[d*Tan[e + f*x]],x]
[Out] ((ArcTan[(-Tan[e + f*x]^2)^(1/4)] - ArcTanh[(-Tan[e + f*x]^2)^(1/4)])*(-Tan
[e + f*x])^(1/4)*Sqrt[d*Tan[e + f*x]])/(f*Tan[e + f*x]^(3/4))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{d\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}{d\tan(fx+e)+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan(fx+e)}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{d\tan(fx+e)}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right.}{4f\left(d^{2}\right)^{\frac{1}{4}}}$
default	$\frac{d\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}{d\tan(fx+e)+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan(fx+e)}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{d\tan(fx+e)}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right.}{4f\left(d^{2}\right)^{\frac{1}{4}}}$

```
[In] int((d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*d/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1
/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(
1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*a
rctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\begin{aligned}\int\sqrt{d\tan(e+fx)}\,dx=&\ \frac{1}{2}\left(-\frac{d^{2}}{f^{4}}\right)^{\frac{1}{4}}\log\left(f^{3}\left(-\frac{d^{2}}{f^{4}}\right)^{\frac{3}{4}}+\sqrt{d\tan\left(fx+e\right)}d\right)\\&-\frac{1}{2}i\left(-\frac{d^{2}}{f^{4}}\right)^{\frac{1}{4}}\log\left(i\,f^{3}\left(-\frac{d^{2}}{f^{4}}\right)^{\frac{3}{4}}+\sqrt{d\tan\left(fx+e\right)}d\right)\\&+\frac{1}{2}i\left(-\frac{d^{2}}{f^{4}}\right)^{\frac{1}{4}}\log\left(-i\,f^{3}\left(-\frac{d^{2}}{f^{4}}\right)^{\frac{3}{4}}+\sqrt{d\tan\left(fx+e\right)}d\right)\\&-\frac{1}{2}\left(-\frac{d^{2}}{f^{4}}\right)^{\frac{1}{4}}\log\left(-f^{3}\left(-\frac{d^{2}}{f^{4}}\right)^{\frac{3}{4}}+\sqrt{d\tan\left(fx+e\right)}d\right)\end{aligned}$$

```
[In] integrate((d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-d^2/f^4)^(1/4)*log(f^3*(-d^2/f^4)^(3/4) + sqrt(d*tan(f*x + e))*d) - 1
/2*I*(-d^2/f^4)^(1/4)*log(I*f^3*(-d^2/f^4)^(3/4) + sqrt(d*tan(f*x + e))*d)
+ 1/2*I*(-d^2/f^4)^(1/4)*log(-I*f^3*(-d^2/f^4)^(3/4) + sqrt(d*tan(f*x + e))
*d) - 1/2*(-d^2/f^4)^(1/4)*log(-f^3*(-d^2/f^4)^(3/4) + sqrt(d*tan(f*x + e))
*d)
```

Sympy [F]

$$\int \sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan{(e+fx)}}\,dx$$

```
[In] integrate((d*tan(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*tan(e + f*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.80

$$\int \sqrt{d\tan(e+fx)}\,dx = \frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+\right.}{\sqrt{d}}\right.}{4f}$$

```
[In] integrate((d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] 1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e
)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*s
qrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) + sqrt(2
)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x + e) -
sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.92

$$\int \sqrt{d\tan(e+fx)}\,dx = \frac{\frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f} + \frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f} - \frac{\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan\right.}{f}}{4d}$$

```
[In] integrate((d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

[Out] 1/4*(2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f + 2*sqrt(2)*abs(d)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f - sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/f + sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/f)/d

Mupad [B] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.26

$$\int \sqrt{d\tan(e+fx)}\,dx = \frac{(-1)^{1/4}\sqrt{d}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)\right)}{f}$$

[In] int((d*tan(e + f*x))^(1/2),x)

[Out] ((-1)^(1/4)*d^(1/2)*(atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)) - atanh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))))/f

3.230 $\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 227

$$
\begin{aligned}
&\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx \\
&= -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}+\frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f} \\
&\quad+\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f} \\
&\quad-\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f} \\
&\quad+\frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df}
\end{aligned}
$$

```
[Out] -1/8*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/8*a
rctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/16*ln(d^(
1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/f*2^(1/2)-1/1
6*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/f*2^(
1/2)+1/2*cos(f*x+e)^2*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2687, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$=-\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}+\frac{\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}f}$$
$$+\frac{\sqrt{d}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{8\sqrt{2}f}$$
$$-\frac{\sqrt{d}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{8\sqrt{2}f}$$
$$+\frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df}$$

```
[In] Int[Cos[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] -1/4*(Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*
f) + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(4*Sqrt[2
]*f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e +
 f*x]]])/(8*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt
[2]*Sqrt[d*Tan[e + f*x]]])/(8*Sqrt[2]*f) + (Cos[e + f*x]^2*(d*Tan[e + f*x])
^(3/2))/(2*d*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{dx}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df} + \frac{\text{Subst}\left(\int \frac{\sqrt{dx}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{4f}$$

$$= \frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df} + \frac{\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2df}$$

$$= \frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df} - \frac{\text{Subst}\left(\int \frac{d-x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{4df} + \frac{\text{Subst}\left(\int \frac{d+x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{4df}$$

$$= \frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f} + \frac{d\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{8f} + \frac{d\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{8f}$$

$$= \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f} + \frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f} - \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}$$

$$= -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}$$
$$+ \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f}$$
$$- \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f}$$
$$+ \frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.45

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx =$$
$$-\frac{\left(\arcsin(\cos(e+fx)-\sin(e+fx))\csc(e+fx)+\csc(e+fx)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(\right.\right.}}{8f}$$

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] -1/8*((ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Csc[e + f*x] + Csc[e + f*x]*Log[
Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]] - 2*Sqrt[Sin[2*(e + f
*x)]])*Sqrt[Sin[2*(e + f*x)]]*Sqrt[d*Tan[e + f*x]])/f
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(171) = 342$.

Time = 1.17 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.33

method	result
default	$\frac{\left(4\sqrt{2}\,\cos(fx+e)\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)+4\sqrt{2}\,\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)-\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)}{}\right.\right.}{}$

```
[In] int(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/f*(4*2^(1/2)*cos(f*x+e)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)*sin(f*x+e)+4*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(
f*x+e)-ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)^3+
3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)
```

```
-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))+ln
((2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(
f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-cot(f*x+e)*cos(f*x+e)+sin(
f*x+e)+2*cos(f*x+e)+2*cot(f*x+e)-csc(f*x+e)-2)/(cos(f*x+e)-1))+2*arctan((2^
(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)
+1)/(cos(f*x+e)-1))+2*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1)))*(d*tan(f*x+e))^(1/2)*c
os(f*x+e)/(cos(f*x+e)+1)/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*2^
(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 924, normalized size of antiderivative = 4.07

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(16*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + f*(-
d^2/f^4)^(1/4)*log(1/2*d^2*cos(f*x + e)*sin(f*x + e) + 1/2*(f^3*(-d^2/f^4)^
(3/4)*cos(f*x + e)^2 - d*f*(-d^2/f^4)^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt
(d*sin(f*x + e)/cos(f*x + e)) - 1/4*(2*d*f^2*cos(f*x + e)^2 - d*f^2)*sqrt(-
d^2/f^4)) - f*(-d^2/f^4)^(1/4)*log(1/2*d^2*cos(f*x + e)*sin(f*x + e) - 1/2*
(f^3*(-d^2/f^4)^(3/4)*cos(f*x + e)^2 - d*f*(-d^2/f^4)^(1/4)*cos(f*x + e)*si
n(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)) - 1/4*(2*d*f^2*cos(f*x + e)^2
 - d*f^2)*sqrt(-d^2/f^4)) - I*f*(-d^2/f^4)^(1/4)*log(1/2*d^2*cos(f*x + e)*s
in(f*x + e) + 1/2*(I*f^3*(-d^2/f^4)^(3/4)*cos(f*x + e)^2 + I*d*f*(-d^2/f^4)
^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)) + 1/4*(
2*d*f^2*cos(f*x + e)^2 - d*f^2)*sqrt(-d^2/f^4)) + I*f*(-d^2/f^4)^(1/4)*log(
1/2*d^2*cos(f*x + e)*sin(f*x + e) + 1/2*(-I*f^3*(-d^2/f^4)^(3/4)*cos(f*x +
e)^2 - I*d*f*(-d^2/f^4)^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e
)/cos(f*x + e)) + 1/4*(2*d*f^2*cos(f*x + e)^2 - d*f^2)*sqrt(-d^2/f^4)) + f*
(-d^2/f^4)^(1/4)*log(d^2 + 2*(f^3*(-d^2/f^4)^(3/4)*cos(f*x + e)*sin(f*x + e
) - d*f*(-d^2/f^4)^(1/4)*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e)))
 - f*(-d^2/f^4)^(1/4)*log(d^2 - 2*(f^3*(-d^2/f^4)^(3/4)*cos(f*x + e)*sin(f*
x + e) - d*f*(-d^2/f^4)^(1/4)*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x +
 e))) + I*f*(-d^2/f^4)^(1/4)*log(d^2 - 2*(I*f^3*(-d^2/f^4)^(3/4)*cos(f*x +
e)*sin(f*x + e) + I*d*f*(-d^2/f^4)^(1/4)*cos(f*x + e)^2)*sqrt(d*sin(f*x + e
)/cos(f*x + e))) - I*f*(-d^2/f^4)^(1/4)*log(d^2 - 2*(-I*f^3*(-d^2/f^4)^(3/4
)*cos(f*x + e)*sin(f*x + e) - I*d*f*(-d^2/f^4)^(1/4)*cos(f*x + e)^2)*sqrt(d
*sin(f*x + e)/cos(f*x + e))))/f
```

Sympy [F]

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(e+fx)}\cos^2(e+fx)\,dx$$

```
[In] integrate(cos(f*x+e)**2*(d*tan(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*tan(e + f*x))*cos(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.85

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}\cdots\right.}{\sqrt{d}}\cdots\right.}{16\,df\cdots}$$

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] 1/16*(d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x
 + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) + sq
rt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x + e
) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) + 8*(d*tan(f*x + e))
^(3/2)*d^2/(d^2*tan(f*x + e)^2 + d^2))/(d*f)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.96

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{\frac{8\sqrt{d\tan(fx+e)}d^3\tan(fx+e)}{\left(d^2\tan(fx+e)^2+d^2\right)f} + \frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f} + \frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f} - \cdots}{16\,d\cdots}$$

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(8*sqrt(d*tan(f*x + e))*d^3*tan(f*x + e)/((d^2*tan(f*x + e)^2 + d^2)*f
) + 2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqr
t(d*tan(f*x + e)))/sqrt(abs(d)))/f + 2*sqrt(2)*abs(d)^(3/2)*arctan(-1/2*sqr
t(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f - sqrt
(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs
(d)) + abs(d))/f + sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d
*tan(f*x + e))*sqrt(abs(d)) + abs(d))/f)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx=\int \cos\left(e+f\,x\right)^2\,\sqrt{d\tan\left(e+f\,x\right)}\,dx$$

```
[In] int(cos(e + f*x)^2*(d*tan(e + f*x))^(1/2),x)
```

```
[Out] int(cos(e + f*x)^2*(d*tan(e + f*x))^(1/2), x)
```

3.231 $\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{4\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{5f\sqrt{\sin(2e+2fx)}} + \frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

```
[Out] 4/5*cos(f*x+e)*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(
e+1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/sin(2*f*x+2*e)^(1/2)+4/5*cos(
f*x+e)*(d*tan(f*x+e))^(3/2)/d/f+2/5*sec(f*x+e)*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2693, 2695, 2652, 2719}

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df} - \frac{4\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{5f\sqrt{\sin(2e+2fx)}}$$

```
[In] Int[Sec[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-4*Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(5*f*Sq
rt[Sin[2*e + 2*f*x]]) + (4*Cos[e + f*x]*(d*Tan[e + f*x])^(3/2))/(5*d*f) + (
2*Sec[e + f*x]*(d*Tan[e + f*x])^(3/2))/(5*d*f)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{2}{5}\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx\\
&= \frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}\\
&\quad -\frac{4}{5}\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx\\
&= \frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}\\
&\quad -\frac{\left(4\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\right)\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{5\sqrt{\sin(e+fx)}}
\end{aligned}
$$

$$= \frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df} - \frac{\left(4\cos(e+fx)\sqrt{d\tan(e+fx)}\right)\int\sqrt{\sin(2e+2fx)}\,dx}{5\sqrt{\sin(2e+2fx)}}$$

$$= -\frac{4\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{5f\sqrt{\sin(2e+2fx)}} + \frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.49 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\sqrt{d\tan(e+fx)}\left(-4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(e+fx)\right)\sec(e+fx)\tan(e+fx)+3\sqrt{\sec^2(e\right.}{15f\sqrt{\sec^2(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
[Out] (2*Sqrt[d*Tan[e + f*x]]*(-4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^
2]*Sec[e + f*x]*Tan[e + f*x] + 3*Sqrt[Sec[e + f*x]^2]*(2*Sin[e + f*x] + Sec
[e + f*x]*Tan[e + f*x])))/(15*f*Sqrt[Sec[e + f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(118) = 236.

Time = 1.46 (sec) , antiderivative size = 401, normalized size of antiderivative = 3.75

method	result
default	$-\frac{\sqrt{d\tan(fx+e)}\left(-4\cot(fx+e)\sqrt{\cot(fx+e)-\csc(fx+e)+1}\sqrt{\cot(fx+e)-\csc(fx+e)}\,E\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)\sqrt{-c\right.}{}$

```
[In] int(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/5/f*(d*tan(f*x+e))^(1/2)*(-4*cot(f*x+e)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*
(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/
2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)+2*cot(f*x+e)*(cot(f*x+e)-csc(f*
x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2
)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))-4*csc(f*x+e)*(cot
```

```
(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot(f
*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)+2*c
sc(f*x+e)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-c
ot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/
2*2^(1/2))+2*2^(1/2)*cot(f*x+e)-csc(f*x+e)*2^(1/2)-sec(f*x+e)^2*csc(f*x+e)*
2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.72

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx =$$
$$-\frac{2\left(i\sqrt{i\,d}\cos\left(fx+e\right)^2 E(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\,|\,-1)-i\sqrt{-i\,d}\cos\left(fx+e\right)^2 E(\arcsin\left(\text{c}\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(I*sqrt(I*d)*cos(f*x + e)^2*elliptic_e(arcsin(cos(f*x + e) + I*sin(f*x
 + e)), -1) - I*sqrt(-I*d)*cos(f*x + e)^2*elliptic_e(arcsin(cos(f*x + e) -
I*sin(f*x + e)), -1) - I*sqrt(I*d)*cos(f*x + e)^2*elliptic_f(arcsin(cos(f*x
 + e) + I*sin(f*x + e)), -1) + I*sqrt(-I*d)*cos(f*x + e)^2*elliptic_f(arcsi
n(cos(f*x + e) - I*sin(f*x + e)), -1) - (2*cos(f*x + e)^2 + 1)*sqrt(d*sin(f
*x + e)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(e+fx\right)}\sec^3\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**3*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**3, x)
```

Maxima [F]

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\sec(fx+e)^3\,dx$$

```
[In] integrate(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(f*x + e))*sec(f*x + e)^3, x)
```

Giac [F]

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\sec(fx+e)^3\,dx$$

```
[In] integrate(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*tan(f*x + e))*sec(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \frac{\sqrt{d\tan(e+f\,x)}}{\cos(e+f\,x)^3}\,dx$$

```
[In] int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^3,x)
[Out] int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^3, x)
```

3.232 $\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{2\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}} + \frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}$$

```
[Out] 2*cos(f*x+e)*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+
1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/sin(2*f*x+2*e)^(1/2)+2*cos(f*x+
e)*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2693, 2695, 2652, 2719}

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df} - \frac{2\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

```
[In] Int[Sec[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(f*Sqrt
[Sin[2*e + 2*f*x]]) + (2*Cos[e + f*x]*(d*Tan[e + f*x])^(3/2))/(d*f)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df} - 2\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx \\
&= \frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df} \\
&\quad - \frac{\left(2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\right)\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{\sqrt{\sin(e+fx)}} \\
&= \frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df} - \frac{\left(2\cos(e+fx)\sqrt{d\tan(e+fx)}\right)\int \sqrt{\sin(2e+2fx)}\,dx}{\sqrt{\sin(2e+2fx)}} \\
&= -\frac{2\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}} + \frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{2\left(-3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\right)\sin(e+fx)\sqrt{d\tan(e+fx)}}{3f}$$

```
[In] Integrate[Sec[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*(-3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sqrt[Sec[e +
f*x]^2])*Sin[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(94) = 188.

Time = 1.42 (sec) , antiderivative size = 367, normalized size of antiderivative = 4.89

method	result
default	$-\frac{\csc(fx+e)\left(-2\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,E\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)\sqrt{-\cot(fx+e)+\csc(fx}}{}$

```
[In] int(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*csc(f*x+e)*(-2*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))
^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*(-cot(f*x+e)
+csc(f*x+e)+1)^(1/2)*cos(f*x+e)+(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)
-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+
csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*cos(f*x+e)-2*(cot(f*x+e)-csc(f*x+e)+1)^(1/
2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2)
,1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)+(cot(f*x+e)-csc(f*x+e)+1)^(1
/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*Elliptic
F((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+2^(1/2)*cos(f*x+e)-2^(1/2))
*(d*tan(f*x+e))^(1/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.75

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{-i\sqrt{i\,d}E(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\,|-1)+i\sqrt{-i\,d}E(\arcsin\left(\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)\,|}{}$$

```
[In] integrate(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] (-I*sqrt(I*d)*elliptic_e(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1) + I*sqr
t(-I*d)*elliptic_e(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1) + I*sqrt(I*d)
*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1) - I*sqrt(-I*d)*ellip
tic_f(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1) + 2*sqrt(d*sin(f*x + e)/co
s(f*x + e))*sin(f*x + e))/f
```

Sympy [F]

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(e+fx\right)}\sec\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)*(d*tan(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*tan(e + f*x))*sec(e + f*x), x)
```

Maxima [F]

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}\sec\left(fx+e\right)\,dx$$

```
[In] integrate(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(f*x + e))*sec(f*x + e), x)
```

Giac [F]

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}\sec\left(fx+e\right)\,dx$$

```
[In] integrate(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*tan(f*x + e))*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{\cos\left(e+f\,x\right)}\,dx$$

```
[In] int((d*tan(e + f*x))^(1/2)/cos(e + f*x),x)
[Out] int((d*tan(e + f*x))^(1/2)/cos(e + f*x), x)
```

3.233 $\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx=\frac{\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

```
[Out] -cos(f*x+e)*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1
/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2695, 2652, 2719}

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx=\frac{\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

```
[In] Int[Cos[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(f*Sqrt[Si
n[2*e + 2*f*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\right)\int\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{\sqrt{\sin(e+fx)}}\\
&= \frac{\left(\cos(e+fx)\sqrt{d\tan(e+fx)}\right)\int\sqrt{\sin(2e+2fx)}\,dx}{\sqrt{\sin(2e+2fx)}}\\
&= \frac{\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int\cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$=\frac{2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\sin(e+fx)\sqrt{d\tan(e+fx)}}{3f}$$

```
[In] Integrate[Cos[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
[Out] (2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sqrt[Sec[e + f*x]^2]*S
in[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(69) = 138$.

Time = 0.95 (sec) , antiderivative size = 377, normalized size of antiderivative = 8.02

method	result
default	$-\frac{\csc(fx+e)\left(2\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,E\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)\sqrt{-\cot(fx+e)+\csc(f\ldots}{}$

```
[In] int(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*csc(f*x+e)*(2*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)
)^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*(-cot(f*x+e
)+csc(f*x+e)+1)^(1/2)*cos(f*x+e)-(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e
)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)
+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*cos(f*x+e)+2*(cot(f*x+e)-csc(f*x+e)+1)^(1
/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2
),1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)-(cot(f*x+e)-csc(f*x+e)+1)^(
1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*Ellipti
cF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+2^(1/2)*cos(f*x+e)^2-2^(1/
2)*cos(f*x+e))*(d*tan(f*x+e))^(1/2)*2^(1/2)
```

Fricas [F]

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\cos(fx+e)\,dx$$

```
[In] integrate(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*cos(f*x + e), x)
```

Sympy [F]

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(e+fx)}\cos(e+fx)\,dx$$

```
[In] integrate(cos(f*x+e)*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))*cos(e + f*x), x)
```

Maxima [F]

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)\,dx$$

```
[In] integrate(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(f*x + e))*cos(f*x + e), x)
```

Giac [F]

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)\,dx$$

```
[In] integrate(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*tan(f*x + e))*cos(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \cos\left(e+f\,x\right)\,\sqrt{d\tan\left(e+f\,x\right)}\,dx$$

```
[In] int(cos(e + f*x)*(d*tan(e + f*x))^(1/2),x)
[Out] int(cos(e + f*x)*(d*tan(e + f*x))^(1/2), x)
```

3.234 $\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{2f\sqrt{\sin(2e+2fx)}} + \frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

```
[Out] -1/2*cos(f*x+e)*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos
(e+1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/sin(2*f*x+2*e)^(1/2)+1/3*cos
(f*x+e)^3*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2692, 2695, 2652, 2719}

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df} + \frac{\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{2f\sqrt{\sin(2e+2fx)}}$$

```
[In] Int[Cos[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(2*f*Sqrt[
Sin[2*e + 2*f*x]]) + (Cos[e + f*x]^3*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df} + \frac{1}{2}\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx \\
&= \frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df} \\
&\quad + \frac{\left(\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\right)\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{2\sqrt{\sin(e+fx)}} \\
&= \frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df} + \frac{\left(\cos(e+fx)\sqrt{d\tan(e+fx)}\right)\int \sqrt{\sin(2e+2fx)}\,dx}{2\sqrt{\sin(2e+2fx)}} \\
&= \frac{\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{2f\sqrt{\sin(2e+2fx)}} + \frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.53 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{\sqrt{d\tan(e+fx)}\left(\sqrt{\sec^2(e+fx)}(\sin(e+fx)+\sin(3(e+fx)))+4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(e\right.\right.}{12f\sqrt{\sec^2(e+fx)}}$$

[In] Integrate[Cos[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]

```
[Out] (Sqrt[d*Tan[e + f*x]]*(Sqrt[Sec[e + f*x]^2]*(Sin[e + f*x] + Sin[3*(e + f*x)
]) + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sec[e + f*x]*Tan[e
 + f*x]))/(12*f*Sqrt[Sec[e + f*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(96) = 192.

Time = 1.08 (sec) , antiderivative size = 390, normalized size of antiderivative = 4.81

method	result
default	$-\frac{\csc(fx+e)\left(2(\cos^4(fx+e))\sqrt{2}+6\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,E\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)\sqrt{\right.}{}$

```
[In] int(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/f*csc(f*x+e)*(2*cos(f*x+e)^4*2^(1/2)+6*(cot(f*x+e)-csc(f*x+e)+1)^(1/2
)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),
1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*cos(f*x+e)-3*(cot(f*x+e)-csc(
f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1
/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*cos(f*x+e)+6*(c
ot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot
(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)-3
*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)
+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2)
)+2^(1/2)*cos(f*x+e)^2-3*2^(1/2)*cos(f*x+e))*(d*tan(f*x+e))^(1/2)*2^(1/2)
```

Fricas [F]

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)^3\,dx$$

```
[In] integrate(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(f*x + e))*cos(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**3*(d*tan(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)^3\,dx$$

```
[In] integrate(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(f*x + e))*cos(f*x + e)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]e
xt_reduce Error: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \cos\left(e+f\,x\right)^3\,\sqrt{d\tan\left(e+f\,x\right)}\,dx$$

```
[In] int(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2),x)
[Out] int(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2), x)
```

3.235 $\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{7\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{20f\sqrt{\sin(2e+2fx)}} + \frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df} + \frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

```
[Out] -7/20*cos(f*x+e)*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(co
s(e+1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/sin(2*f*x+2*e)^(1/2)+7/30*c
os(f*x+e)^3*(d*tan(f*x+e))^(3/2)/d/f+1/5*cos(f*x+e)^5*(d*tan(f*x+e))^(3/2)/
d/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2692, 2695, 2652, 2719}

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df} + \frac{7\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{20f\sqrt{\sin(2e+2fx)}}$$

```
[In] Int[Cos[e + f*x]^5*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (7*Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(20*f*Sq
rt[Sin[2*e + 2*f*x]]) + (7*Cos[e + f*x]^3*(d*Tan[e + f*x])^(3/2))/(30*d*f)
+ (Cos[e + f*x]^5*(d*Tan[e + f*x])^(3/2))/(5*d*f)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{7}{10}\int\cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx\\
&= \frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df}+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}\\
&\quad+\frac{7}{20}\int\cos(e+fx)\sqrt{d\tan(e+fx)}\,dx\\
&= \frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df}+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}\\
&\quad+\frac{\left(7\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\right)\int\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{20\sqrt{\sin(e+fx)}}
\end{aligned}$$

$$= \frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df} + \frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{\left(7\cos(e+fx)\sqrt{d\tan(e+fx)}\right)\int\sqrt{\sin(2e+2fx)}\,dx}{20\sqrt{\sin(2e+2fx)}}$$

$$= \frac{7\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{20f\sqrt{\sin(2e+2fx)}} + \frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df} + \frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.83 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int\cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\left(20\sin(2(e+fx))+3\sin(4(e+fx))+28\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2\right.\right.}{120f}$$

```
[In] Integrate[Cos[e + f*x]^5*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[d*Tan[e + f*x]]*(20*Sin[2*(e + f*x)] + 3*Sin[4*(e + f*x)
] + 28*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sqrt[Sec[e + f*x]^
2]*Tan[e + f*x]))/(120*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(122) = 244.

Time = 1.38 (sec) , antiderivative size = 404, normalized size of antiderivative = 3.64

method	result
default	$-\frac{\csc(fx+e)\left(12\left(\cos^6(fx+e)\right)\sqrt{2}+2\left(\cos^4(fx+e)\right)\sqrt{2}+42\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,E\left(\sqrt{-\cot(fx+e)+c\right.\right.}{}$

```
[In] int(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120/f*csc(f*x+e)*(12*cos(f*x+e)^6*2^(1/2)+2*cos(f*x+e)^4*2^(1/2)+42*(cot
(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot(f
*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*cos
(f*x+e)-21*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-
cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1
```

```
/2*2^(1/2))*cos(f*x+e)+42*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f
*x+e))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*(-cot(
f*x+e)+csc(f*x+e)+1)^(1/2)-21*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-c
sc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+cs
c(f*x+e)+1)^(1/2),1/2*2^(1/2))+7*2^(1/2)*cos(f*x+e)^2-21*2^(1/2)*cos(f*x+e)
)*(d*tan(f*x+e))^(1/2)*2^(1/2)
```

Fricas [F]

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx=\int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)^5\,dx$$

```
[In] integrate(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(f*x + e))*cos(f*x + e)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx=\text{Timed out}$$

```
[In] integrate(cos(f*x+e)**5*(d*tan(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx=\int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)^5\,dx$$

```
[In] integrate(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(f*x + e))*cos(f*x + e)^5, x)
```

Giac [F]

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)^5\,dx$$

```
[In] integrate(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*tan(f*x + e))*cos(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \cos\left(e+f\,x\right)^5\,\sqrt{d\tan\left(e+f\,x\right)}\,dx$$

```
[In] int(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2),x)
[Out] int(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2), x)
```

3.236 $\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2(d\tan(a+bx))^{5/2}}{5bd}+\frac{4(d\tan(a+bx))^{9/2}}{9bd^3}+\frac{2(d\tan(a+bx))^{13/2}}{13bd^5}$$

```
[Out] 2/5*(d*tan(b*x+a))^(5/2)/b/d+4/9*(d*tan(b*x+a))^(9/2)/b/d^3+2/13*(d*tan(b*x
+a))^(13/2)/b/d^5
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2687, 276}

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2(d\tan(a+bx))^{13/2}}{13bd^5}+\frac{4(d\tan(a+bx))^{9/2}}{9bd^3}+\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

```
[In] Int[Sec[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*(d*Tan[a + b*x])^(5/2))/(5*b*d) + (4*(d*Tan[a + b*x])^(9/2))/(9*b*d^3) +
 (2*(d*Tan[a + b*x])^(13/2))/(13*b*d^5)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (dx)^{3/2}\left(1+x^2\right)^2\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left((dx)^{3/2}+\frac{2(dx)^{7/2}}{d^2}+\frac{(dx)^{11/2}}{d^4}\right)\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{2(d\tan(a+bx))^{5/2}}{5bd}+\frac{4(d\tan(a+bx))^{9/2}}{9bd^3}+\frac{2(d\tan(a+bx))^{13/2}}{13bd^5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\left(-32-8\sec^2(a+bx)-5\sec^4(a+bx)+45\sec^6(a+bx)\right)\sqrt{d\tan(a+bx)}}{585b}$$

```
[In] Integrate[Sec[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
[Out] (2*d*(-32 - 8*Sec[a + b*x]^2 - 5*Sec[a + b*x]^4 + 45*Sec[a + b*x]^6)*Sqrt[d
*Tan[a + b*x]])/(585*b)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{2(d\tan(bx+a))^{\frac{13}{2}}}{13}+\frac{4d^2(d\tan(bx+a))^{\frac{9}{2}}}{9}+\frac{2d^4(d\tan(bx+a))^{\frac{5}{2}}}{5}}{d^5b}$	52
default	$\frac{\frac{2(d\tan(bx+a))^{\frac{13}{2}}}{13}+\frac{4d^2(d\tan(bx+a))^{\frac{9}{2}}}{9}+\frac{2d^4(d\tan(bx+a))^{\frac{5}{2}}}{5}}{d^5b}$	52

```
[In] int(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^5/b*(1/13*(d*tan(b*x+a))^(13/2)+2/9*d^2*(d*tan(b*x+a))^(9/2)+1/5*d^4*(d
*tan(b*x+a))^(5/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\left(32\,d\cos\left(bx+a\right)^6+8\,d\cos\left(bx+a\right)^4+5\,d\cos\left(bx+a\right)^2-45\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{585\,b\cos\left(bx+a\right)^6}$$

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/585*(32*d*cos(b*x + a)^6 + 8*d*cos(b*x + a)^4 + 5*d*cos(b*x + a)^2 - 45*
d)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(b*x + a)^6)
```

Sympy [F]

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}\sec^6\left(a+bx\right)dx$$

```
[In] integrate(sec(b*x+a)**6*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(45\left(d\tan\left(bx+a\right)\right)^{\frac{13}{2}}+130\left(d\tan\left(bx+a\right)\right)^{\frac{9}{2}}d^2+117\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}d^4\right)}{585\,bd^5}$$

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/585*(45*(d*tan(b*x + a))^(13/2) + 130*(d*tan(b*x + a))^(9/2)*d^2 + 117*(d
*tan(b*x + a))^(5/2)*d^4)/(b*d^5)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(45\sqrt{d\tan(bx+a)}d^6\tan(bx+a)^6+130\sqrt{d\tan(bx+a)}d^6\tan(bx+a)^4+117\sqrt{d\tan(bx$$
$$585\,bd^5$$

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/585*(45*sqrt(d*tan(b*x + a))*d^6*tan(b*x + a)^6 + 130*sqrt(d*tan(b*x + a)
)*d^6*tan(b*x + a)^4 + 117*sqrt(d*tan(b*x + a))*d^6*tan(b*x + a)^2)/(b*d^5)
```

Mupad [B] (verification not implemented)

Time = 11.54 (sec) , antiderivative size = 392, normalized size of antiderivative = 5.85

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{64\,d\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{585\,b} - \frac{64\,d\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{585\,b\left(e^{a\,2i+b\,x\,2i}+1\right)} - \frac{32\,d\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{195\,b\left(e^{a\,2i+b\,x\,2i}+1\right)^2} + \frac{1216\,d\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{117\,b\left(e^{a\,2i+b\,x\,2i}+1\right)^3} - \frac{3488\,d\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{117\,b\left(e^{a\,2i+b\,x\,2i}+1\right)^4} + \frac{384\,d\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{13\,b\left(e^{a\,2i+b\,x\,2i}+1\right)^5} - \frac{128\,d\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{13\,b\left(e^{a\,2i+b\,x\,2i}+1\right)^6}$$

```
[In] int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^6,x)
```

```
[Out] (1216*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))
/(117*b*(exp(a*2i + b*x*2i) + 1)^3) - (64*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1
i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(585*b*(exp(a*2i + b*x*2i) + 1)) - (32
*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(195
*b*(exp(a*2i + b*x*2i) + 1)^2) - (64*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2))/(585*b) - (3488*d*(-(d*(exp(a*2i + b*x*2i)*
1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(117*b*(exp(a*2i + b*x*2i) + 1)^
4) + (384*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1
/2))/(13*b*(exp(a*2i + b*x*2i) + 1)^5) - (128*d*(-(d*(exp(a*2i + b*x*2i)*1i
 - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(13*b*(exp(a*2i + b*x*2i) + 1)^6)
```

3.237 $\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2(d\tan(a+bx))^{5/2}}{5bd}+\frac{2(d\tan(a+bx))^{9/2}}{9bd^3}$$

```
[Out] 2/5*(d*tan(b*x+a))^(5/2)/b/d+2/9*(d*tan(b*x+a))^(9/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2687, 14}

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2(d\tan(a+bx))^{9/2}}{9bd^3}+\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

```
[In] Int[Sec[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*(d*Tan[a + b*x])^(5/2))/(5*b*d) + (2*(d*Tan[a + b*x])^(9/2))/(9*b*d^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (dx)^{3/2}\left(1+x^2\right)\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left((dx)^{3/2}+\frac{(dx)^{7/2}}{d^2}\right)\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{2(d\tan(a+bx))^{5/2}}{5bd}+\frac{2(d\tan(a+bx))^{9/2}}{9bd^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\left(-4-\sec^2(a+bx)+5\sec^4(a+bx)\right)\sqrt{d\tan(a+bx)}}{45b}$$

```
[In] Integrate[Sec[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
[Out] (2*d*(-4 - Sec[a + b*x]^2 + 5*Sec[a + b*x]^4)*Sqrt[d*Tan[a + b*x]])/(45*b)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{2(d\tan(bx+a))^{\frac{9}{2}}}{9}+\frac{2d^2(d\tan(bx+a))^{\frac{5}{2}}}{5}}{b\,d^3}$	37
default	$\frac{\frac{2(d\tan(bx+a))^{\frac{9}{2}}}{9}+\frac{2d^2(d\tan(bx+a))^{\frac{5}{2}}}{5}}{b\,d^3}$	37

```
[In] int(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/b/d^3*(1/9*(d*tan(b*x+a))^(9/2)+1/5*d^2*(d*tan(b*x+a))^(5/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\left(4\,d\cos\left(bx+a\right)^4+d\cos\left(bx+a\right)^2-5\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{45\,b\cos\left(bx+a\right)^4}$$

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] -2/45*(4*d*cos(b*x + a)^4 + d*cos(b*x + a)^2 - 5*d)*sqrt(d*sin(b*x + a)/cos
(b*x + a))/(b*cos(b*x + a)^4)
```

Sympy [F]

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}\sec^4\left(a+bx\right)\,dx$$

```
[In] integrate(sec(b*x+a)**4*(d*tan(b*x+a))**(3/2),x)
[Out] Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(5\left(d\tan\left(bx+a\right)\right)^{\frac{9}{2}}+9\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}d^2\right)}{45\,bd^3}$$

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] 2/45*(5*(d*tan(b*x + a))^(9/2) + 9*(d*tan(b*x + a))^(5/2)*d^2)/(b*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(5\sqrt{d\tan(bx+a)}d^4\tan(bx+a)^4+9\sqrt{d\tan(bx+a)}d^4\tan(bx+a)^2\right)}{45\,bd^3}$$

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/45*(5*sqrt(d*tan(b*x + a))*d^4*tan(b*x + a)^4 + 9*sqrt(d*tan(b*x + a))*d^
4*tan(b*x + a)^2)/(b*d^3)
```

Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 276, normalized size of antiderivative = 6.13

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{8\,d\sqrt{-\frac{d\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{45\,b} - \frac{8\,d\sqrt{-\frac{d\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{45\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)} + \frac{56\,d\sqrt{-\frac{d\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{15\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)^2} - \frac{64\,d\sqrt{-\frac{d\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{9\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)^3} + \frac{32\,d\sqrt{-\frac{d\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{9\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)^4}$$

```
[In] int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^4,x)
```

```
[Out] (56*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(
15*b*(exp(a*2i + b*x*2i) + 1)^2) - (8*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/
(exp(a*2i + b*x*2i) + 1))^(1/2))/(45*b*(exp(a*2i + b*x*2i) + 1)) - (8*d*(-(
d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(45*b) - (
64*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(9
*b*(exp(a*2i + b*x*2i) + 1)^3) + (32*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2))/(9*b*(exp(a*2i + b*x*2i) + 1)^4)
```

3.238 $\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

```
[Out] 2/5*(d*tan(b*x+a))^(5/2)/b/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2687, 32}

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

```
[In] Int[Sec[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*(d*Tan[a + b*x])^(5/2))/(5*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (dx)^{3/2}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

```
[In] Integrate[Sec[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
[Out] (2*(d*Tan[a + b*x])^(5/2))/(5*b*d)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2(d\tan(bx+a))^{\frac{5}{2}}}{5bd}$	19
default	$\frac{2(d\tan(bx+a))^{\frac{5}{2}}}{5bd}$	19

```
[In] int(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/5*(d*tan(b*x+a))^(5/2)/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\left(d\cos\left(bx+a\right)^2-d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{5\,b\cos\left(bx+a\right)^2}$$

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] -2/5*(d*cos(b*x + a)^2 - d)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(b*x +
a)^2)
```

Sympy [F]

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(a+bx)})^{\frac{3}{2}}\sec^2{(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
[Out] Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,(d\tan{(bx+a)})^{\frac{5}{2}}}{5\,bd}$$

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] 2/5*(d*tan(b*x + a))^(5/2)/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,\sqrt{d\tan{(bx+a)}}d\tan{(bx+a)}^2}{5\,b}$$

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] 2/5*sqrt(d*tan(b*x + a))*d*tan(b*x + a)^2/b
```

Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.55

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,d\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}\,\left(\cos{(2\,a+2\,b\,x)}-2\,\cos{(4\,a+4\,b\,x)}-\cos{(6\,a+6\,b\,x)}+2\right)}{5\,b\,\left(15\,\cos{(2\,a+2\,b\,x)}+6\,\cos{(4\,a+4\,b\,x)}+\cos{(6\,a+6\,b\,x)}+10\right)}$$

```
[In] int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^2,x)
[Out] (2*d*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2)*(cos(2*a + 2*b*x)
- 2*cos(4*a + 4*b*x) - cos(6*a + 6*b*x) + 2))/(5*b*(15*cos(2*a + 2*b*x) + 6
*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) + 10))
```

3.239 $\int (d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 210

$$\int (d\tan(a+bx))^{3/2}\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}b} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}b} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}b} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}b} + \frac{2d\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] 1/2*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-1/2*d^
(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)+1/4*d^(3/2)*
ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)-1/4*d
^(3/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2
)+2*d*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used

$= \{3554, 3557, 335, 217, 1179, 642, 1176, 631, 210\}$

$$\int (d\tan(a+bx))^{3/2}\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}b} - \frac{d^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}b} + \frac{d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{2\sqrt{2}b} - \frac{d^{3/2}\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{2\sqrt{2}b} + \frac{2d\sqrt{d\tan(a+bx)}}{b}$$

```
[In] Int[(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(Sqrt[2]*b) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(Sqrt[2]*b) +
(d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]
)/(2*Sqrt[2]*b) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqr
t[d*Tan[a + b*x]]])/(2*Sqrt[2]*b) + (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d\sqrt{d\tan(a+bx)}}{b} - d^2\int \frac{1}{\sqrt{d\tan(a+bx)}}\,dx \\
&= \frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\tan(a+bx)\right)}{b} \\
&= \frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{b}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{d^2\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{b} \\
&\quad - \frac{d^2\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{b} \\
&= \frac{2d\sqrt{d\tan(a+bx)}}{b} + \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}b} \\
&\quad + \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}b} \\
&\quad - \frac{d^2\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2b} \\
&\quad - \frac{d^2\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2b} \\
&= \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}b} \\
&\quad - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}b} \\
&\quad + \frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}b} \\
&\quad + \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}b} \\
&= \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}b} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}b} \\
&\quad + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}b} \\
&\quad - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}b} + \frac{2d\sqrt{d\tan(a+bx)}}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.76

$$\int (d\tan(a+bx))^{3/2}\,dx = \frac{\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(a-\right.}{2\sqrt{}}\right.}{b\tan^{\frac{3}{2}}(a+bx)}$$

[In] Integrate[(d*Tan[a + b*x])^(3/2),x]

[Out] ((ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/(2*Sqrt[2]) + 2*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(3/2))/(b*Tan[a + b*x]^(3/2))

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{2d\left(\sqrt{d\tan(bx+a)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(bx+a)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(bx+a)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8}\right.}{b}$
default	$\frac{2d\left(\sqrt{d\tan(bx+a)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(bx+a)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(bx+a)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8}\right.}{b}$

[In] int((d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/b*d*((d*tan(b*x+a))^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(b*x+a)+(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(b*x+a)-(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)+1))
)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.84

$$\int (d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{\left(-\frac{d^6}{b^4}\right)^{\frac{1}{4}} b\log\left(\sqrt{d\tan\left(bx+a\right)}d+\left(-\frac{d^6}{b^4}\right)^{\frac{1}{4}} b\right)+i\left(-\frac{d^6}{b^4}\right)^{\frac{1}{4}} b\log\left(\sqrt{d\tan\left(bx+a\right)}d+i\left(-\frac{d^6}{b^4}\right)^{\frac{1}{4}} b\right)-i\left(\cdots\right.}{}$$

```
[In] integrate((d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*((-d^6/b^4)^(1/4)*b*log(sqrt(d*tan(b*x + a))*d + (-d^6/b^4)^(1/4)*b) +
 I*(-d^6/b^4)^(1/4)*b*log(sqrt(d*tan(b*x + a))*d + I*(-d^6/b^4)^(1/4)*b) -
I*(-d^6/b^4)^(1/4)*b*log(sqrt(d*tan(b*x + a))*d - I*(-d^6/b^4)^(1/4)*b) - (
-d^6/b^4)^(1/4)*b*log(sqrt(d*tan(b*x + a))*d - (-d^6/b^4)^(1/4)*b) - 4*sqrt
(d*tan(b*x + a))*d)/b
```

Sympy [F]

$$\int (d\tan(a+bx))^{3/2}\,dx = \int \left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.81

$$\int (d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{2\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+\sqrt{2}d^{\frac{5}{2}}\log\left(d\tan\cdots\right.}{}$$

```
[In] integrate((d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
b*x + a)))/sqrt(d)) + 2*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + sqrt(2)*d^(5/2)*log(d*tan(b*x + a) +
 sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - sqrt(2)*d^(5/2)*log(d*tan(b*x
+ a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan(b*x + a))*d
^2)/(b*d)
```

Giac [F]

$$\int (d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\,dx$$

[In] integrate((d*tan(b*x+a))^(3/2),x, algorithm="giac")

[Out] integrate((d*tan(b*x + a))^(3/2), x)

Mupad [B] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

$$\int (d\tan(a+bx))^{3/2}\,dx = \frac{2\,d\,\sqrt{d\tan(a+b\,x)}}{b} + \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(a+b\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{b} + \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(a+b\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{b}$$

[In] int((d*tan(a + b*x))^(3/2),x)

[Out] (2*d*(d*tan(a + b*x))^(1/2))/b + ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*tan(a + b*x))^(1/2))/d^(1/2))*1i)/b + ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)*(d*tan(a + b*x))^(1/2))/d^(1/2))*1i)/b

3.240 $\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 225

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}+\frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} - \frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b}$$

```
[Out] -1/8*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)+1/8*d
^(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-1/16*d^(3/2
)*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)+1/1
6*d^(3/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(
1/2)-1/2*d*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2687, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}+\frac{d^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}b} - \frac{d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b} + \frac{d^{3/2}\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b} - \frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b}$$

```
[In] Int[Cos[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/4*(d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(Sqrt[2]*
b) + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2
]*b) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a +
 b*x]]])/(8*Sqrt[2]*b) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt
[2]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) - (d*Cos[a + b*x]^2*Sqrt[d*Tan[a +
 b*x]])/(2*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(dx)^{3/2}}{(1+x^2)^2}\,dx, x, \tan(a+bx)\right)}{b}$$

$$= -\frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b} + \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{dx}(1+x^2)}\,dx, x, \tan(a+bx)\right)}{4b}$$

$$= -\frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b} + \frac{d\text{Subst}\left(\int \frac{1}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2b}$$

$$= -\frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b} + \frac{\text{Subst}\left(\int \frac{d-x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{d+x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4b}$$

$$= -\frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b} - \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} - \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{d^2\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8b} + \frac{d^2\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8b}$$

$$= -\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} - \frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b} + \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}$$

$$= -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}+\frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}$$
$$-\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b}$$
$$+\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b}$$
$$-\frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.49

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$
$$-\frac{d\csc(a+bx)\left(\sin(a+bx)+\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-\log\left(\cos(a+bx)+\sin(\right.\right.}{8b}$$

```
[In] Integrate[Cos[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/8*(d*Csc[a + b*x]*(Sin[a + b*x] + ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sq
rt[Sin[2*(a + b*x)]] - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*
x)]]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)])*Sqrt[d*Tan[a + b*x]])/b
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(169) = 338$.

Time = 2.17 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.33

method	result
default	$-\frac{\cos(bx+a)\left(4\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\sqrt{2}\left(\cos^2(bx+a)\right)+4\cos(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}-\ln\left(\frac{2\sin(bx+a)\sqrt{-\left(\cot^3(bx+a)\right)+}}{}\right.\right.}{}$

```
[In] int(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/b*cos(b*x+a)*(4*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/
2)*cos(b*x+a)^2+4*cos(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)
^2)^(1/2)-ln((2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b
*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-cot(b*x+a)*cos
```

```
(b*x+a)+2*cot(b*x+a)+2*cos(b*x+a)+sin(b*x+a)-csc(b*x+a)-2)/(-1+cos(b*x+a)))
+ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(
b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b
*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))+2*arcta
n((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b
*x+a)-1)/(-1+cos(b*x+a)))+2*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x
+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a))))*d*(d*tan(b*x+a)
)^(1/2)/(cos(b*x+a)+1)/(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1
/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 927, normalized size of antiderivative = 4.12

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/32*(16*d*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2 - (-d^6/b^4)^(
1/4)*b*log(-2*d^5*cos(b*x + a)^2 + 2*sqrt(-d^6/b^4)*b^2*d^2*cos(b*x + a)*si
n(b*x + a) + d^5 + 2*((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) + (-
d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + (-d
^6/b^4)^(1/4)*b*log(-2*d^5*cos(b*x + a)^2 + 2*sqrt(-d^6/b^4)*b^2*d^2*cos(b*
x + a)*sin(b*x + a) + d^5 - 2*((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x
+ a) + (-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a
))) + I*(-d^6/b^4)^(1/4)*b*log(-2*d^5*cos(b*x + a)^2 - 2*sqrt(-d^6/b^4)*b^2
*d^2*cos(b*x + a)*sin(b*x + a) + d^5 - 2*(I*(-d^6/b^4)^(1/4)*b*d^3*cos(b*x
+ a)*sin(b*x + a) - I*(-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x +
 a)/cos(b*x + a))) - I*(-d^6/b^4)^(1/4)*b*log(-2*d^5*cos(b*x + a)^2 - 2*sqr
t(-d^6/b^4)*b^2*d^2*cos(b*x + a)*sin(b*x + a) + d^5 - 2*(-I*(-d^6/b^4)^(1/4
)*b*d^3*cos(b*x + a)*sin(b*x + a) + I*(-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*
sqrt(d*sin(b*x + a)/cos(b*x + a))) + (-d^6/b^4)^(1/4)*b*log(-d^5 + 2*((-d^6
/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) - (-d^6/b^4)^(3/4)*b^3*cos(b*x
+ a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - (-d^6/b^4)^(1/4)*b*log(-d^5 -
2*((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) - (-d^6/b^4)^(3/4)*b^3*
cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - I*(-d^6/b^4)^(1/4)*b*l
og(-d^5 - 2*(I*(-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) + I*(-d^6/b
^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + I*(-d^6/
b^4)^(1/4)*b*log(-d^5 - 2*(-I*(-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x +
 a) - I*(-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x +
a))))/b
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.84

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+\sqrt{2}d}{}$$

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] 1/16*(2*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
b*x + a)))/sqrt(d)) + 2*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + sqrt(2)*d^(5/2)*log(d*tan(b*x + a) +
 sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - sqrt(2)*d^(5/2)*log(d*tan(b*x
+ a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan(b*x + a))*d
^4/(d^2*tan(b*x + a)^2 + d^2))/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{1}{16}d\left(\frac{2\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{2\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(b}\right.}{2\sqrt{|d|}}\right.}{b}\right.$$

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*d*(2*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2
*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 2*sqrt(2)*sqrt(abs(d))*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b +
sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt
(abs(d)) + abs(d))/b - sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) - sqrt(2)*sq
rt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 8*sqrt(d*tan(b*x + a))*d^2/((
d^2*tan(b*x + a)^2 + d^2)*b))
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \cos\left(a+b\,x\right)^2\,{\left(d\,\tan(a+b\,x)\right)}^{3/2}\,dx$$

```
[In] int(cos(a + b*x)^2*(d*tan(a + b*x))^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2*(d*tan(a + b*x))^(3/2), x)
```

3.241 $\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{4d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{77b\sqrt{d\tan(a+bx)}} - \frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b} - \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b} + \frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}$$

```
[Out] 4/77*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*
Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)-4/7
7*d*sec(b*x+a)*(d*tan(b*x+a))^(1/2)/b-2/77*d*sec(b*x+a)^3*(d*tan(b*x+a))^(1
/2)/b+2/11*d*sec(b*x+a)^5*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used

$= \{2691, 2693, 2694, 2653, 2720\}$

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{4d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{77b\sqrt{d\tan(a+bx)}} + \frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b} - \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b} - \frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b}$$

```
[In] Int[Sec[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-4*d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
77*b*Sqrt[d*Tan[a + b*x]]) - (4*d*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(77*b)
 - (2*d*Sec[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(77*b) + (2*d*Sec[a + b*x]^5*S
qrt[d*Tan[a + b*x]])/(11*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\int\frac{\sec^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\
&= -\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b}+\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}\\
&\quad-\frac{1}{77}\left(6d^2\right)\int\frac{\sec^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\
&= -\frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b}-\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b}\\
&\quad+\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{77}\left(4d^2\right)\int\frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\
&= -\frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b}-\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b}\\
&\quad+\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{\left(4d^2\sqrt{\sin(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{77\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b}-\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b}\\
&\quad+\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}\\
&\quad-\frac{\left(4d^2\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{77\sqrt{d\tan(a+bx)}}\\
&= -\frac{4d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{77b\sqrt{d\tan(a+bx)}}\\
&\quad-\frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b}-\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b}\\
&\quad+\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.77 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$
$$-\frac{d\sec^5(a+bx)\left(-23+6\cos(2(a+bx))+\cos(4(a+bx))+16\cos^6(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\text{t}\right.\right.}{154b}$$

```
[In] Integrate[Sec[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/154*(d*Sec[a + b*x]^5*(-23 + 6*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 16*
Cos[a + b*x]^6*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[Sec[a
 + b*x]^2])*Sqrt[d*Tan[a + b*x]])/b
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.95

method	result
default	$\frac{d\sqrt{d\tan(bx+a)}\left(4\sin(bx+a)\cos(bx+a)\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,F\left(\sqrt{1+\mathrm{c}}\right.\right.}{}$

```
[In] int(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/77/b*d*(d*tan(b*x+a))^(1/2)/(cos(b*x+a)^2-1)*(4*sin(b*x+a)*cos(b*x+a)*(1+
csc(b*x+a)-cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+c
ot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+4*s
in(b*x+a)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1
+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2
*2^(1/2))+2*sin(b*x+a)*tan(b*x+a)*2^(1/2)+tan(b*x+a)^2*sec(b*x+a)*2^(1/2)-7
*tan(b*x+a)^2*sec(b*x+a)^3*2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \sec^5(a+bx)(d\tan(a +bx))^{3/2}\,dx = \frac{2\left(2\sqrt{i}\,dd\cos(bx+a)^5\,F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+2\sqrt{-i}\,dd\cos(bx+a)^5\right.}{77\,b\,\mathrm{c}}$$

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/77*(2*sqrt(I*d)*d*cos(b*x + a)^5*elliptic_f(arcsin(cos(b*x + a) + I*sin(b
*x + a)), -1) + 2*sqrt(-I*d)*d*cos(b*x + a)^5*elliptic_f(arcsin(cos(b*x + a
) - I*sin(b*x + a)), -1) - (2*d*cos(b*x + a)^4 + d*cos(b*x + a)^2 - 7*d)*sq
rt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x + a)^5)
```

Sympy [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(a+bx)})^{\frac{3}{2}}\sec^5{(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**5*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**5, x)
```

Maxima [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(bx+a)})^{\frac{3}{2}}\sec{(bx+a)}^5\,dx$$

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^5, x)
```

Giac [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(bx+a)})^{\frac{3}{2}}\sec{(bx+a)}^5\,dx$$

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \frac{(d\tan(a+b\,x))^{3/2}}{\cos\left(a+b\,x\right)^5}\,dx$$

```
[In] int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^5,x)
[Out] int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^5, x)
```

3.242 $\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{21b\sqrt{d\tan(a+bx)}} - \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}$$

```
[Out] 2/21*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*
Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)-2/2
1*d*sec(b*x+a)*(d*tan(b*x+a))^(1/2)/b+2/7*d*sec(b*x+a)^3*(d*tan(b*x+a))^(1/
2)/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2691, 2693, 2694, 2653, 2720}

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{21b\sqrt{d\tan(a+bx)}} + \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b} - \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b}$$

```
[In] Int[Sec[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
21*b*Sqrt[d*Tan[a + b*x]]) - (2*d*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(21*b)
 + (2*d*Sec[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(7*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b} - \frac{1}{7}d^2\int\frac{\sec^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\ &= -\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}\\ &\quad - \frac{1}{21}\left(2d^2\right)\int\frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\end{aligned}$$

$$= -\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b} - \frac{\left(2d^2\sqrt{\sin(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{21\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$= -\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b} - \frac{\left(2d^2\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{21\sqrt{d\tan(a+bx)}}$$

$$= -\frac{2d^2\,\mathrm{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{21b\sqrt{d\tan(a+bx)}} - \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.74

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{d\sec^3(a+bx)\left(-5+\cos(2(a+bx))+4\cos^4(a+bx)\,\mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2\right.}{21b}$$

```
[In] Integrate[Sec[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
[Out] -1/21*(d*Sec[a + b*x]^3*(-5 + Cos[2*(a + b*x)] + 4*Cos[a + b*x]^4*Hypergeom
etric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a
 + b*x]])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(119) = 238.

Time = 1.47 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.26

method	result
default	$\frac{d\sqrt{d\tan(bx+a)}\left(2\sin(bx+a)\cos(bx+a)\sqrt{1+\csc(bx+a)-\cot(bx+a)}\sqrt{\cot(bx+a)-\csc(bx+a)}\sqrt{-\csc(bx+a)+1+\cot(bx+a)}F\left(\sqrt{1+\right.\right.}{}$

```
[In] int(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/b*d*(d*tan(b*x+a))^(1/2)/(cos(b*x+a)^2-1)*(2*sin(b*x+a)*cos(b*x+a)*(1+
csc(b*x+a)-cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+c
ot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*s
in(b*x+a)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1
+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2
*2^(1/2))+sin(b*x+a)*tan(b*x+a)*2^(1/2)-3*tan(b*x+a)^2*sec(b*x+a)*2^(1/2))*
2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(\sqrt{i\,d}d\cos\left(bx+a\right)^3 F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\,|-1)+\sqrt{-i\,d}d\cos\left(bx+a\right)^3 F(a\right.}{21\,b\cos\left(bx+a\right)}$$

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(sqrt(I*d)*d*cos(b*x + a)^3*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x
 + a)), -1) + sqrt(-I*d)*d*cos(b*x + a)^3*elliptic_f(arcsin(cos(b*x + a) -
I*sin(b*x + a)), -1) - (d*cos(b*x + a)^2 - 3*d)*sqrt(d*sin(b*x + a)/cos(b*x
 + a)))/(b*cos(b*x + a)^3)
```

Sympy [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}\sec^3\left(a+bx\right)dx$$

```
[In] integrate(sec(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**3, x)
```

Maxima [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sec(bx+a)^3\,dx$$

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^3, x)
```

Giac [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sec(bx+a)^3\,dx$$

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \frac{(d\tan(a+b\,x))^{3/2}}{\cos(a+b\,x)^3}\,dx$$

```
[In] int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^3,x)
[Out] int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^3, x)
```

3.243 $\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{3b\sqrt{d\tan(a+bx)}}+\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

```
[Out] 1/3*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*P
i+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)+2/3*
d*sec(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2691, 2694, 2653, 2720}

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Sec[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/3*(d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])
/(b*Sqrt[d*Tan[a + b*x]]) + (2*d*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b} - \frac{1}{3}d^2\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx \\
&= \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b} - \frac{\left(d^2\sqrt{\sin(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b} - \frac{\left(d^2\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{3\sqrt{d\tan(a+bx)}} \\
&= -\frac{d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{3b\sqrt{d\tan(a+bx)}} + \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2d\cos(a+bx)\left(\sec^2(a+bx)-\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{\ }}{3b}$$

```
[In] Integrate[Sec[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*d*Cos[a + b*x]*(Sec[a + b*x]^2 - Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a
 + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(95) = 190.

Time = 1.39 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.76

method	result
default	$-\frac{\left(-\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)(\cos^2(b}{}$

```
[In] int(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*(-(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(
cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*
2^(1/2))*cos(b*x+a)^2-(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+
a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a
))^(1/2),1/2*2^(1/2))*cos(b*x+a)+sin(b*x+a)*2^(1/2))*tan(b*x+a)*d*(d*tan(b*
x+a))^(1/2)/(cos(b*x+a)^2-1)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.21

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{\sqrt{i\,d}d\cos(bx+a)\,F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+\sqrt{-i\,d}d\cos(bx+a)\,F(\text{arcsin}}{3\,b\cos(bx+a)}$$

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(I*d)*d*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x +
a)), -1) + sqrt(-I*d)*d*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) - I*sin
(b*x + a)), -1) + 2*d*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x + a))
```

Sympy [F]

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(a+bx)})^{\frac{3}{2}}\sec{(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x), x)
```

Maxima [F]

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sec(bx+a)\,dx$$

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a), x)
```

Giac [F]

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sec(bx+a)\,dx$$

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \frac{(d\tan(a+b\,x))^{3/2}}{\cos(a+b\,x)}\,dx$$

```
[In] int((d*tan(a + b*x))^(3/2)/cos(a + b*x),x)
```

```
[Out] int((d*tan(a + b*x))^(3/2)/cos(a + b*x), x)
```

3.244 $\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{d^2\operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] -1/2*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*
Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)-d*c
os(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2690, 2694, 2653, 2720}

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[In] Int[Cos[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b
*Sqrt[d*Tan[a + b*x]]) - (d*Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}+\frac{1}{2}d^2\int\frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\
&= -\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}+\frac{\left(d^2\sqrt{\sin(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}+\frac{\left(d^2\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{2\sqrt{d\tan(a+bx)}}\\
&= \frac{d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d\cos(a+bx)\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{d\tan(a+}}{b}$$

```
[In] Integrate[Cos[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
[Out] (d*Cos[a + b*x]*(-1 + Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqr
t[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(95) = 190.

Time = 3.02 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.81

method	result
default	$\frac{\sin(bx+a)\left(-\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)\right.}{}$

```
[In] int(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2/b*sin(b*x+a)*(-(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))
^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^
(1/2),1/2*2^(1/2))*cos(b*x+a)-(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+
cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-c
ot(b*x+a))^(1/2),1/2*2^(1/2))+sin(b*x+a)*2^(1/2)*cos(b*x+a))*(d*tan(b*x+a))
^(1/2)*d/(cos(b*x+a)^2-1)*2^(1/2)
```

Fricas [F]

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)\,dx$$

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(b*x + a))*d*cos(b*x + a)*tan(b*x + a), x)
```

Sympy [F]

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(a+bx)})^{\frac{3}{2}}\cos{(a+bx)}\,dx$$

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))**(3/2),x)
[Out] Integral((d*tan(a + b*x))**(3/2)*cos(a + b*x), x)
```

Maxima [F]

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(bx+a)})^{\frac{3}{2}}\cos{(bx+a)}\,dx$$

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]e
xt_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0]ext_r
educe
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \cos{(a+b\,x)}\,\left(d\tan(a+b\,x)\right)^{3/2}\,dx$$

```
[In] int(cos(a + b*x)*(d*tan(a + b*x))^(3/2),x)
[Out] int(cos(a + b*x)*(d*tan(a + b*x))^(3/2), x)
```

3.245 $\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{d^2\operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{12b\sqrt{d\tan(a+bx)}}+\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b}-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

```
[Out] -1/12*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4
*Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)+1/
6*d*cos(b*x+a)*(d*tan(b*x+a))^(1/2)/b-1/3*d*cos(b*x+a)^3*(d*tan(b*x+a))^(1/
2)/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2690, 2692, 2694, 2653, 2720}

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{12b\sqrt{d\tan(a+bx)}}-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}+\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b}$$

```
[In] Int[Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(12*
b*Sqrt[d*Tan[a + b*x]]) + (d*Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/(6*b) - (d*
Cos[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}+\frac{1}{6}d^2\int\frac{\cos(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\
&= \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b}-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}+\frac{1}{12}d^2\int\frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx
\end{aligned}
$$

$$= \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b} - \frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b} + \frac{\left(d^2\sqrt{\sin(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{12\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$= \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b} - \frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b} + \frac{\left(d^2\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{12\sqrt{d\tan(a+bx)}}$$

$$= \frac{d^2\operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{12b\sqrt{d\tan(a+bx)}} + \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b} - \frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{\cos(a+bx)\left(\sqrt[4]{-1}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sqrt{\sec^2(a+bx)}+\cos(2(a+bx))\sqrt{\tan(a}\right.}{6b\tan^{\frac{3}{2}}(a+bx)}$$

```
[In] Integrate[Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/6*(Cos[a + b*x]*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a +
b*x]]], -1]*Sqrt[Sec[a + b*x]^2] + Cos[2*(a + b*x)]*Sqrt[Tan[a + b*x]])*(d*
Tan[a + b*x])^(3/2))/(b*Tan[a + b*x]^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.34 (sec) , antiderivative size = 1939, normalized size of antiderivative = 17.95

method	result	size
default	Expression too large to display	1939

```
[In] int(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

[Out] 1/48/b*sin(b*x+a)*(6*I*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*I*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)+6*I*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*cos(b*x+a)-6*I*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+8*2^(1/2)*cos(b*x+a)^3*sin(b*x+a)-6*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)-6*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)+8*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-6*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+8*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-4*sin(b*x+a)*2^(1/2)*cos(b*x+a)+3*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))*cos(b*x+a)-3*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln((2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-cot(b*x+a)*cos(b*x+a)+2*cot(b*x+a)+2*cos(b*x+a)+sin(b*x+a)-csc(b*x+a)-2)/(-1+cos(b*x+a)))*cos(b*x+a)+6*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))*cos(b*x+a)+6*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*cos(b*x+a)+3*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))-3*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln((2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-cot(b*x+a)*cos(b*x+a)+2*cot(b*x+a)+2*cos(b*x+a)+sin(b*x+a)-csc(b*x+a)-2)/(-1+cos(b*x+a)))+6*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))+6*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^

```
2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^
2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a))))*(d*tan(b*x+a))^(1/2)*d/(-1+cos(b*x
+a))/(cos(b*x+a)+1)*2^(1/2)
```

Fricas [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)^3\,dx$$

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(b*x + a))*d*cos(b*x + a)^3*tan(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)^3\,dx$$

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]e
xt_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0]ext_r
educe
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \cos{(a+b\,x)}^3\,\left(d\tan(a+b\,x)\right)^{3/2}\,dx$$

```
[In] int(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2),x)
[Out] int(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2), x)
```

3.246 $\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{24b\sqrt{d\tan(a+bx)}} + \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b} + \frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b} - \frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

```
[Out] -1/24*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4
*Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)+1/
12*d*cos(b*x+a)*(d*tan(b*x+a))^(1/2)/b+1/30*d*cos(b*x+a)^3*(d*tan(b*x+a))^(
1/2)/b-1/5*d*cos(b*x+a)^5*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used

$= \{2690, 2692, 2694, 2653, 2720\}$

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{24b\sqrt{d\tan(a+bx)}} - \frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b} + \frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b} + \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b}$$

```
[In] Int[Cos[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(24*
b*Sqrt[d*Tan[a + b*x]]) + (d*Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/(12*b) + (d
*Cos[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(30*b) - (d*Cos[a + b*x]^5*Sqrt[d*Tan
[a + b*x]])/(5*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}+\frac{1}{10}d^2\int\frac{\cos^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$=\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}+\frac{1}{12}d^2\int\frac{\cos(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$=\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b}+\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}+\frac{1}{24}d^2\int\frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$=\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b}+\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}+\frac{\left(d^2\sqrt{\sin(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{24\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$=\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b}+\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}+\frac{\left(d^2\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{24\sqrt{d\tan(a+bx)}}$$

$$=\frac{d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{24b\sqrt{d\tan(a+bx)}}+\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b}+\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\cos(2(a+bx))\csc(a+bx)\left(10\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sqrt{\sec^2(a+bx)} \ldots}{120b\sqrt{\tan(a+bx)}\,(-1 \ldots}$$

```
[In] Integrate[Cos[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Cos[2*(a + b*x)]*Csc[a + b*x]*(10*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4
)*Sqrt[Tan[a + b*x]]], -1]*Sqrt[Sec[a + b*x]^2] + (-3 + 10*Cos[2*(a + b*x)]
 + 3*Cos[4*(a + b*x)])*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(3/2))/(120*b*S
qrt[Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.39 (sec) , antiderivative size = 1958, normalized size of antiderivative = 14.40

method	result	size
default	Expression too large to display	1958

```
[In] int(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/240/b*sin(b*x+a)*(24*2^(1/2)*cos(b*x+a)^5*sin(b*x+a)+30*I*(1+csc(b*x+a)-c
ot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(
1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-30*I
*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+
a)-csc(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1
/2*2^(1/2))*cos(b*x+a)-4*2^(1/2)*cos(b*x+a)^3*sin(b*x+a)+30*I*(1+csc(b*x+a)
-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))
^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*co
s(b*x+a)-30*I*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1
/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2
),1/2-1/2*I,1/2*2^(1/2))+50*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+co
t(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot
(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-30*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-c
sc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1
+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)-30*(cot(b*x
+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*
x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2
```

```
))*cos(b*x+a)+50*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(
1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1
/2),1/2*2^(1/2))-30*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a)
)^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))-30*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a
)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x
+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-10*sin(b*x+a)*2^(1/2)*cos(b*x+
a)+15*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cot(b*x+a)*cos(b
*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*
cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x
+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))*cos(b*x+a)-15*(-cos(b*x+a)*si
n(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln((2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+
a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a
))^(1/2)-cot(b*x+a)*cos(b*x+a)+2*cot(b*x+a)+2*cos(b*x+a)+sin(b*x+a)-csc(b*x
+a)-2)/(-1+cos(b*x+a)))*cos(b*x+a)+30*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1
)^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1
)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))*cos(b*x+a)+30*(-cos(b*x+a)*sin(b*
x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*
x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*cos(b*x+a)+15*(
-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2
*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x
+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin
(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))-15*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+
a)+1)^2)^(1/2)*ln((2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*
cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-cot(b*x+a
)*cos(b*x+a)+2*cot(b*x+a)+2*cos(b*x+a)+sin(b*x+a)-csc(b*x+a)-2)/(-1+cos(b*x
+a)))+30*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b*x+a)
*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+
cos(b*x+a)))+30*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin
(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-
1)/(-1+cos(b*x+a))))*(d*tan(b*x+a))^(1/2)*d/(-1+cos(b*x+a))/(cos(b*x+a)+1)*
2^(1/2)
```

Fricas [F]

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx=\int (d\tan{(bx+a)})^{\frac{3}{2}}\cos{(bx+a)}^5\,dx$$

```
[In] integrate(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(b*x + a))*d*cos(b*x + a)^5*tan(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(b*x+a)**5*(d*tan(b*x+a))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)^5\,dx$$

```
[In] integrate(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a)^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0]e
xt_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0]ext_r
educe
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \cos(a+b\,x)^5\,(d\tan(a+b\,x))^{3/2}\,dx$$

```
[In] int(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2),x)
[Out] int(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2), x)
```

3.247 $\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx=\frac{2(d\tan(e+fx))^{7/2}}{7df}+\frac{4(d\tan(e+fx))^{11/2}}{11d^3f}+\frac{2(d\tan(e+fx))^{15/2}}{15d^5f}$$

```
[Out] 2/7*(d*tan(f*x+e))^(7/2)/d/f+4/11*(d*tan(f*x+e))^(11/2)/d^3/f+2/15*(d*tan(f
*x+e))^(15/2)/d^5/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2687, 276}

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx=\frac{2(d\tan(e+fx))^{15/2}}{15d^5f}+\frac{4(d\tan(e+fx))^{11/2}}{11d^3f}+\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

```
[In] Int[Sec[e + f*x]^6*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*(d*Tan[e + f*x])^(7/2))/(7*d*f) + (4*(d*Tan[e + f*x])^(11/2))/(11*d^3*f)
 + (2*(d*Tan[e + f*x])^(15/2))/(15*d^5*f)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (dx)^{5/2}\left(1+x^2\right)^2\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left((dx)^{5/2}+\frac{2(dx)^{9/2}}{d^2}+\frac{(dx)^{13/2}}{d^4}\right)\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{2(d\tan(e+fx))^{7/2}}{7df}+\frac{4(d\tan(e+fx))^{11/2}}{11d^3f}+\frac{2(d\tan(e+fx))^{15/2}}{15d^5f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2(117+44\cos(2(e+fx))+4\cos(4(e+fx)))\sec^4(e+fx)(d\tan(e+fx))^{7/2}}{1155df}$$

```
[In] Integrate[Sec[e + f*x]^6*(d*Tan[e + f*x])^(5/2),x]
[Out] (2*(117 + 44*Cos[2*(e + f*x)] + 4*Cos[4*(e + f*x)])*Sec[e + f*x]^4*(d*Tan[e
 + f*x])^(7/2))/(1155*d*f)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\frac{\frac{2(d\tan(fx+e))^{\frac{15}{2}}}{15}+\frac{4d^2(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{2d^4(d\tan(fx+e))^{\frac{7}{2}}}{7}}{d^5f}$$

```
[In] int(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x)
[Out] 2/d^5/f*(1/15*(d*tan(f*x+e))^(15/2)+2/11*d^2*(d*tan(f*x+e))^(11/2)+1/7*d^4*
(d*tan(f*x+e))^(7/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{2\left(32\,d^2\cos\left(fx+e\right)^6+24\,d^2\cos\left(fx+e\right)^4+21\,d^2\cos\left(fx+e\right)^2-77\,d^2\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin\left(fx+e\right)}{1155\,f\cos\left(fx+e\right)^7}$$

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
[Out] -2/1155*(32*d^2*cos(f*x + e)^6 + 24*d^2*cos(f*x + e)^4 + 21*d^2*cos(f*x + e
)^2 - 77*d^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x + e
)^7)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)**6*(d*tan(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\left(77\left(d\tan\left(fx+e\right)\right)^{\frac{15}{2}}+210\left(d\tan\left(fx+e\right)\right)^{\frac{11}{2}}d^2+165\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}d^4\right)}{1155\,d^5f}$$

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
[Out] 2/1155*(77*(d*tan(f*x + e))^(15/2) + 210*(d*tan(f*x + e))^(11/2)*d^2 + 165*
(d*tan(f*x + e))^(7/2)*d^4)/(d^5*f)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\left(77\sqrt{d\tan(fx+e)}d^7\tan(fx+e)^7 + 210\sqrt{d\tan(fx+e)}d^7\tan(fx+e)^5 + 165\sqrt{d\tan(\right.}{1155\,d^5 f}$$

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 2/1155*(77*sqrt(d*tan(f*x + e))*d^7*tan(f*x + e)^7 + 210*sqrt(d*tan(f*x + e
))*d^7*tan(f*x + e)^5 + 165*sqrt(d*tan(f*x + e))*d^7*tan(f*x + e)^3)/(d^5*f
)
```

Mupad [B] (verification not implemented)

Time = 18.33 (sec) , antiderivative size = 474, normalized size of antiderivative = 7.07

$$\begin{aligned}\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx &= \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,64i}{1155\,f} \\ &+ \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,64i}{1155\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)} + \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,32i}{385\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^2} - \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,2432i}{231\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^3} \\ &+ \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,1504i}{33\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^4} - \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,4288i}{55\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^5} \\ &+ \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,896i}{15\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^6} - \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,256i}{15\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^7}\end{aligned}$$

```
[In] int((d*tan(e + f*x))^(5/2)/cos(e + f*x)^6,x)
```

```
[Out] (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*64i
)/(1155*f) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) +
1))^(1/2)*64i)/(1155*f*(exp(e*2i + f*x*2i) + 1)) + (d^2*(-(d*(exp(e*2i + f*
x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*32i)/(385*f*(exp(e*2i + f*x
*2i) + 1)^2) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i)
+ 1))^(1/2)*2432i)/(231*f*(exp(e*2i + f*x*2i) + 1)^3) + (d^2*(-(d*(exp(e*2i
 + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*1504i)/(33*f*(exp(e*2i
 + f*x*2i) + 1)^4) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*
x*2i) + 1))^(1/2)*4288i)/(55*f*(exp(e*2i + f*x*2i) + 1)^5) + (d^2*(-(d*(exp
(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*896i)/(15*f*(exp(
e*2i + f*x*2i) + 1)^6) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i
+ f*x*2i) + 1))^(1/2)*256i)/(15*f*(exp(e*2i + f*x*2i) + 1)^7)
```

3.248 $\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx=\frac{2(d\tan(e+fx))^{7/2}}{7df}+\frac{2(d\tan(e+fx))^{11/2}}{11d^3f}$$

```
[Out] 2/7*(d*tan(f*x+e))^(7/2)/d/f+2/11*(d*tan(f*x+e))^(11/2)/d^3/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2687, 14}

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx=\frac{2(d\tan(e+fx))^{11/2}}{11d^3f}+\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

```
[In] Int[Sec[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*(d*Tan[e + f*x])^(7/2))/(7*d*f) + (2*(d*Tan[e + f*x])^(11/2))/(11*d^3*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (dx)^{5/2}\left(1+x^2\right)\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left((dx)^{5/2}+\frac{(dx)^{9/2}}{d^2}\right)\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{2(d\tan(e+fx))^{7/2}}{7df}+\frac{2(d\tan(e+fx))^{11/2}}{11d^3f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2(9+2\cos(2(e+fx)))\sec^2(e+fx)(d\tan(e+fx))^{7/2}}{77df}$$

```
[In] Integrate[Sec[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
[Out] (2*(9 + 2*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(d*Tan[e + f*x])^(7/2))/(77*d*f)
```

Maple [A] (verified)

Time = 236.85 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{2(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{2d^2(d\tan(fx+e))^{\frac{7}{2}}}{7}}{f\,d^3}$	37
default	$\frac{\frac{2(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{2d^2(d\tan(fx+e))^{\frac{7}{2}}}{7}}{f\,d^3}$	37

```
[In] int(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/f/d^3*(1/11*(d*tan(f*x+e))^(11/2)+1/7*d^2*(d*tan(f*x+e))^(7/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.53

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{2\left(4\,d^2\cos\left(fx+e\right)^4+3\,d^2\cos\left(fx+e\right)^2-7\,d^2\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin\left(fx+e\right)}{77\,f\cos\left(fx+e\right)^5}$$

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*d^2*cos(f*x + e)^4 + 3*d^2*cos(f*x + e)^2 - 7*d^2)*sqrt(d*sin(f*x
+ e)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)**4*(d*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\left(7\left(d\tan\left(fx+e\right)\right)^{\frac{11}{2}}+11\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}d^2\right)}{77\,d^3 f}$$

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/77*(7*(d*tan(f*x + e))^(11/2) + 11*(d*tan(f*x + e))^(7/2)*d^2)/(d^3*f)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\left(7\sqrt{d\tan(fx+e)}d^5\tan(fx+e)^5+11\sqrt{d\tan(fx+e)}d^5\tan(fx+e)^3\right)}{77\,d^3f}$$

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 2/77*(7*sqrt(d*tan(f*x + e))*d^5*tan(f*x + e)^5 + 11*sqrt(d*tan(f*x + e))*d
^5*tan(f*x + e)^3)/(d^3*f)
```

Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 352, normalized size of antiderivative = 7.82

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,8i}{77\,f} + \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,8i}{77\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)} - \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,296i}{77\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^2} + \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,944i}{77\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^3} - \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,160i}{11\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^4} + \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,64i}{11\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^5}$$

```
[In] int((d*tan(e + f*x))^(5/2)/cos(e + f*x)^4,x)
```

```
[Out] (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*8i)
/(77*f) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))
^(1/2)*8i)/(77*f*(exp(e*2i + f*x*2i) + 1)) - (d^2*(-(d*(exp(e*2i + f*x*2i)*
1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*296i)/(77*f*(exp(e*2i + f*x*2i) +
 1)^2) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^
(1/2)*944i)/(77*f*(exp(e*2i + f*x*2i) + 1)^3) - (d^2*(-(d*(exp(e*2i + f*x*2
i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*160i)/(11*f*(exp(e*2i + f*x*2i
) + 1)^4) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1
))^(1/2)*64i)/(11*f*(exp(e*2i + f*x*2i) + 1)^5)
```

3.249 $\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2(d\tan(e+fx))^{7/2}}{7df}$$

```
[Out] 2/7*(d*tan(f*x+e))^(7/2)/d/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2687, 32}

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2(d\tan(e+fx))^{7/2}}{7df}$$

```
[In] Int[Sec[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*(d*Tan[e + f*x])^(7/2))/(7*d*f)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (dx)^{5/2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{2(d\tan(e+fx))^{7/2}}{7df}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2(d\tan(e+fx))^{7/2}}{7df}$$

```
[In] Integrate[Sec[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
[Out] (2*(d*Tan[e + f*x])^(7/2))/(7*d*f)
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2(d\tan(fx+e))^{\frac{7}{2}}}{7df}$	19
default	$\frac{2(d\tan(fx+e))^{\frac{7}{2}}}{7df}$	19

```
[In] int(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/7*(d*tan(f*x+e))^(7/2)/d/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(18) = 36.

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{2\left(d^2\cos\left(fx+e\right)^2-d^2\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin\left(fx+e\right)}{7\,f\cos\left(fx+e\right)^3}$$

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
[Out] -2/7*(d^2*cos(f*x + e)^2 - d^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x +
 e)/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \int (d\tan(e+fx))^{\frac{5}{2}}\sec^2(e+fx)\,dx$$

```
[In] integrate(sec(f*x+e)**2*(d*tan(f*x+e))**(5/2),x)
[Out] Integral((d*tan(e + f*x))**(5/2)*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\,(d\tan(fx+e))^{\frac{7}{2}}}{7\,df}$$

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
[Out] 2/7*(d*tan(f*x + e))^(7/2)/(d*f)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\,\sqrt{d\tan(fx+e)}d^2\tan(fx+e)^3}{7\,f}$$

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
[Out] 2/7*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e)^3/f
```

Mupad [B] (verification not implemented)

Time = 6.71 (sec) , antiderivative size = 230, normalized size of antiderivative = 10.45

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{d^2\sqrt{-\frac{d\,(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,2\mathrm{i}}{7\,f} - \frac{d^2\sqrt{-\frac{d\,(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,12\mathrm{i}}{7\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)} + \frac{d^2\sqrt{-\frac{d\,(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,24\mathrm{i}}{7\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2} - \frac{d^2\sqrt{-\frac{d\,(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3}$$

```
[In] int((d*tan(e + f*x))^(5/2)/cos(e + f*x)^2,x)
```

```
[Out] (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*2i)
/(7*f) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^
(1/2)*12i)/(7*f*(exp(e*2i + f*x*2i) + 1)) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1
i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*24i)/(7*f*(exp(e*2i + f*x*2i) + 1)
^2) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/
2)*16i)/(7*f*(exp(e*2i + f*x*2i) + 1)^3)
```

3.250 $\int (d\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 212

$$\int (d\tan(e+fx))^{5/2}\,dx = \frac{d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} + \frac{2d(d\tan(e+fx))^{3/2}}{3f}$$

```
[Out] 1/2*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/2*d^
(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/4*d^(5/2)*
ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f*2^(1/2)+1/4*d
^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f*2^(1/2
)+2/3*d*(d*tan(f*x+e))^(3/2)/f
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used

$= \{3554, 3557, 335, 303, 1176, 631, 210, 1179, 642\}$

$$\int (d\tan(e+fx))^{5/2}\,dx = \frac{d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f} - \frac{d^{5/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} + \frac{d^{5/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f} + \frac{2d(d\tan(e+fx))^{3/2}}{3f}$$

```
[In] Int[(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]]
)/(2*Sqrt[2]*f) + (d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqr
t[d*Tan[e + f*x]]])/(2*Sqrt[2]*f) + (2*d*(d*Tan[e + f*x])^(3/2))/(3*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{2d(d\tan(e+fx))^{3/2}}{3f} - d^2\int\sqrt{d\tan(e+fx)}\,dx$$

$$= \frac{2d(d\tan(e+fx))^{3/2}}{3f} - \frac{d^3\text{Subst}\left(\int\frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(e+fx)\right)}{f}$$

$$= \frac{2d(d\tan(e+fx))^{3/2}}{3f} - \frac{(2d^3)\,\text{Subst}\left(\int\frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}$$

$$= \frac{2d(d\tan(e+fx))^{3/2}}{3f} + \frac{d^3 \text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f} - \frac{d^3 \text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}$$

$$= \frac{2d(d\tan(e+fx))^{3/2}}{3f} - \frac{d^{5/2} \text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{5/2} \text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^3 \text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2f} - \frac{d^3 \text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2f}$$

$$= -\frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} + \frac{2d(d\tan(e+fx))^{3/2}}{3f} - \frac{d^{5/2} \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{5/2} \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}$$

$$= \frac{d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} + \frac{2d(d\tan(e+fx))^{3/2}}{3f}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.48

$$\int (d\tan(e+fx))^{5/2}\,dx = \frac{d(d\tan(e+fx))^{3/2}\left(-3\arctan\left(\sqrt[4]{-\tan^2(e+fx)}\right)\sqrt[4]{-\tan(e+fx)}+3\text{arctanh}\left(\sqrt[4]{-\tan^2(e+fx)}\right)\ldots\right.}{3f\tan^{\frac{7}{4}}(e+fx)}$$

[In] Integrate[(d*Tan[e + f*x])^(5/2),x]

[Out] (d*(d*Tan[e + f*x])^(3/2)*(-3*ArcTan[(-Tan[e + f*x]^2)^(1/4)]*(-Tan[e + f*x])^(1/4) + 3*ArcTanh[(-Tan[e + f*x]^2)^(1/4)]*(-Tan[e + f*x])^(1/4) + 2*Tan[e + f*x]^(7/4)))/(3*f*Tan[e + f*x]^(7/4))

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{2d\left(\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\ldots}{}\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{f}$
default	$\frac{2d\left(\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\ldots}{}\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{f}$

[In] int((d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/f*d*(1/3*(d*tan(f*x+e))^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95

$$\int (d\tan(e+fx))^{5/2}\,dx = \frac{4\sqrt{d\tan(fx+e)}d^2\tan(fx+e) - 3\left(-\frac{d^{10}}{f^4}\right)^{\frac{1}{4}} f\log\left(\sqrt{d\tan(fx+e)}d^7 + \left(-\frac{d^{10}}{f^4}\right)^{\frac{3}{4}} f^3\right) + 3}{}$$

```
[In] integrate((d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e) - 3*(-d^10/f^4)^(1/4)*f*log(sq
rt(d*tan(f*x + e))*d^7 + (-d^10/f^4)^(3/4)*f^3) + 3*I*(-d^10/f^4)^(1/4)*f*l
og(sqrt(d*tan(f*x + e))*d^7 + I*(-d^10/f^4)^(3/4)*f^3) - 3*I*(-d^10/f^4)^(1
/4)*f*log(sqrt(d*tan(f*x + e))*d^7 - I*(-d^10/f^4)^(3/4)*f^3) + 3*(-d^10/f^
4)^(1/4)*f*log(sqrt(d*tan(f*x + e))*d^7 - (-d^10/f^4)^(3/4)*f^3))/f
```

Sympy [F]

$$\int (d\tan(e+fx))^{5/2}\,dx = \int (d\tan(e+fx))^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((d*tan(e + f*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.83

$$\int (d\tan(e+fx))^{5/2}\,dx = -\frac{3d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{}\right.}{\sqrt{d}}\right.}{12\,df}$$

```
[In] integrate((d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) +
 sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x
+ e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) - 8*(d*tan(f*x +
e))^(3/2)*d^2)/(d*f)
```

Giac [F(-1)]

Timed out.

$$\int (d\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

$$\int (d\tan(e+fx))^{5/2}\,dx = \frac{2\,d\,(d\tan(e+f\,x))^{3/2}}{3\,f} - \frac{(-1)^{1/4}\,d^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)}{f} + \frac{(-1)^{1/4}\,d^{5/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)}{f}$$

```
[In] int((d*tan(e + f*x))^(5/2),x)
```

```
[Out] (2*d*(d*tan(e + f*x))^(3/2))/(3*f) - ((-1)^(1/4)*d^(5/2)*atan(((-1)^(1/4)*(
d*tan(e + f*x))^(1/2))/d^(1/2)))/f + ((-1)^(1/4)*d^(5/2)*atanh(((-1)^(1/4)*
(d*tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.251 $\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 225

$$\begin{aligned}
\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = &-\frac{3d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}+\frac{3d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}\\
&+\frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f}\\
&-\frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f}\\
&-\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f}
\end{aligned}$$

```
[Out] -3/8*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+3/8*d
^(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+3/16*d^(5/2
)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f*2^(1/2)-3/1
6*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f*2^(
1/2)-1/2*d*cos(f*x+e)^2*(d*tan(f*x+e))^(3/2)/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2687, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{3d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}+\frac{3d^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}f}+\frac{3d^{5/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{8\sqrt{2}f}-\frac{3d^{5/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{8\sqrt{2}f}-\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f}$$

```
[In] Int[Cos[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-3*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(4*Sqrt[2]*
f) + (3*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(4*Sqrt
[2]*f) + (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan
[e + f*x]]])/(8*Sqrt[2]*f) - (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x]
+ Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(8*Sqrt[2]*f) - (d*Cos[e + f*x]^2*(d*Tan[e
 + f*x])^(3/2))/(2*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(dx)^{5/2}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f} + \frac{(3d^2)\,\text{Subst}\left(\int \frac{\sqrt{dx}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{4f}$$

$$= -\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f} + \frac{(3d)\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2f}$$

$$= -\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f} - \frac{(3d)\text{Subst}\left(\int \frac{d-x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{4f} + \frac{(3d)\text{Subst}\left(\int \frac{d+x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{4f}$$

$$= -\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f} + \frac{(3d^{5/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f} + \frac{(3d^{5/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f} + \frac{(3d^3)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{8f} + \frac{(3d^3)\,\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{8f}$$

$$= \frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f} - \frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f} - \frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f} + \frac{(3d^{5/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f} - \frac{(3d^{5/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}$$

$$= -\frac{3d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f} + \frac{3d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}$$
$$+ \frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f}$$
$$- \frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f}$$
$$- \frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.48

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx =$$
$$-\frac{d^2\left(3\arcsin(\cos(e+fx)-\sin(e+fx))\csc(e+fx)+3\csc(e+fx)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\mathrm{si}}\right.\right.}{8f}$$

```
[In] Integrate[Cos[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -1/8*(d^2*(3*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Csc[e + f*x] + 3*Csc[e + f
*x]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]] + 2*Sqrt[Sin[
2*(e + f*x)]])*Sqrt[Sin[2*(e + f*x)]]*Sqrt[d*Tan[e + f*x]])/f
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(169) = 338.

Time = 2.48 (sec) , antiderivative size = 551, normalized size of antiderivative = 2.45

method	result
default	$\frac{(\sin^2(fx+e))\cos(fx+e)\left(4\sqrt{2}\cos(fx+e)\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)+4\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)-3\ln\left(-\frac{\cot(f}{}\right.\right.}{}$

```
[In] int(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/f*sin(f*x+e)^2*cos(f*x+e)*(4*2^(1/2)*cos(f*x+e)*(-sin(f*x+e)*cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+4*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-3*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*
sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+
```

```
e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*
x+e)+2)/(cos(f*x+e)-1))+3*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x
+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(
f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)
/(cos(f*x+e)-1))-6*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))-6*arctan((2^(1/2)*(-sin(f*
x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e
)-1)))*(d*tan(f*x+e))^(1/2)*d^2/(cos(f*x+e)-1)/(cos(f*x+e)+1)^2/(-sin(f*x+e
)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 969, normalized size of antiderivative = 4.31

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(16*d^2*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) -
 3*I*(-d^10/f^4)^(1/4)*f*log(27/2*d^8*cos(f*x + e)*sin(f*x + e) + 27/4*(2*d
^3*f^2*cos(f*x + e)^2 - d^3*f^2)*sqrt(-d^10/f^4) - 27/2*(I*(-d^10/f^4)^(1/4
)*d^5*f*cos(f*x + e)*sin(f*x + e) + I*(-d^10/f^4)^(3/4)*f^3*cos(f*x + e)^2)
*sqrt(d*sin(f*x + e)/cos(f*x + e))) + 3*I*(-d^10/f^4)^(1/4)*f*log(27/2*d^8*
cos(f*x + e)*sin(f*x + e) + 27/4*(2*d^3*f^2*cos(f*x + e)^2 - d^3*f^2)*sqrt(
-d^10/f^4) - 27/2*(-I*(-d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)*sin(f*x + e) - I
*(-d^10/f^4)^(3/4)*f^3*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))) +
 3*(-d^10/f^4)^(1/4)*f*log(27/2*d^8*cos(f*x + e)*sin(f*x + e) - 27/4*(2*d^3
*f^2*cos(f*x + e)^2 - d^3*f^2)*sqrt(-d^10/f^4) + 27/2*((-d^10/f^4)^(1/4)*d^
5*f*cos(f*x + e)*sin(f*x + e) - (-d^10/f^4)^(3/4)*f^3*cos(f*x + e)^2)*sqrt(
d*sin(f*x + e)/cos(f*x + e))) - 3*(-d^10/f^4)^(1/4)*f*log(27/2*d^8*cos(f*x
+ e)*sin(f*x + e) - 27/4*(2*d^3*f^2*cos(f*x + e)^2 - d^3*f^2)*sqrt(-d^10/f^
4) - 27/2*((-d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)*sin(f*x + e) - (-d^10/f^4)^
(3/4)*f^3*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))) + 3*(-d^10/f^4
)^(1/4)*f*log(27*d^8 + 54*((-d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)^2 - (-d^10/
f^4)^(3/4)*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e))
) - 3*(-d^10/f^4)^(1/4)*f*log(27*d^8 - 54*((-d^10/f^4)^(1/4)*d^5*f*cos(f*x
+ e)^2 - (-d^10/f^4)^(3/4)*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x +
e)/cos(f*x + e))) - 3*I*(-d^10/f^4)^(1/4)*f*log(27*d^8 - 54*(I*(-d^10/f^4)^
(1/4)*d^5*f*cos(f*x + e)^2 + I*(-d^10/f^4)^(3/4)*f^3*cos(f*x + e)*sin(f*x +
 e))*sqrt(d*sin(f*x + e)/cos(f*x + e))) + 3*I*(-d^10/f^4)^(1/4)*f*log(27*d^
8 - 54*(-I*(-d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)^2 - I*(-d^10/f^4)^(3/4)*f^3
*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e))))/f
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**2*(d*tan(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.86

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{\vphantom{d}}\right.}{\sqrt{\vphantom{d}}}\right.}{16\,df}$$

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
[Out] 1/16*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f
*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) +
sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x +
 e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) - 8*(d*tan(f*x + e
))^(3/2)*d^4/(d^2*tan(f*x + e)^2 + d^2))/(d*f)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.03

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{1}{16}\left(\frac{8\sqrt{d\tan(fx+e)}d^2\tan(fx+e)}{\left(d^2\tan(fx+e)^2+d^2\right)f}-\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{df}-\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\right.}{}\right.$$

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*(8*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e)/((d^2*tan(f*x + e)^2 + d^2)*
f) - 6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sq
rt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) - 6*sqrt(2)*abs(d)^(3/2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*
f) + 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e
))*sqrt(abs(d)) + abs(d))/(d*f) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e)
 - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f))*d^2
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx=\int \cos\left(e+f\,x\right)^2\,\left(d\tan(e+f\,x)\right)^{5/2}\,dx$$

```
[In] int(cos(e + f*x)^2*(d*tan(e + f*x))^(5/2),x)
[Out] int(cos(e + f*x)^2*(d*tan(e + f*x))^(5/2), x)
```

3.252 $\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 253

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{3d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f}+\frac{3d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f}+\frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{64\sqrt{2}f}-\frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{64\sqrt{2}f}+\frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f}-\frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f}$$

```
[Out] -3/64*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+3/64
*d^(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+3/128*d^(
5/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f*2^(1/2)-
3/128*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f
*2^(1/2)+3/16*d*cos(f*x+e)^2*(d*tan(f*x+e))^(3/2)/f-1/4*d*cos(f*x+e)^4*(d*t
an(f*x+e))^(3/2)/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2687, 294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{3d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f}+\frac{3d^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}f}+\frac{3d^{5/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{64\sqrt{2}f}-\frac{3d^{5/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{64\sqrt{2}f}-\frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f}+\frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f}$$

```
[In] Int[Cos[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-3*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(32*Sqrt[2]
*f) + (3*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(32*Sq
rt[2]*f) + (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*T
an[e + f*x]]])/(64*Sqrt[2]*f) - (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*
x] + Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(64*Sqrt[2]*f) + (3*d*Cos[e + f*x]^2*(d
*Tan[e + f*x])^(3/2))/(16*f) - (d*Cos[e + f*x]^4*(d*Tan[e + f*x])^(3/2))/(4
*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(dx)^{5/2}}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f} + \frac{(3d^2)\,\text{Subst}\left(\int \frac{\sqrt{dx}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{8f}$$

$$= \frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f} - \frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f} + \frac{(3d^2)\,\text{Subst}\left(\int \frac{\sqrt{dx}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{32f}$$

$$= \frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f} - \frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f} + \frac{(3d)\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{16f}$$

$$= \frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f} - \frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f} - \frac{(3d)\text{Subst}\left(\int \frac{d-x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{32f} + \frac{(3d)\text{Subst}\left(\int \frac{d+x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{32f}$$

$$= \frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f} - \frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f} + \frac{(3d^{5/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{64\sqrt{2}f} + \frac{(3d^{5/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{64\sqrt{2}f} + \frac{(3d^3)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{64f} + \frac{(3d^3)\,\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{64f}$$

$$= \frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{64\sqrt{2}f} - \frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{64\sqrt{2}f} + \frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f} - \frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f} + \frac{(3d^{5/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f} - \frac{(3d^{5/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f}$$

$$= -\frac{3d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f} + \frac{3d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f} + \frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{64\sqrt{2}f} - \frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{64\sqrt{2}f} + \frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f} - \frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.49

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx =$$

$$-\frac{d^2\Big(3\arcsin(\cos(e+fx)-\sin(e+fx))\csc(e+fx)\sqrt{\sin(2(e+fx))}+3\csc(e+fx)\log\Big(\cos(e+fx)+\sin}{64f}$$

```
[In] Integrate[Cos[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -1/64*(d^2*(3*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Csc[e + f*x]*Sqrt[Sin[2*(
e + f*x)]] + 3*Csc[e + f*x]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e
 + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] - 2*Sin[2*(e + f*x)] + 2*Sin[4*(e + f*x)]
)*Sqrt[d*Tan[e + f*x]])/f
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(193) = 386.

Time = 59.81 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.53

method	result
default	$\frac{\cos(fx+e)(\sin^2(fx+e))\Bigg(16\sqrt{2}\,(\cos^3(fx+e))\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)+16\sqrt{2}\,(\cos^2(fx+e))\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(fx-}{}$

```
[In] int(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/f*cos(f*x+e)*sin(f*x+e)^2*(16*2^(1/2)*cos(f*x+e)^3*(-sin(f*x+e)*cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+16*2^(1/2)*cos(f*x+e)^2*(-sin(f*x+
e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-12*2^(1/2)*cos(f*x+e)*(-si
n(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-12*2^(1/2)*(-sin(f*x
+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+3*ln(-(cot(f*x+e)*cos(f*x
+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*cs
c(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e
)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))-3*ln((2*sin(f*x+e)*(-cot(f*x+e)^
3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+
e)-csc(f*x+e))^(1/2)-cot(f*x+e)*cos(f*x+e)+sin(f*x+e)+2*cos(f*x+e)+2*cot(f*
x+e)-csc(f*x+e)-2)/(cos(f*x+e)-1))-6*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))-6*arctan
((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*
x+e)-1)/(cos(f*x+e)-1)))*(d*tan(f*x+e))^(1/2)*d^2/(cos(f*x+e)-1)/(cos(f*x+e
)+1)^2/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 985, normalized size of antiderivative = 3.89

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/256*(16*(4*d^2*cos(f*x + e)^3 - 3*d^2*cos(f*x + e))*sqrt(d*sin(f*x + e)/
cos(f*x + e))*sin(f*x + e) - 3*I*(-d^10/f^4)^(1/4)*f*log(27/2*d^8*cos(f*x +
 e)*sin(f*x + e) + 27/4*(2*d^3*f^2*cos(f*x + e)^2 - d^3*f^2)*sqrt(-d^10/f^4
) - 27/2*(I*(-d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)*sin(f*x + e) + I*(-d^10/f^
4)^(3/4)*f^3*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))) + 3*I*(-d^1
0/f^4)^(1/4)*f*log(27/2*d^8*cos(f*x + e)*sin(f*x + e) + 27/4*(2*d^3*f^2*cos
(f*x + e)^2 - d^3*f^2)*sqrt(-d^10/f^4) - 27/2*(-I*(-d^10/f^4)^(1/4)*d^5*f*c
os(f*x + e)*sin(f*x + e) - I*(-d^10/f^4)^(3/4)*f^3*cos(f*x + e)^2)*sqrt(d*s
in(f*x + e)/cos(f*x + e))) + 3*(-d^10/f^4)^(1/4)*f*log(27/2*d^8*cos(f*x + e
)*sin(f*x + e) - 27/4*(2*d^3*f^2*cos(f*x + e)^2 - d^3*f^2)*sqrt(-d^10/f^4)
+ 27/2*((-d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)*sin(f*x + e) - (-d^10/f^4)^(3/
4)*f^3*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))) - 3*(-d^10/f^4)^(
1/4)*f*log(27/2*d^8*cos(f*x + e)*sin(f*x + e) - 27/4*(2*d^3*f^2*cos(f*x + e
)^2 - d^3*f^2)*sqrt(-d^10/f^4) - 27/2*((-d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)
*sin(f*x + e) - (-d^10/f^4)^(3/4)*f^3*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/c
os(f*x + e))) + 3*(-d^10/f^4)^(1/4)*f*log(27*d^8 + 54*((-d^10/f^4)^(1/4)*d^
5*f*cos(f*x + e)^2 - (-d^10/f^4)^(3/4)*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(
d*sin(f*x + e)/cos(f*x + e))) - 3*(-d^10/f^4)^(1/4)*f*log(27*d^8 - 54*((-d^
10/f^4)^(1/4)*d^5*f*cos(f*x + e)^2 - (-d^10/f^4)^(3/4)*f^3*cos(f*x + e)*sin
(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e))) - 3*I*(-d^10/f^4)^(1/4)*f*log
(27*d^8 - 54*(I*(-d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)^2 + I*(-d^10/f^4)^(3/4
)*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e))) + 3*I*(
-d^10/f^4)^(1/4)*f*log(27*d^8 - 54*(-I*(-d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)
^2 - I*(-d^10/f^4)^(3/4)*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)
/cos(f*x + e))))/f
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**4*(d*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.89

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\cdots\right.}{\sqrt{d}}\cdots}{128\,\cdots}$$

[In] integrate(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")

[Out] 1/128*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) + 8*(3*(d*tan(f*x + e))^(7/2)*d^4 - (d*tan(f*x + e))^(3/2)*d^6)/(d^4*tan(f*x + e)^4 + 2*d^4*tan(f*x + e)^2 + d^4))/(d*f)

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{1}{128}\,d^2\left(\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{df}+\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(f\cdots}\right.}{2\sqrt{|d|}}\cdots\right.}{df}\cdots\right.$$

[In] integrate(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="giac")

[Out] 1/128*d^2*(6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) + 6*sqrt(2)*abs(d)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) + 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) + 8*(3*sqrt(d*tan(f*x + e))*d^4*tan(f*x + e)^3 - sqrt(d*tan(f*x + e))*d^4*tan(f*x + e))/((d^2*tan(f*x + e)^2 + d^2)^2*f))

Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \int \cos\left(e+f\,x\right)^4\left(d\tan(e+f\,x)\right)^{5/2}\,dx$$

```
[In] int(cos(e + f*x)^4*(d*tan(e + f*x))^(5/2),x)
[Out] int(cos(e + f*x)^4*(d*tan(e + f*x))^(5/2), x)
```

3.253 $\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{4\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{7f\sqrt{d\tan(e+fx)}} + \frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df} + \frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

```
[Out] -4/7*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f
*x),2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d*tan(f*x+e))^(1/2)+4/7*sec
(f*x+e)*(d*tan(f*x+e))^(1/2)/d/f+2/7*sec(f*x+e)^3*(d*tan(f*x+e))^(1/2)/d/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2693, 2694, 2653, 2720}

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df} + \frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df} + \frac{4\sqrt{\sin(2e+2fx)}\sec(e+fx)\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)}{7f\sqrt{d\tan(e+fx)}}$$

```
[In] Int[Sec[e + f*x]^5/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (4*EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(7*f*S
qrt[d*Tan[e + f*x]]) + (4*Sec[e + f*x]*Sqrt[d*Tan[e + f*x]])/(7*d*f) + (2*S
ec[e + f*x]^3*Sqrt[d*Tan[e + f*x]])/(7*d*f)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df} + \frac{6}{7}\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx \\
&= \frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df} + \frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df} + \frac{4}{7}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx \\
&= \frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df} + \frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df} \\
&\quad + \frac{\left(4\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}\,dx}{7\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}} \\
&= \frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df} + \frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df} \\
&\quad + \frac{\left(4\sec(e+fx)\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{7\sqrt{d\tan(e+fx)}}
\end{aligned}$$

$$= \frac{4\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{7f\sqrt{d\tan(e+fx)}} + \frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df} + \frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.68 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\left((2+\cos(2(e+fx)))\sec^4(e+fx)+4\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\right)\sin}{7f\sqrt{d\tan(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]^5/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*((2 + Cos[2*(e + f*x)])*Sec[e + f*x]^4 + 4*Hypergeometric2F1[1/4, 1/2, 5
/4, -Tan[e + f*x]^2]*Sqrt[Sec[e + f*x]^2])*Sin[e + f*x])/(7*f*Sqrt[d*Tan[e
+ f*x]])
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.01

method	result
default	$\frac{\left(4\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,F\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)+4\sec(\right.}{}$

```
[In] int(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7/f/(d*tan(f*x+e))^(1/2)*(4*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-c
sc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+cs
c(f*x+e)+1)^(1/2),1/2*2^(1/2))+4*sec(f*x+e)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2
)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((
-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+2*tan(f*x+e)*sec(f*x+e)*2^(1/2
)+tan(f*x+e)*sec(f*x+e)^3*2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{2\left(2\sqrt{i\,d}\cos\left(fx+e\right)^3 F(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\mid -1)+2\sqrt{-i\,d}\cos\left(fx+e\right)^3 F(\arcsin}{7\,df\,\cos\left(fx+e\right)^3}$$

```
[In] integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] -2/7*(2*sqrt(I*d)*cos(f*x + e)^3*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x
 + e)), -1) + 2*sqrt(-I*d)*cos(f*x + e)^3*elliptic_f(arcsin(cos(f*x + e) -
I*sin(f*x + e)), -1) - (2*cos(f*x + e)^2 + 1)*sqrt(d*sin(f*x + e)/cos(f*x +
 e)))/(d*f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec^5\left(e+fx\right)}{\sqrt{d\tan\left(e+fx\right)}}\,dx$$

```
[In] integrate(sec(f*x+e)**5/(d*tan(f*x+e))**(1/2),x)
[Out] Integral(sec(e + f*x)**5/sqrt(d*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)^5}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^5/sqrt(d*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)^5}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^5/sqrt(d*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)^5\,\sqrt{d\tan\left(e+f\,x\right)}}\,dx$$

```
[In] int(1/(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2)),x)
[Out] int(1/(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2)), x)
```

3.254 $\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{3f\sqrt{d\tan(e+fx)}} + \frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

```
[Out] -2/3*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f
*x),2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d*tan(f*x+e))^(1/2)+2/3*sec
(f*x+e)*(d*tan(f*x+e))^(1/2)/d/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2693, 2694, 2653, 2720}

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df} + \frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)}{3f\sqrt{d\tan(e+fx)}}$$

```
[In] Int[Sec[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(3*f*S
qrt[d*Tan[e + f*x]]) + (2*Sec[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*d*f)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{2}{3}\int\frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx\\
&= \frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{\left(2\sqrt{\sin(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}\,dx}{3\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}\\
&= \frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{\left(2\sec(e+fx)\sqrt{\sin(2e+2fx)}\right)\int\frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{3\sqrt{d\tan(e+fx)}}\\
&= \frac{2\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{3f\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\left(\sec^2(e+fx)+2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\right)\sin(e+fx)}{3f\sqrt{d\tan(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(Sec[e + f*x]^2 + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[e + f*x]^2]*Sq
rt[Sec[e + f*x]^2])*Sin[e + f*x])/(3*f*Sqrt[d*Tan[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(94) = 188.

Time = 1.44 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.53

method	result
default	$\frac{\left(2\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,F\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)+2\,\text{se}}{}$

```
[In] int(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f/(d*tan(f*x+e))^(1/2)*(2*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-c
sc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+cs
c(f*x+e)+1)^(1/2),1/2*2^(1/2))+2*sec(f*x+e)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2
)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((
-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+tan(f*x+e)*sec(f*x+e)*2^(1/2))
*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.23

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{2\left(\sqrt{i\,d}\cos\left(fx+e\right)F(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)|-1)+\sqrt{-i\,d}\cos\left(fx+e\right)F(\arcsin\left(\cos\left(f\right.\right.\right.}{3\,df\,\cos\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(I*d)*cos(f*x + e)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e
)), -1) + sqrt(-I*d)*cos(f*x + e)*elliptic_f(arcsin(cos(f*x + e) - I*sin(f*
x + e)), -1) - sqrt(d*sin(f*x + e)/cos(f*x + e)))/(d*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec^3\left(e+fx\right)}{\sqrt{d\tan\left(e+fx\right)}}\,dx$$

```
[In] integrate(sec(f*x+e)**3/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/sqrt(d*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)^3}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)^3}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)^3\,\sqrt{d\tan\left(e+f\,x\right)}}\,dx$$

```
[In] int(1/(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2)),x)
[Out] int(1/(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2)), x)
```

3.255 $\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{f\sqrt{d\tan(e+fx)}}$$

```
[Out] -(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),
2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d*tan(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2694, 2653, 2720}

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)}{f\sqrt{d\tan(e+fx)}}$$

```
[In] Int[Sec[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(f*Sqrt[
d*Tan[e + f*x]])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}\,dx}{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}\\ &= \frac{\left(\sec(e+fx)\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{\sqrt{d\tan(e+fx)}}\\ &= \frac{\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{f\sqrt{d\tan(e+fx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{2\sqrt[4]{-1}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(e+fx)}\right),-1\right)\sec^3(e+fx)\sqrt{\tan(e+fx)}}{f\sqrt{d\tan(e+fx)}\,(1+\tan^2(e+fx))^{3/2}}$$

```
[In] Integrate[Sec[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[e + f*x]]], -1]*Sec[
e + f*x]^3*Sqrt[Tan[e + f*x]])/(f*Sqrt[d*Tan[e + f*x]]*(1 + Tan[e + f*x]^2)
^(3/2))
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.19

method	result
default	$\frac{F\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)\sqrt{\cot(fx+e)-\csc(fx+e)}\,\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,(1+\sec(f}{f\sqrt{d\tan(fx+e)}}$

[In] int(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/f*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*(cot(f*x+e)-csc(f*x+e))^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)/(d*tan(f*x+e))^(1/2)*(1+sec(f*x+e))*2^(1/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{\sqrt{i\,d}F(\arcsin(\cos(fx+e)+i\,\sin(fx+e))\,|-1)+\sqrt{-i\,d}F(\arcsin(\cos(fx+e)-i\,\sin(fx+e))\,|-1}{df}$$

[In] integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")

[Out] -(sqrt(I*d)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1) + sqrt(-I*d)*elliptic_f(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1))/(d*f)

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$$

[In] integrate(sec(f*x+e)/(d*tan(f*x+e))**(1/2),x)

[Out] Integral(sec(e + f*x)/sqrt(d*tan(e + f*x)), x)

Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{d\tan\left(e+f\,x\right)}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(d*tan(e + f*x))^(1/2)),x)
[Out] int(1/(cos(e + f*x)*(d*tan(e + f*x))^(1/2)), x)
```

3.256 $\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\operatorname{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{2f\sqrt{d\tan(e+fx)}} + \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

```
[Out] -1/2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f
*x),2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d*tan(f*x+e))^(1/2)+cos(f*x
+e)*(d*tan(f*x+e))^(1/2)/d/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2692, 2694, 2653, 2720}

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df} + \frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\operatorname{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)}{2f\sqrt{d\tan(e+fx)}}$$

```
[In] Int[Cos[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sqr
t[d*Tan[e + f*x]]) + (Cos[e + f*x]*Sqrt[d*Tan[e + f*x]])/(d*f)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{1}{2}\int\frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx\\
&= \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{\sqrt{\sin(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}\,dx}{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}\\
&= \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{\left(\sec(e+fx)\sqrt{\sin(2e+2fx)}\right)\int\frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{2\sqrt{d\tan(e+fx)}}\\
&= \frac{\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{2f\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.66

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$$
$$= \frac{\cos(2(e+fx))\sec(e+fx)\left(\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(e+fx)}\right),-1\right)\sec^2(e+fx)-\sqrt{\sec^2(e}{f\sqrt{\sec^2(e+fx)}\sqrt{d\tan(e+fx)}\left(-1+\tan^2(e+fx)\right)}$$

```
[In] Integrate[Cos[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (Cos[2*(e + f*x)]*Sec[e + f*x]*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*S
qrt[Tan[e + f*x]]], -1]*Sec[e + f*x]^2 - Sqrt[Sec[e + f*x]^2]*Sqrt[Tan[e +
f*x]])*Sqrt[Tan[e + f*x]])/(f*Sqrt[Sec[e + f*x]^2]*Sqrt[d*Tan[e + f*x]]*(-1
 + Tan[e + f*x]^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(93) = 186.

Time = 3.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.53

method	result
default	$\frac{\left(\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,F\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)+\sec(fx}{2}$

```
[In] int(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/(d*tan(f*x+e))^(1/2)*((cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc
(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+csc(
f*x+e)+1)^(1/2),1/2*2^(1/2))+sec(f*x+e)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(c
ot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot
(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+2^(1/2)*sin(f*x+e))*2^(1/2)
```

Fricas [F]

$$\int \frac{\cos(e + fx)}{\sqrt{d \tan(e + fx)}} \, dx = \int \frac{\cos(fx + e)}{\sqrt{d \tan(fx + e)}} \, dx$$

```
[In] integrate(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(f*x + e))*cos(f*x + e)/(d*tan(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\cos(e + fx)}{\sqrt{d \tan(e + fx)}} \, dx = \int \frac{\cos(e + fx)}{\sqrt{d \tan(e + fx)}} \, dx$$

```
[In] integrate(cos(f*x+e)/(d*tan(f*x+e))**(1/2),x)
[Out] Integral(cos(e + f*x)/sqrt(d*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\cos(e + fx)}{\sqrt{d \tan(e + fx)}} \, dx = \int \frac{\cos(fx + e)}{\sqrt{d \tan(fx + e)}} \, dx$$

```
[In] integrate(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{\cos(e + fx)}{\sqrt{d \tan(e + fx)}} \, dx = \int \frac{\cos(fx + e)}{\sqrt{d \tan(fx + e)}} \, dx$$

```
[In] integrate(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos(e+f\,x)}{\sqrt{d\tan(e+f\,x)}}\,dx$$

```
[In] int(cos(e + f*x)/(d*tan(e + f*x))^(1/2),x)
[Out] int(cos(e + f*x)/(d*tan(e + f*x))^(1/2), x)
```

3.257 $\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{5\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{12f\sqrt{d\tan(e+fx)}} + \frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df} + \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

```
[Out] -5/12*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+
f*x),2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d*tan(f*x+e))^(1/2)+5/6*co
s(f*x+e)*(d*tan(f*x+e))^(1/2)/d/f+1/3*cos(f*x+e)^3*(d*tan(f*x+e))^(1/2)/d/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2692, 2694, 2653, 2720}

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df} + \frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df} + \frac{5\sqrt{\sin(2e+2fx)}\sec(e+fx)\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)}{12f\sqrt{d\tan(e+fx)}}$$

```
[In] Int[Cos[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (5*EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(12*f*
Sqrt[d*Tan[e + f*x]]) + (5*Cos[e + f*x]*Sqrt[d*Tan[e + f*x]])/(6*d*f) + (Co
s[e + f*x]^3*Sqrt[d*Tan[e + f*x]])/(3*d*f)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df} + \frac{5}{6}\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx \\
&= \frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df} + \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df} + \frac{5}{12}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx \\
&= \frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df} + \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df} \\
&\quad + \frac{\left(5\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}\,dx}{12\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}} \\
&= \frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df} + \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df} \\
&\quad + \frac{\left(5\sec(e+fx)\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{12\sqrt{d\tan(e+fx)}}
\end{aligned}
$$

$$= \frac{5\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{12f\sqrt{d\tan(e+fx)}} + \frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df} + \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{11\sin(e+fx)+\sin(3(e+fx))-10\sqrt[4]{-1}\cos(e+fx)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(e+fx)}\right),-1\right)\sqrt{}}{12f\sqrt{d\tan(e+fx)}}$$

```
[In] Integrate[Cos[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (11*Sin[e + f*x] + Sin[3*(e + f*x)] - 10*(-1)^(1/4)*Cos[e + f*x]*EllipticF[
I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[e + f*x]]], -1]*Sqrt[Sec[e + f*x]^2]*Sqrt[Tan
[e + f*x]])/(12*f*Sqrt[d*Tan[e + f*x]])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.23 (sec) , antiderivative size = 1906, normalized size of antiderivative = 17.49

method	result	size
default	Expression too large to display	1906

```
[In] int(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/f/(d*tan(f*x+e))^(1/2)*(6*I*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+
e)-csc(f*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2-1/2*I,
1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)-6*I*sec(f*x+e)*(cot(f*x+e)-cs
c(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^
(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*
I*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e
)+csc(f*x+e)+1)^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2+1/2*I
,1/2*2^(1/2))+6*I*sec(f*x+e)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-cs
c(f*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2-1/2*I,1/2*2
^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)+6*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*
(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1
```

/2-1/2*I,1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)-32*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+6*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)+6*sec(f*x+e)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)-32*sec(f*x+e)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+6*sec(f*x+e)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-3*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))-6*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-6*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-20*2^(1/2)*sin(f*x+e)+3*sec(f*x+e)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-3*sec(f*x+e)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))-6*sec(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-6*sec(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*2^(1/2)

Fricas [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^3}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(f*x + e))*cos(f*x + e)^3/(d*tan(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**3/(d*tan(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^3}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^3}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos\left(e+f\,x\right)^3}{\sqrt{d\tan\left(e+f\,x\right)}}\,dx$$

```
[In] int(cos(e + f*x)^3/(d*tan(e + f*x))^(1/2),x)
[Out] int(cos(e + f*x)^3/(d*tan(e + f*x))^(1/2), x)
```

3.258 $\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{bd\sqrt{d\tan(a+bx)}}+\frac{4(d\tan(a+bx))^{3/2}}{3bd^3}+\frac{2(d\tan(a+bx))^{7/2}}{7bd^5}$$

```
[Out] -2/b/d/(d*tan(b*x+a))^(1/2)+4/3*(d*tan(b*x+a))^(3/2)/b/d^3+2/7*(d*tan(b*x+a
))^(7/2)/b/d^5
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2687, 276}

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2(d\tan(a+bx))^{7/2}}{7bd^5}+\frac{4(d\tan(a+bx))^{3/2}}{3bd^3}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Sec[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -2/(b*d*Sqrt[d*Tan[a + b*x]]) + (4*(d*Tan[a + b*x])^(3/2))/(3*b*d^3) + (2*(
d*Tan[a + b*x])^(7/2))/(7*b*d^5)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(dx)^{3/2}}\,dx, x, \tan(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{(dx)^{3/2}} + \frac{2\sqrt{dx}}{d^2} + \frac{(dx)^{5/2}}{d^4}\right)\,dx, x, \tan(a+bx)\right)}{b}\\
&= -\frac{2}{bd\sqrt{d\tan(a+bx)}} + \frac{4(d\tan(a+bx))^{3/2}}{3bd^3} + \frac{2(d\tan(a+bx))^{7/2}}{7bd^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{-42 + (22 + 6\sec^2(a+bx))\tan^2(a+bx)}{21bd\sqrt{d\tan(a+bx)}}$$

```
[In] Integrate[Sec[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
[Out] (-42 + (22 + 6*Sec[a + b*x]^2)*Tan[a + b*x]^2)/(21*b*d*Sqrt[d*Tan[a + b*x]]
)
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{2\left(32-8\left(\sec^2(bx+a)\right)-3\left(\sec^4(bx+a)\right)\right)}{21b\sqrt{d\tan(bx+a)}\,d}$	41

```
[In] int(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/21/b/(d*tan(b*x+a))^(1/2)/d*(32-8*sec(b*x+a)^2-3*sec(b*x+a)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(32\cos\left(bx+a\right)^4-8\cos\left(bx+a\right)^2-3\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{21\,bd^2\cos\left(bx+a\right)^3\sin\left(bx+a\right)}$$

```
[In] integrate(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] -2/21*(32*cos(b*x + a)^4 - 8*cos(b*x + a)^2 - 3)*sqrt(d*sin(b*x + a)/cos(b*
x + a))/(b*d^2*cos(b*x + a)^3*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec^6\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)**6/(d*tan(b*x+a))**(3/2),x)
[Out] Integral(sec(a + b*x)**6/(d*tan(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(\frac{21}{\sqrt{d\tan(bx+a)}}-\frac{3\left(d\tan(bx+a)\right)^{\frac{7}{2}}+14\left(d\tan(bx+a)\right)^{\frac{3}{2}}d^2}{d^4}\right)}{21\,bd}$$

```
[In] integrate(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] -2/21*(21/sqrt(d*tan(b*x + a)) - (3*(d*tan(b*x + a))^(7/2) + 14*(d*tan(b*x
+ a))^(3/2)*d^2)/d^4)/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(\frac{21}{\sqrt{d\tan(bx+a)}b}-\frac{3\sqrt{d\tan(bx+a)}b^6d^{27}\tan(bx+a)^3+14\sqrt{d\tan(bx+a)}b^6d^{27}\tan(bx+a)}{b^7d^{28}}\right)}{21\,d}$$

```
[In] integrate(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/21*(21/(sqrt(d*tan(b*x + a))*b) - (3*sqrt(d*tan(b*x + a))*b^6*d^27*tan(b
*x + a)^3 + 14*sqrt(d*tan(b*x + a))*b^6*d^27*tan(b*x + a))/(b^7*d^28))/d
```

Mupad [B] (verification not implemented)

Time = 7.66 (sec) , antiderivative size = 268, normalized size of antiderivative = 4.12

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\left(\frac{20\mathrm{i}}{21\,b\,d^2}+\frac{e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,64\mathrm{i}}{21\,b\,d^2}\right)\sqrt{-\frac{d\,\left(e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1} + \frac{\sqrt{-\frac{d\,\left(e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,20\mathrm{i}}{21\,b\,d^2\,\left(e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)} + \frac{\sqrt{-\frac{d\,\left(e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,24\mathrm{i}}{7\,b\,d^2\,{\left(e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2} - \frac{\sqrt{-\frac{d\,\left(e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,b\,d^2\,{\left(e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^3}$$

```
[In] int(1/(cos(a + b*x)^6*(d*tan(a + b*x))^(3/2)),x)
```

```
[Out] ((-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*20i)/(2
1*b*d^2*(exp(a*2i + b*x*2i) + 1)) - ((20i/(21*b*d^2) + (exp(a*2i + b*x*2i)*
64i)/(21*b*d^2))*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1
))^(1/2))/(exp(a*2i + b*x*2i) - 1) + ((-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(e
xp(a*2i + b*x*2i) + 1))^(1/2)*24i)/(7*b*d^2*(exp(a*2i + b*x*2i) + 1)^2) - (
(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*16i)/(7*
b*d^2*(exp(a*2i + b*x*2i) + 1)^3)
```

3.259 $\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{bd\sqrt{d\tan(a+bx)}}+\frac{2(d\tan(a+bx))^{3/2}}{3bd^3}$$

```
[Out] -2/b/d/(d*tan(b*x+a))^(1/2)+2/3*(d*tan(b*x+a))^(3/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2687, 14}

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2(d\tan(a+bx))^{3/2}}{3bd^3}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Sec[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -2/(b*d*Sqrt[d*Tan[a + b*x]]) + (2*(d*Tan[a + b*x])^(3/2))/(3*b*d^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(dx)^{3/2}}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{(dx)^{3/2}} + \frac{\sqrt{dx}}{d^2}\right)\,dx, x, \tan(a+bx)\right)}{b}\\ &= -\frac{2}{bd\sqrt{d\tan(a+bx)}} + \frac{2(d\tan(a+bx))^{3/2}}{3bd^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2(-3+\tan^2(a+bx))}{3bd\sqrt{d\tan(a+bx)}}$$

```
[In] Integrate[Sec[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
[Out] (2*(-3 + Tan[a + b*x]^2))/(3*b*d*Sqrt[d*Tan[a + b*x]])
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{2\left(4-\left(\sec^2(bx+a)\right)\right)}{3b\sqrt{d\tan(bx+a)}\,d}$	31

```
[In] int(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3/b/(d*tan(b*x+a))^(1/2)/d*(4-sec(b*x+a)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(4\cos\left(bx+a\right)^2-1\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{3\,bd^2\cos\left(bx+a\right)\sin\left(bx+a\right)}$$

```
[In] integrate(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] -2/3*(4*cos(b*x + a)^2 - 1)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*d^2*cos(b*
x + a)*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec^4\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)**4/(d*tan(b*x+a))**(3/2),x)
[Out] Integral(sec(a + b*x)**4/(d*tan(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(\frac{3}{\sqrt{d\tan(bx+a)}}-\frac{(d\tan(bx+a))^{\frac{3}{2}}}{d^2}\right)}{3\,bd}$$

```
[In] integrate(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] -2/3*(3/sqrt(d*tan(b*x + a)) - (d*tan(b*x + a))^(3/2)/d^2)/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{d\tan(bx+a)}\tan(bx+a)}{bd} - \frac{3}{\sqrt{d\tan(bx+a)}b}\right)}{3\,d}$$

```
[In] integrate(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(d*tan(b*x + a))*tan(b*x + a)/(b*d) - 3/(sqrt(d*tan(b*x + a))*b))/
d
```

Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{4\left(\sin\left(2\,a+2\,b\,x\right)+\sin\left(4\,a+4\,b\,x\right)\right)\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}}{3\,b\,d^2\,\sin\left(2\,a+2\,b\,x\right)^2}$$

```
[In] int(1/(cos(a + b*x)^4*(d*tan(a + b*x))^(3/2)),x)
```

```
[Out] -(4*(sin(2*a + 2*b*x) + sin(4*a + 4*b*x))*((d*sin(2*a + 2*b*x))/(cos(2*a +
2*b*x) + 1))^(1/2))/(3*b*d^2*sin(2*a + 2*b*x)^2)
```

3.260 $\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

```
[Out] -2/b/d/(d*tan(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2687, 32}

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Sec[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -2/(b*d*Sqrt[d*Tan[a + b*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(dx)^{3/2}}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

```
[In] Integrate[Sec[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
[Out] -2/(b*d*Sqrt[d*Tan[a + b*x]])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{2}{bd\sqrt{d\tan(bx+a)}}$	19
default	$-\frac{2}{bd\sqrt{d\tan(bx+a)}}$	19

```
[In] int(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/b/d/(d*tan(b*x+a))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)}{bd^2\sin(bx+a)}$$

```
[In] integrate(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)/(b*d^2*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
[Out] Integral(sec(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{\sqrt{d\tan(bx+a)}bd}$$

```
[In] integrate(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] -2/(sqrt(d*tan(b*x + a))*b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{\sqrt{d\tan(bx+a)}bd}$$

```
[In] integrate(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] -2/(sqrt(d*tan(b*x + a))*b*d)
```

Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\sin\left(2\,a+2\,b\,x\right)\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}}{b\,d^2\,\sin\left(a+b\,x\right)^2}$$

```
[In] int(1/(cos(a + b*x)^2*(d*tan(a + b*x))^(3/2)),x)
[Out] -(sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2))/(b*
d^2*sin(a + b*x)^2)
```

3.261 $\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 212

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}bd^{3/2}} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}bd^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}bd^{3/2}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}bd^{3/2}} - \frac{2}{bd\sqrt{d\tan(a+bx)}}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)-1/2*ar
ctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)-1/4*ln(d^(1/
2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1/2)+1/4*l
n(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1/2
)-2/b/d/(d*tan(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used

$= \{3555, 3557, 335, 303, 1176, 631, 210, 1179, 642\}$

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}bd^{3/2}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}bd^{3/2}} - \frac{\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{2\sqrt{2}bd^{3/2}} + \frac{\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{2\sqrt{2}bd^{3/2}} - \frac{2}{bd\sqrt{d\tan(a+bx)}}$$

```
[In] Int[(d*Tan[a + b*x])^(-3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*b*d^(3/2)) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*b*d^(3/2)) - Log[
Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]/(2*Sqrt[2]*b
*d^(3/2)) + Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x
]]]/(2*Sqrt[2]*b*d^(3/2)) - 2/(b*d*Sqrt[d*Tan[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{2}{bd\sqrt{d\tan(a+bx)}} - \frac{\int\sqrt{d\tan(a+bx)}\,dx}{d^2}$$

$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}} - \frac{\text{Subst}\left(\int\frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}} - \frac{2\text{Subst}\left(\int\frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{bd} - \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}bd^{3/2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}bd^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2bd} - \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2bd}$$

$$= -\frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}bd^{3/2}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}bd^{3/2}} - \frac{2}{bd\sqrt{d\tan(a+bx)}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}bd^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}bd^{3/2}}$$

$$= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}bd^{3/2}} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}bd^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}bd^{3/2}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}bd^{3/2}} - \frac{2}{bd\sqrt{d\tan(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.39

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \frac{-2-\arctan\left(\sqrt[4]{-\tan^2(a+bx)}\right)\sqrt[4]{-\tan^2(a+bx)}+\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(a+bx}\right.}{bd\sqrt{d\tan(a+bx)}}$$

[In] Integrate[(d*Tan[a + b*x])^(-3/2),x]

[Out] (-2 - ArcTan[(-Tan[a + b*x]^2)^(1/4)]*(-Tan[a + b*x]^2)^(1/4) + ArcTanh[(-Tan[a + b*x]^2)^(1/4)]*(-Tan[a + b*x]^2)^(1/4))/(b*d*Sqrt[d*Tan[a + b*x]])

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{2d\left(-\frac{1}{d^2\sqrt{d\tan(bx+a)}}-\frac{\sqrt{2}\left(\ln\left(\frac{d\tan(bx+a)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(bx+a)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{}}{}\right.\right.}{8d^2(d^2)^{\frac{1}{4}}}\right.}{b}$
default	$\frac{2d\left(-\frac{1}{d^2\sqrt{d\tan(bx+a)}}-\frac{\sqrt{2}\left(\ln\left(\frac{d\tan(bx+a)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(bx+a)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{}}{}\right.\right.}{8d^2(d^2)^{\frac{1}{4}}}\right.}{b}$

[In] int(1/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/b*d*(-1/d^2/(d*tan(b*x+a))^(1/2)-1/8/d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(b*x+a)-(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(b*x+a)+(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)+1)))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx =$$

$$-\frac{bd^2\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{4}}\log\left(b^3d^5\left(-\frac{1}{b^4d^6}\right)^{\frac{3}{4}}+\sqrt{d\tan(bx+a)}\right)\tan(bx+a)-i\,bd^2\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{4}}\log\left(i\,b^3d^5\left(-\frac{1}{b^4d^6}\right)^{\frac{3}{4}}+\sqrt{d}\right.}{}$$

```
[In] integrate(1/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*d^2*(-1/(b^4*d^6))^(1/4)*log(b^3*d^5*(-1/(b^4*d^6))^(3/4) + sqrt(d*
tan(b*x + a)))*tan(b*x + a) - I*b*d^2*(-1/(b^4*d^6))^(1/4)*log(I*b^3*d^5*(-
1/(b^4*d^6))^(3/4) + sqrt(d*tan(b*x + a)))*tan(b*x + a) + I*b*d^2*(-1/(b^4*
d^6))^(1/4)*log(-I*b^3*d^5*(-1/(b^4*d^6))^(3/4) + sqrt(d*tan(b*x + a)))*tan
(b*x + a) - b*d^2*(-1/(b^4*d^6))^(1/4)*log(-b^3*d^5*(-1/(b^4*d^6))^(3/4) +
sqrt(d*tan(b*x + a)))*tan(b*x + a) + 4*sqrt(d*tan(b*x + a)))/(b*d^2*tan(b*x
 + a))
```

Sympy [F]

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.79

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx =$$

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)}{\sqrt{d}}+}{4\,bd}$$

```
[In] integrate(1/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(b*x + a)
))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sq
rt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a) + sqrt(2)
*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x + a) - s
qrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + 8/sqrt(d*tan(b*x + a)))/
(b*d)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(a+b\,x)}}{\sqrt{d}}\right)}{b\,d^{3/2}} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(a+b\,x)}}{\sqrt{d}}\right)}{b\,d^{3/2}} - \frac{2}{b\,d\,\sqrt{d\tan\left(a+b\,x\right)}}$$

```
[In] int(1/(d*tan(a + b*x))^(3/2),x)
```

```
[Out] ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*tan(a + b*x))^(1/2))/d^(1/2)))/(b*d^(3/2))
 - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*tan(a + b*x))^(1/2))/d^(1/2)))/(b*d^(3/2
)) - 2/(b*d*(d*tan(a + b*x))^(1/2))
```

3.262 $\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 249

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} - \frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} - \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} + \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} - \frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}}$$

```
[Out] 5/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)-5/8*ar
ctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)-5/16*ln(d^(1
/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1/2)+5/16
*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1
/2)-5/2/b/d/(d*tan(b*x+a))^(1/2)+1/2*cos(b*x+a)^2/b/d/(d*tan(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2687, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} - \frac{5\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}bd^{3/2}} - \frac{5\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}bd^{3/2}} + \frac{5\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}bd^{3/2}} - \frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Cos[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (5*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^(3/2)
) - (5*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^(
3/2)) - (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]
]])/(8*Sqrt[2]*b*d^(3/2)) + (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]
*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b*d^(3/2)) - 5/(2*b*d*Sqrt[d*Tan[a + b*x
]]) + Cos[a + b*x]^2/(2*b*d*Sqrt[d*Tan[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(dx)^{3/2}(1+x^2)^2}\,dx, x, \tan(a+bx)\right)}{b}$$

$$= \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} + \frac{5\text{Subst}\left(\int \frac{1}{(dx)^{3/2}(1+x^2)}\,dx, x, \tan(a+bx)\right)}{4b}$$

$$= -\frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} - \frac{5\text{Subst}\left(\int \frac{\sqrt{dx}}{1+x^2}\,dx, x, \tan(a+bx)\right)}{4bd^2}$$

$$= -\frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} - \frac{5\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2bd^3}$$

$$= -\frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} + \frac{5\text{Subst}\left(\int \frac{d-x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4bd^3} - \frac{5\text{Subst}\left(\int \frac{d+x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4bd^3}$$

$$= -\frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} - \frac{5\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} - \frac{5\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} - \frac{5\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8bd} - \frac{5\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8bd}$$

$$= -\frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} + \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} - \frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} - \frac{5\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} + \frac{5\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}}$$

$$= \frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} - \frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} - \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} + \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} - \frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.46

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\csc(a+bx)\left(-17\cos(a+bx)+\cos(3(a+bx))+5\arcsin(\cos(a+bx)-\sin(a+b$$

```
[In] Integrate[Cos[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Csc[a + b*x]*(-17*Cos[a + b*x] + Cos[3*(a + b*x)] + 5*ArcSin[Cos[a + b*x]
- Sin[a + b*x]]*Sqrt[Sin[2*(a + b*x)]] + 5*Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x)]])*Sqrt[d*Tan[a + b*x]])/(8*
b*d^2)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(189) = 378$.

Time = 19.77 (sec) , antiderivative size = 937, normalized size of antiderivative = 3.76

method	result	size
default	Expression too large to display	937

```
[In] int(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/b*csc(b*x+a)*(4*cos(b*x+a)^2*sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a
)/(cos(b*x+a)+1)^2)^(1/2)-20*sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(co
s(b*x+a)+1)^2)^(1/2)+10*cos(b*x+a)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*
sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))+10*cos(b*
x+a)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(
1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))-5*cos(b*x+a)*ln(-(cot(b*x+a)*cos(b*x+a)
-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b
*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-s
in(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))+5*cos(b*x+a)*ln((2*sin(b*x+a)*(-co
t(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3
+cot(b*x+a)-csc(b*x+a))^(1/2)-cot(b*x+a)*cos(b*x+a)+2*cot(b*x+a)+2*cos(b*x+
a)+sin(b*x+a)-csc(b*x+a)-2)/(-1+cos(b*x+a)))-10*arctan((sin(b*x+a)*2^(1/2)*
(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a
)))-10*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)
^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+5*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*
x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc
(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)
+csc(b*x+a)+2)/(-1+cos(b*x+a)))-5*ln((2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x
+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+
a))^(1/2)-cot(b*x+a)*cos(b*x+a)+2*cot(b*x+a)+2*cos(b*x+a)+sin(b*x+a)-csc(b*
x+a)-2)/(-1+cos(b*x+a))))/(d*tan(b*x+a))^(1/2)/(-cos(b*x+a)*sin(b*x+a)/(cos
(b*x+a)+1)^2)^(1/2)/d*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 1034, normalized size of antiderivative = 4.15

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(5*b*d^2*(-1/(b^4*d^6))^(1/4)*log(-1/2*cos(b*x + a)*sin(b*x + a) + 1/2
*(b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 - b*d*(-1/(b^4*d^6))^(1/4)*co
```

```
s(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1/4*(2*b^2*d^3
*cos(b*x + a)^2 - b^2*d^3)*sqrt(-1/(b^4*d^6)))*sin(b*x + a) - 5*b*d^2*(-1/(
b^4*d^6))^(1/4)*log(-1/2*cos(b*x + a)*sin(b*x + a) - 1/2*(b^3*d^4*(-1/(b^4*
d^6))^(3/4)*cos(b*x + a)^2 - b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)*sin(b*x
+ a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1/4*(2*b^2*d^3*cos(b*x + a)^2 - b
^2*d^3)*sqrt(-1/(b^4*d^6)))*sin(b*x + a) - 5*I*b*d^2*(-1/(b^4*d^6))^(1/4)*l
og(-1/2*cos(b*x + a)*sin(b*x + a) + 1/2*(I*b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos
(b*x + a)^2 + I*b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*
sin(b*x + a)/cos(b*x + a)) - 1/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(
-1/(b^4*d^6)))*sin(b*x + a) + 5*I*b*d^2*(-1/(b^4*d^6))^(1/4)*log(-1/2*cos(b
*x + a)*sin(b*x + a) + 1/2*(-I*b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)^2
- I*b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)
/cos(b*x + a)) - 1/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-1/(b^4*d^6)
))*sin(b*x + a) - 5*b*d^2*(-1/(b^4*d^6))^(1/4)*log(2*(b^3*d^4*(-1/(b^4*d^6)
)^(3/4)*cos(b*x + a)*sin(b*x + a) - b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)^2
)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*sin(b*x + a) + 5*b*d^2*(-1/(b^4*d^
6))^(1/4)*log(-2*(b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) -
b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))
+ 1)*sin(b*x + a) - 5*I*b*d^2*(-1/(b^4*d^6))^(1/4)*log(-2*(I*b^3*d^4*(-1/(b
^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + I*b*d*(-1/(b^4*d^6))^(1/4)*cos(b
*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*sin(b*x + a) + 5*I*b*d^2*
(-1/(b^4*d^6))^(1/4)*log(-2*(-I*b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)*s
in(b*x + a) - I*b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a
)/cos(b*x + a)) + 1)*sin(b*x + a) + 16*(cos(b*x + a)^3 - 5*cos(b*x + a))*sq
rt(d*sin(b*x + a)/cos(b*x + a)))/(b*d^2*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos^2{(a+bx)}}{(d\tan{(a+bx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
[Out] Integral(cos(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.82

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\frac{10\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{10\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{5\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right.}{\sqrt{d}}}{16\,bd}$$

```
[In] integrate(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(b*x +
a)))/sqrt(d))/sqrt(d) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2
*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - 5*sqrt(2)*log(d*tan(b*x + a) + sq
rt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + 5*sqrt(2)*log(d*tan(b*x +
 a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + 8*(5*d^2*tan(b*x
+ a)^2 + 4*d^2)/((d*tan(b*x + a))^(5/2) + sqrt(d*tan(b*x + a))*d^2))/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.01

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\frac{10\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2} + \frac{10\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2} - \frac{5\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan\right.}{bd^2}}{16\,d}$$

```
[In] integrate(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*(10*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2
*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^2) + 10*sqrt(2)*abs(d)^(3/2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)
))/(b*d^2) - 5*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan
(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2) + 5*sqrt(2)*abs(d)^(3/2)*log(d*ta
n(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2) +
8*(5*d^2*tan(b*x + a)^2 + 4*d^2)/((sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)^2
+ sqrt(d*tan(b*x + a))*d^2)*b))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos{(a+b\,x)}^2}{{(d\tan{(a+b\,x)})}^{3/2}}\,dx$$

```
[In] int(cos(a + b*x)^2/(d*tan(a + b*x))^(3/2),x)
[Out] int(cos(a + b*x)^2/(d*tan(a + b*x))^(3/2), x)
```

3.263 $\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{24\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^2\sqrt{\sin(2a+2bx)}} + \frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}$$

```
[Out] -2*sec(b*x+a)^3/b/d/(d*tan(b*x+a))^(1/2)+24/5*cos(b*x+a)*(sin(a+1/4*Pi+b*x)
^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*x
+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)+24/5*cos(b*x+a)*(d*tan(b*x+a))^(3/2)/
b/d^3+12/5*sec(b*x+a)*(d*tan(b*x+a))^(3/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2688, 2693, 2695, 2652, 2719}

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} - \frac{24\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^2\sqrt{\sin(2a+2bx)}} - \frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Sec[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Sec[a + b*x]^3)/(b*d*Sqrt[d*Tan[a + b*x]]) - (24*Cos[a + b*x]*EllipticE
[a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(5*b*d^2*Sqrt[Sin[2*a + 2*b*x]])
+ (24*Cos[a + b*x]*(d*Tan[a + b*x])^(3/2))/(5*b*d^3) + (12*Sec[a + b*x]*(d*
Tan[a + b*x])^(3/2))/(5*b*d^3)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{6\int \sec^3(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2}$$

$$= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\int\sec(a+bx)\sqrt{d\tan(a+bx)}\,dx}{5d^2}$$

$$= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} - \frac{24\int\cos(a+bx)\sqrt{d\tan(a+bx)}\,dx}{5d^2}$$

$$= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} - \frac{\left(24\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{5d^2\sqrt{\sin(a+bx)}}$$

$$= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} - \frac{\left(24\cos(a+bx)\sqrt{d\tan(a+bx)}\right)\int\sqrt{\sin(2a+2bx)}\,dx}{5d^2\sqrt{\sin(2a+2bx)}}$$

$$= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{24\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^2\sqrt{\sin(2a+2bx)}} + \frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.88 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.75

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\csc(a+bx)\sqrt{d\tan(a+bx)}\left(-8\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\text{ta}\right.}{5bd^2\sqrt{\sec^2(a+}}$$

```
[In] Integrate[Sec[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]]*(-8*Hypergeometric2F1[3/4, 3/2, 7/4, -
Tan[a + b*x]^2]*Tan[a + b*x]^2 + Sqrt[Sec[a + b*x]^2]*(-5 + 12*Sin[a + b*x]
^2 + Tan[a + b*x]^2)))/(5*b*d^2*Sqrt[Sec[a + b*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(147) = 294.

Time = 1.62 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.75

method	result
default	$-\frac{\left(12\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)-24\sqrt{1}\right.}{}$

```
[In] int(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b/(d*tan(b*x+a))^(1/2)/d*(12*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a
)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+
a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-24*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(
b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b
*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+12*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a)
)^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Elli
pticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-24*sec(b*x+a)*(1+csc(b*x
+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+
a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+12*2^(1/2)
-6*sec(b*x+a)*2^(1/2)-sec(b*x+a)^3*2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.61

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx =$$

$$-\frac{2\left(6i\sqrt{i\,d}\cos\left(bx+a\right)^2E(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\,|\,-1)\sin\left(bx+a\right)-6i\sqrt{-i\,d}\cos\left(bx+a\right)^2E\right.}{}$$

```
[In] integrate(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(6*I*sqrt(I*d)*cos(b*x + a)^2*elliptic_e(arcsin(cos(b*x + a) + I*sin(b
*x + a)), -1)*sin(b*x + a) - 6*I*sqrt(-I*d)*cos(b*x + a)^2*elliptic_e(arcsi
n(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) - 6*I*sqrt(I*d)*cos(b*x
+ a)^2*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) +
 6*I*sqrt(-I*d)*cos(b*x + a)^2*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x +
 a)), -1)*sin(b*x + a) + (12*cos(b*x + a)^4 - 6*cos(b*x + a)^2 - 1)*sqrt(d*
sin(b*x + a)/cos(b*x + a)))/(b*d^2*cos(b*x + a)^2*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec^5{\left(a+bx \right)}}{\left(d\tan{\left(a+bx \right)}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)**5/(d*tan(b*x+a))**(3/2),x)
[Out] Integral(sec(a + b*x)**5/(d*tan(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec\left(bx+a\right)^{5}}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec\left(bx+a\right)^{5}}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(sec(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{{\cos\left(a+b\,x\right)}^5\,{\left(d\,\mathrm{tan}\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int(1/(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2)),x)
[Out] int(1/(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2)), x)
```

3.264 $\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{4\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}} + \frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}$$

```
[Out] -2*sec(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+4*cos(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(
1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*x+a))^
(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)+4*cos(b*x+a)*(d*tan(b*x+a))^(3/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2688, 2693, 2695, 2652, 2719}

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3} - \frac{4\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}} - \frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Sec[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Sec[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (4*Cos[a + b*x]*EllipticE[a
- Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]]) + (4*
Cos[a + b*x]*(d*Tan[a + b*x])^(3/2))/(b*d^3)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{2\int\sec(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2}\\
&= -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}-\frac{4\int\cos(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2}\\
&= -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}\\
&\quad-\frac{\left(4\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{d^2\sqrt{\sin(a+bx)}}
\end{aligned}$$

$$= -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3} - \frac{\left(4\cos(a+bx)\sqrt{d\tan(a+bx)}\right)\int\sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}}$$

$$= -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{4\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}} + \frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int\frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\csc(a+bx)\sqrt{d\tan(a+bx)}\left(3\cos(2(a+bx))\sqrt{\sec^2(a+bx)} + 4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+b\right.\right.}{3bd^2\sqrt{\sec^2(a+bx)}}$$

```
[In] Integrate[Sec[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]]*(3*Cos[2*(a + b*x)]*Sqrt[Sec[a + b*x]
^2] + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Tan[a + b*x]^2))/
(3*b*d^2*Sqrt[Sec[a + b*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(121) = 242.

Time = 1.48 (sec) , antiderivative size = 367, normalized size of antiderivative = 3.53

method	result
default	$-\frac{\left(-4\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)+2\sqrt{c}\right.}{}$

```
[In] int(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(d*tan(b*x+a))^(1/2)/d*(-4*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a
)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)
-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a
)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+
a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-4*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1
```

```
/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Elliptic
E((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*sec(b*x+a)*(1+csc(b*x+a)-c
ot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(
1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*2^(1/2)-sec(b
*x+a)*2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.65

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx =$$
$$-\frac{2\left(i\sqrt{i\,d}E(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\,|-1)\sin\left(bx+a\right)-i\sqrt{-i\,d}E(\arcsin\left(\cos\left(bx+a\right)-i\,\sin\right.\right.}{}$$

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(I*sqrt(I*d)*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b
*x + a) - I*sqrt(-I*d)*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1
)*sin(b*x + a) - I*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a
)), -1)*sin(b*x + a) + I*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(
b*x + a)), -1)*sin(b*x + a) + (2*cos(b*x + a)^2 - 1)*sqrt(d*sin(b*x + a)/co
s(b*x + a)))/(b*d^2*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec^3\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)**3/(d*tan(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^3\,\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2)),x)
[Out] int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2)), x)
```

3.265 $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{2\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

```
[Out] -2*cos(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+2*cos(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(
1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*x+a))^
(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2688, 2695, 2652, 2719}

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}} - \frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Sec[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (2*Cos[a + b*x]*EllipticE[a
- Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{2\int \cos(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2} \\
&= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{\left(2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{d^2\sqrt{\sin(a+bx)}} \\
&= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{\left(2\cos(a+bx)\sqrt{d\tan(a+bx)}\right)\int \sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}} \\
&= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{2\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.43 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sin(a+bx)\left(3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\tan^2(a+bx)\right)}{3b(d\tan(a+bx))^{3/2}}$$

```
[In] Integrate[Sec[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Sin[a + b*x]*(3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*S
qrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(3*b*(d*Tan[a + b*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(97) = 194.

Time = 1.11 (sec) , antiderivative size = 352, normalized size of antiderivative = 4.51

method	result
default	$-\frac{\left(-2\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)+\sqrt{\cdots}\right.}{}$

```
[In] int(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/d/(d*tan(b*x+a))^(1/2)*(-2*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a
)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)
-cot(b*x+a))^(1/2),1/2*2^(1/2))+(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+
1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)
-cot(b*x+a))^(1/2),1/2*2^(1/2))-2*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2
)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE(
(1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+sec(b*x+a)*(1+csc(b*x+a)-cot(b
*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)
*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.17

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx =$$

$$-\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)^2 + i\sqrt{i\,d}E(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)\sin(bx+a) - i\sqrt{-i\,d}E(\mathrm{ar}$$

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2 + I*sqrt(I*d)*elliptic
_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) - I*sqrt(-I*d)*e
lliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) - I*sqrt(
I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + I
*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x +
 a))/(b*d^2*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(sec(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)\,{\left(d\tan\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int(1/(cos(a + b*x)*(d*tan(a + b*x))^(3/2)),x)
[Out] int(1/(cos(a + b*x)*(d*tan(a + b*x))^(3/2)), x)
```

3.266 $\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{3\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

```
[Out] -2*cos(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+3*cos(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(
1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*x+a))^
(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2689, 2695, 2652, 2719}

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{3\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}} - \frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Cos[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (3*Cos[a + b*x]*EllipticE[a
- Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{3\int \cos(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2} \\
&= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{\left(3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{d^2\sqrt{\sin(a+bx)}} \\
&= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{\left(3\cos(a+bx)\sqrt{d\tan(a+bx)}\right)\int \sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}} \\
&= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{3\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sin(a+bx)\left(1+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\tan^2(a+bx)\right)}{b(d\tan(a+bx))^{3/2}}$$

```
[In] Integrate[Cos[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Sin[a + b*x]*(1 + Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqr
t[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(b*(d*Tan[a + b*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(97) = 194.

Time = 1.11 (sec) , antiderivative size = 366, normalized size of antiderivative = 4.69

method	result
default	$\frac{\left(6\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)-3\sqrt{\cot(b}\right.}{}$

```
[In] int(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b/(d*tan(b*x+a))^(1/2)/d*(6*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a
)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)
-cot(b*x+a))^(1/2),1/2*2^(1/2))-3*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a
)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+
a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+6*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1
/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Elliptic
E((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-3*sec(b*x+a)*(1+csc(b*x+a)-c
ot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(
1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(b*x
+a)-3*2^(1/2))*2^(1/2)
```

Fricas [F]

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(b*x + a))*cos(b*x + a)/(d^2*tan(b*x + a)^2), x)
```

Sympy [F]

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*tan(b*x+a))**(3/2),x)
[Out] Integral(cos(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(cos(a + b*x)/(d*tan(a + b*x))^(3/2),x)
[Out] int(cos(a + b*x)/(d*tan(a + b*x))^(3/2), x)
```

3.267 $\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{2bd^2\sqrt{\sin(2a+2bx)}} - \frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3}$$

```
[Out] -2*cos(b*x+a)^3/b/d/(d*tan(b*x+a))^(1/2)+7/2*cos(b*x+a)*(sin(a+1/4*Pi+b*x)^
2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*x+
a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)-7/3*cos(b*x+a)^3*(d*tan(b*x+a))^(3/2)/
b/d^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2689, 2692, 2695, 2652, 2719}

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3} - \frac{7\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{2bd^2\sqrt{\sin(2a+2bx)}} - \frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Cos[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]^3)/(b*d*Sqrt[d*Tan[a + b*x]]) - (7*Cos[a + b*x]*EllipticE[
a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(2*b*d^2*Sqrt[Sin[2*a + 2*b*x]]) -
 (7*Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(3*b*d^3)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\int \cos^3(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2} \\
&= -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3} - \frac{7\int \cos(a+bx)\sqrt{d\tan(a+bx)}\,dx}{2d^2} \\
&= -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3} \\
&\quad - \frac{\left(7\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{2d^2\sqrt{\sin(a+bx)}}
\end{aligned}
$$

$$= -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3} - \frac{\left(7\cos(a+bx)\sqrt{d\tan(a+bx)}\right)\int\sqrt{\sin(2a+2bx)}\,dx}{2d^2\sqrt{\sin(2a+2bx)}}$$

$$= -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{2bd^2\sqrt{\sin(2a+2bx)}} - \frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.64 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sin(a+bx)\left(-13+\cos(2(a+bx)) - 14\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+\right.\right.}{6b(d\tan(a+bx))^{3/2}}$$

```
[In] Integrate[Cos[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Sin[a + b*x]*(-13 + Cos[2*(a + b*x)] - 14*Hypergeometric2F1[3/4, 3/2, 7/4,
 -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(6*b*(d*Tan[a + b*x
])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(125) = 250$.

Time = 1.49 (sec) , antiderivative size = 380, normalized size of antiderivative = 3.39

method	result
default	$\frac{\left(2(\cos^3(bx+a))\sqrt{2}-21\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\cot(\right.\right.}{}$

```
[In] int(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/b/(d*tan(b*x+a))^(1/2)/d*(2*cos(b*x+a)^3*2^(1/2)-21*(cot(b*x+a)-csc(b*
x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2
)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+42*(1+csc(b*x+a)-c
ot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(
1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-21*sec(b*x+a)*(
1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)
-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+4
```

```
2*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/
2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),
1/2*2^(1/2))+7*2^(1/2)*cos(b*x+a)-21*2^(1/2))*2^(1/2)
```

Fricas [F]

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos{(bx+a)}^3}{(d\tan{(bx+a)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(b*x + a))*cos(b*x + a)^3/(d^2*tan(b*x + a)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos{(bx+a)}^3}{(d\tan{(bx+a)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos{(bx+a)}^3}{(d\tan{(bx+a)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^3}{\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(cos(a + b*x)^3/(d*tan(a + b*x))^(3/2),x)
[Out] int(cos(a + b*x)^3/(d*tan(a + b*x))^(3/2), x)
```

3.268 $\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 142

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{77\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{20bd^2\sqrt{\sin(2a+2bx)}} - \frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3} - \frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}$$

```
[Out] -2*cos(b*x+a)^5/b/d/(d*tan(b*x+a))^(1/2)+77/20*cos(b*x+a)*(sin(a+1/4*Pi+b*x
)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*
x+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)-77/30*cos(b*x+a)^3*(d*tan(b*x+a))^(3
/2)/b/d^3-11/5*cos(b*x+a)^5*(d*tan(b*x+a))^(3/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2689, 2692, 2695, 2652, 2719}

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} - \frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3} - \frac{77\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{20bd^2\sqrt{\sin(2a+2bx)}} - \frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

```
[In] Int[Cos[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]^5)/(b*d*Sqrt[d*Tan[a + b*x]]) - (77*Cos[a + b*x]*EllipticE
[a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(20*b*d^2*Sqrt[Sin[2*a + 2*b*x]])
 - (77*Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(30*b*d^3) - (11*Cos[a + b*x]
^5*(d*Tan[a + b*x])^(3/2))/(5*b*d^3)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{11\int \cos^5(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2} \\
&= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} \\
&\quad - \frac{77\int \cos^3(a+bx)\sqrt{d\tan(a+bx)}\,dx}{10d^2}
\end{aligned}$$

$$= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3} - \frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} - \frac{77\int\cos(a+bx)\sqrt{d\tan(a+bx)}\,dx}{20d^2}$$

$$= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3} - \frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} - \frac{\left(77\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{20d^2\sqrt{\sin(a+bx)}}$$

$$= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3} - \frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} - \frac{\left(77\cos(a+bx)\sqrt{d\tan(a+bx)}\right)\int\sqrt{\sin(2a+2bx)}\,dx}{20d^2\sqrt{\sin(2a+2bx)}}$$

$$= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{77\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{20bd^2\sqrt{\sin(2a+2bx)}} - \frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3} - \frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.91 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.63

$$\int\frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sin(a+bx)\left(-277+34\cos(2(a+bx))+3\cos(4(a+bx))-308\,\text{Hypergeometric2}\right.}{120b(d\tan(a+bx))^{3/2}}$$

```
[In] Integrate[Cos[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Sin[a + b*x]*(-277 + 34*Cos[2*(a + b*x)] + 3*Cos[4*(a + b*x)] - 308*Hyperg
eometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*Tan[a + b*
x]^2))/(120*b*(d*Tan[a + b*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(151) = 302.

Time = 1.39 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.77

method	result
default	$\frac{\left(12\left(\cos^5(bx+a)\right)\sqrt{2}+22\left(\cos^3(bx+a)\right)\sqrt{2}+462\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E}{}$

```
[In] int(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120/b/(d*tan(b*x+a))^(1/2)/d*(12*cos(b*x+a)^5*2^(1/2)+22*cos(b*x+a)^3*2^(
1/2)+462*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(
cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*
2^(1/2))-231*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)
*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),
1/2*2^(1/2))+462*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+
cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot
(b*x+a))^(1/2),1/2*2^(1/2))-231*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*
(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1
+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+77*2^(1/2)*cos(b*x+a)-231*2^(1/2
))*2^(1/2)
```

Fricas [F]

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*cos(b*x + a)^5/(d^2*tan(b*x + a)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(b*x+a)**5/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^5}{\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(cos(a + b*x)^5/(d*tan(a + b*x))^(3/2),x)
[Out] int(cos(a + b*x)^5/(d*tan(a + b*x))^(3/2), x)
```

3.269 $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}} - \frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\tan(a+bx)}}$$

```
[Out] 1/3*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*
x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/d^2/(d*tan(b*x+a))^(1/2)-2/3*
sec(b*x+a)/b/d/(d*tan(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2689, 2694, 2653, 2720}

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3bd^2\sqrt{d\tan(a+bx)}} - \frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

```
[In] Int[Sec[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*Sec[a + b*x])/(3*b*d*(d*Tan[a + b*x])^(3/2)) - (EllipticF[a - Pi/4 + b*
x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Tan[a + b*x]])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}} - \frac{\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{3d^2} \\
&= -\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}} - \frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{3d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= -\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}} - \frac{\left(\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{3d^2\sqrt{d\tan(a+bx)}} \\
&= -\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}} - \frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\tan(a+bx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.38

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{2\cos(2(a+bx))\csc(a+bx)\sqrt{\sec^2(a+bx)}\left(\sqrt{\sec^2(a+bx)}-\sqrt[4]{-1}\,\text{EllipticF}\left(ia\right.\right.}{3bd^2\sqrt{d\tan(a+bx)}\,(-1+\tan^2(a+}$$

```
[In] Integrate[Sec[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*Cos[2*(a + b*x)]*Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]*(Sqrt[Sec[a + b*x]^2]
 - (-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Tan[a
 + b*x]^(3/2)))/(3*b*d^2*Sqrt[d*Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(97) = 194.

Time = 1.10 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.65

method	result
default	$\frac{(\csc^2(bx+a))(1-\cos(bx+a))^2\left(2\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{2-2\csc(bx+a)+2\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,F\left(\sqrt{1+\csc(bx\right.\right.}{6b\sqrt{(\csc^3(bx+a))(1-\cos(bx+a))^3-\csc(bx+a)+\cot(bx+a)}\,\sqrt{\csc(bx+a)(1-\cos(bx+a))\left((\csc^2(bx+a))(1-\cos(bx+a))^2-1\right)}}$

```
[In] int(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b*csc(b*x+a)^2*(1-cos(b*x+a))^2*(2*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(2-2
*csc(b*x+a)+2*cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+
csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*(csc(b*x+a)-cot(b*x+a))-csc(b*x+a
)^4*(1-cos(b*x+a))^4+1)/(csc(b*x+a)^3*(1-cos(b*x+a))^3-csc(b*x+a)+cot(b*x+a
))^(1/2)/(csc(b*x+a)*(1-cos(b*x+a))*(csc(b*x+a)^2*(1-cos(b*x+a))^2-1))^(1/2
)/(csc(b*x+a)^2*(1-cos(b*x+a))^2-1)^2/(-d/(csc(b*x+a)^2*(1-cos(b*x+a))^2-1)
*(csc(b*x+a)-cot(b*x+a)))^(5/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.45

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\left(\cos\left(bx+a\right)^2-1\right)\sqrt{i\,d}F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\,|-1)+\left(\cos\left(bx+\right.\right.}{3\left(bd^3\cos\right.}$$

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((cos(b*x + a)^2 - 1)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(
b*x + a)), -1) + (cos(b*x + a)^2 - 1)*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x
+ a) - I*sin(b*x + a)), -1) + 2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x +
 a))/(b*d^3*cos(b*x + a)^2 - b*d^3)
```

Sympy [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sec\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Integral(sec(a + b*x)/(d*tan(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sec\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sec(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(sec(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{1}{\cos(a+b\,x)\,\left(d\tan(a+b\,x)\right)^{5/2}}\,dx$$

```
[In] int(1/(cos(a + b*x)*(d*tan(a + b*x))^(5/2)),x)
[Out] int(1/(cos(a + b*x)*(d*tan(a + b*x))^(5/2)), x)
```

3.270 $\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}} - \frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}} - \frac{4\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}$$

```
[Out] -4/5*cos(b*x+a)/b/d^3/(d*tan(b*x+a))^(1/2)+4/5*cos(b*x+a)*(sin(a+1/4*Pi+b*x
)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*
x+a))^(1/2)/b/d^4/sin(2*b*x+2*a)^(1/2)-2/5*sec(b*x+a)/b/d/(d*tan(b*x+a))^(5
/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2688, 2695, 2652, 2719}

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = -\frac{4\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}} - \frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}} - \frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

```
[In] Int[Sec[a + b*x]^3/(d*Tan[a + b*x])^(7/2),x]
```

```
[Out] (-2*Sec[a + b*x])/(5*b*d*(d*Tan[a + b*x])^(5/2)) - (4*Cos[a + b*x])/(5*b*d^
3*Sqrt[d*Tan[a + b*x]]) - (4*Cos[a + b*x]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt
[d*Tan[a + b*x]])/(5*b*d^4*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}+\frac{2\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx}{5d^2}\\
&= -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}-\frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}}-\frac{4\int \cos(a+bx)\sqrt{d\tan(a+bx)}\,dx}{5d^4}\\
&= -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}-\frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}}\\
&\quad -\frac{\left(4\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{5d^4\sqrt{\sin(a+bx)}}\\
&= -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}-\frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}}\\
&\quad -\frac{\left(4\cos(a+bx)\sqrt{d\tan(a+bx)}\right)\int \sqrt{\sin(2a+2bx)}\,dx}{5d^4\sqrt{\sin(2a+2bx)}}
\end{aligned}
$$

$$= -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}} - \frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}} - \frac{4\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)\sqrt{d\tan(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.99 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx =$$

$$-\frac{2\left(4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sec^2(a+bx)+3(-2+\csc^2(a+bx)+\csc^4(a+bx))\sqrt{\sec}\right.}{15bd^4\sqrt{\sec^2(a+bx)}}$$

```
[In] Integrate[Sec[a + b*x]^3/(d*Tan[a + b*x])^(7/2),x]
```

```
[Out] (-2*(4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]^2 + 3
*(-2 + Csc[a + b*x]^2 + Csc[a + b*x]^4)*Sqrt[Sec[a + b*x]^2])*Sin[a + b*x]*
Sqrt[d*Tan[a + b*x]])/(15*b*d^4*Sqrt[Sec[a + b*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(121) = 242.

Time = 1.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.38

method	result
default	$-\frac{\left(-4\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)+2\sqrt{c}\right.}{}$

```
[In] int(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b/d^3/(d*tan(b*x+a))^(1/2)*(-4*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b
*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*
x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b
*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(
b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-4*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a)
)^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Elli
pticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*sec(b*x+a)*(1+csc(b*x+
a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a
))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*2^(1/2)+c
ot(b*x+a)*csc(b*x+a)*2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.19

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx =$$
$$-\frac{2\left(\left(i\cos(bx+a)^2-i\right)\sqrt{i\,d}E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a)+\left(-i\cos(bx+a)^2\right.\right.}{}$$

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*((I*cos(b*x + a)^2 - I)*sqrt(I*d)*elliptic_e(arcsin(cos(b*x + a) + I*s
in(b*x + a)), -1)*sin(b*x + a) + (-I*cos(b*x + a)^2 + I)*sqrt(-I*d)*ellipti
c_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (-I*cos(b*x +
 a)^2 + I)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*
sin(b*x + a) + (I*cos(b*x + a)^2 - I)*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x
+ a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (2*cos(b*x + a)^4 - 3*cos(b*x +
a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/((b*d^4*cos(b*x + a)^2 - b*d^4)*si
n(b*x + a))
```

Sympy [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = \int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)**3/(d*tan(b*x+a))**(7/2),x)
```

```
[Out] Integral(sec(a + b*x)**3/(d*tan(a + b*x))**(7/2), x)
```

Maxima [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = \int \frac{\sec(bx+a)^3}{(d\tan(bx+a))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(7/2), x)
```

Giac [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = \int \frac{\sec\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x, algorithm="giac")
[Out] integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^3\,\left(d\tan\left(a+b\,x\right)\right)^{7/2}}\,dx$$

```
[In] int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(7/2)),x)
[Out] int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(7/2)), x)
```

3.271 $\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{7}{6},-\frac{1}{2},-\frac{1}{6},\cos^2(e+fx)\right)\sec^{\frac{7}{3}}(e+fx)\sin(e+fx)}{7f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/7*hypergeom([-7/6, -1/2],[-1/6],cos(f*x+e)^2)*sec(f*x+e)^(7/3)*sin(f*x+e)
/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2712, 2656}

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\sin(e+fx)\sec^{\frac{7}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{7}{6},-\frac{1}{2},-\frac{1}{6},\cos^2(e+fx)\right)}{7f\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[Sec[e + f*x]^(10/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Hypergeometric2F1[-7/6, -1/2, -1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(7/3)*S
in[e + f*x])/(7*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \left(\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\right)\int \frac{\sin^2(e+fx)}{\cos^{\frac{10}{3}}(e+fx)}\,dx$$
$$= \frac{3\,\text{Hypergeometric2F1}\left(-\frac{7}{6},-\frac{1}{2},-\frac{1}{6},\cos^2(e+fx)\right)\sec^{\frac{7}{3}}(e+fx)\sin(e+fx)}{7f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx$$
$$= \frac{3\sqrt[3]{\sec(e+fx)}\left(-3\sin(e+fx)+2\sqrt[6]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sin(e+fx)\right.}{7f}$$

```
[In] Integrate[Sec[e + f*x]^(10/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Sec[e + f*x]^(1/3)*(-3*Sin[e + f*x] + 2*(Cos[e + f*x]^2)^(1/6)*Hypergeom
etric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*Tan[e +
 f*x]))/(7*f)
```

Maple [F]

$$\int \left(\sec^{\frac{4}{3}}(fx+e)\right)\left(\tan^2(fx+e)\right)dx$$

```
[In] int(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x)
```

```
[Out] int(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x)
```

Fricas [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{4}{3}}\tan\left(fx+e\right)^2\,dx$$

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral(sec(f*x + e)^(4/3)*tan(f*x + e)^2, x)
```

Sympy [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \tan^2\left(e+fx\right)\sec^{\frac{4}{3}}\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**(4/3)*tan(f*x+e)**2,x)
[Out] Integral(tan(e + f*x)**2*sec(e + f*x)**(4/3), x)
```

Maxima [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{4}{3}}\tan\left(fx+e\right)^2\,dx$$

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^(4/3)*tan(f*x + e)^2, x)
```

Giac [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{4}{3}}\tan\left(fx+e\right)^2\,dx$$

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sec(f*x + e)^(4/3)*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \tan(e+f\,x)^2\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{4/3}\,dx$$

```
[In] int(tan(e + f*x)^2*(1/cos(e + f*x))^(4/3),x)
[Out] int(tan(e + f*x)^2*(1/cos(e + f*x))^(4/3), x)
```

3.272 $\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},\cos^2(e+fx)\right)\sec^{\frac{5}{3}}(e+fx)\sin(e+fx)}{5f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/5*hypergeom([-5/6, -1/2],[1/6],cos(f*x+e)^2)*sec(f*x+e)^(5/3)*sin(f*x+e)/
f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2712, 2656}

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\sin(e+fx)\sec^{\frac{5}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},\cos^2(e+fx)\right)}{5f\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[Sec[e + f*x]^(8/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Hypergeometric2F1[-5/6, -1/2, 1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(5/3)*Si
n[e + f*x])/(5*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \left(\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\right)\int \frac{\sin^2(e+fx)}{\cos^{\frac{8}{3}}(e+fx)}\,dx$$
$$= \frac{3\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},\cos^2(e+fx)\right)\sec^{\frac{5}{3}}(e+fx)\sin(e+fx)}{5f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx =$$
$$-\frac{3\left(-1+\cos^2(e+fx)\right)^{5/6}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\sin^2(e+fx)\right)\sec^{\frac{5}{3}}(e+fx)\sin(e+fx)}{5f}$$

```
[In] Integrate[Sec[e + f*x]^(8/3)*Sin[e + f*x]^2,x]
[Out] (-3*(-1 + (Cos[e + f*x]^2)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, Sin[e + f
*x]^2])*Sec[e + f*x]^(5/3)*Sin[e + f*x])/(5*f)
```

Maple [F]

$$\int \left(\sec^{\frac{2}{3}}(fx+e)\right)\left(\tan^2(fx+e)\right)dx$$

```
[In] int(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x)
[Out] int(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x)
```

Fricas [F]

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{2}{3}}\tan\left(fx+e\right)^2\,dx$$

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral(sec(f*x + e)^(2/3)*tan(f*x + e)^2, x)
```

Sympy [F]

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \tan^2\left(e+fx\right)\sec^{\frac{2}{3}}\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**(2/3)*tan(f*x+e)**2,x)
[Out] Integral(tan(e + f*x)**2*sec(e + f*x)**(2/3), x)
```

Maxima [F]

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{2}{3}}\tan\left(fx+e\right)^2\,dx$$

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^2, x)
```

Giac [F]

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{2}{3}}\tan\left(fx+e\right)^2\,dx$$

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \tan(e+f\,x)^2\,\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{2/3}\,dx$$

```
[In] int(tan(e + f*x)^2*(1/cos(e + f*x))^(2/3),x)
[Out] int(tan(e + f*x)^2*(1/cos(e + f*x))^(2/3), x)
```

3.273 $\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{2},\frac{1}{3},\cos^2(e+fx)\right)\sec^{\frac{4}{3}}(e+fx)\sin(e+fx)}{4f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/4*hypergeom([-2/3, -1/2],[1/3],cos(f*x+e)^2)*sec(f*x+e)^(4/3)*sin(f*x+e)/
f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2712, 2656}

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\sin(e+fx)\sec^{\frac{4}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{2},\frac{1}{3},\cos^2(e+fx)\right)}{4f\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[Sec[e + f*x]^(7/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Hypergeometric2F1[-2/3, -1/2, 1/3, Cos[e + f*x]^2]*Sec[e + f*x]^(4/3)*Si
n[e + f*x])/(4*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \left(\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\right)\int\frac{\sin^2(e+fx)}{\cos^{\frac{7}{3}}(e+fx)}\,dx$$
$$= \frac{3\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{2},\frac{1}{3},\cos^2(e+fx)\right)\sec^{\frac{4}{3}}(e+fx)\sin(e+fx)}{4f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int\sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = -\frac{3\left(-1+\cos^2(e+fx)\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\sin^2(e+fx)\right)\sec^{\frac{4}{3}}(e+fx)\sin(e+fx)}{4f}$$

```
[In] Integrate[Sec[e + f*x]^(7/3)*Sin[e + f*x]^2,x]
[Out] (-3*(-1 + (Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, Sin[e + f
*x]^2])*Sec[e + f*x]^(4/3)*Sin[e + f*x])/(4*f)
```

Maple [F]

$$\int\left(\sec^{\frac{1}{3}}(fx+e)\right)\left(\tan^2(fx+e)\right)dx$$

```
[In] int(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x)
[Out] int(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x)
```

Fricas [F]

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{1}{3}}\tan\left(fx+e\right)^2\,dx$$

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral(sec(f*x + e)^(1/3)*tan(f*x + e)^2, x)
```

Sympy [F]

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \tan^2\left(e+fx\right)\sqrt[3]{\sec\left(e+fx\right)}\,dx$$

```
[In] integrate(sec(f*x+e)**(1/3)*tan(f*x+e)**2,x)
[Out] Integral(tan(e + f*x)**2*sec(e + f*x)**(1/3), x)
```

Maxima [F]

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{1}{3}}\tan\left(fx+e\right)^2\,dx$$

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^2, x)
```

Giac [F]

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{1}{3}}\tan\left(fx+e\right)^2\,dx$$

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \tan(e+f\,x)^2\left(\frac{1}{\cos(e+f\,x)}\right)^{1/3}\,dx$$

```
[In] int(tan(e + f*x)^2*(1/cos(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^2*(1/cos(e + f*x))^(1/3), x)
```

3.274 $\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\cos^2(e+fx)\right)\sec^{\frac{2}{3}}(e+fx)\sin(e+fx)}{2f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/2*hypergeom([-1/2, -1/3],[2/3],cos(f*x+e)^2)*sec(f*x+e)^(2/3)*sin(f*x+e)/
f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2712, 2656}

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\sin(e+fx)\sec^{\frac{2}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[Sec[e + f*x]^(5/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Hypergeometric2F1[-1/2, -1/3, 2/3, Cos[e + f*x]^2]*Sec[e + f*x]^(2/3)*Si
n[e + f*x])/(2*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \left(\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\right)\int \frac{\sin^2(e+fx)}{\cos^{\frac{5}{3}}(e+fx)}\,dx$$
$$= \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\cos^2(e+fx)\right)\sec^{\frac{2}{3}}(e+fx)\sin(e+fx)}{2f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = -\frac{3\left(-1+\sqrt[3]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\right)\sec^{\frac{2}{3}}(e+fx)\sin(e+fx)}{2f}$$

```
[In] Integrate[Sec[e + f*x]^(5/3)*Sin[e + f*x]^2,x]
[Out] (-3*(-1 + (Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, Sin[e + f
*x]^2])*Sec[e + f*x]^(2/3)*Sin[e + f*x])/(2*f)
```

Maple [F]

$$\int \frac{\tan^2(fx+e)}{\sec(fx+e)^{\frac{1}{3}}}dx$$

```
[In] int(tan(f*x+e)^2/sec(f*x+e)^(1/3),x)
[Out] int(tan(f*x+e)^2/sec(f*x+e)^(1/3),x)
```

Fricas [F]

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \frac{\tan(fx+e)^2}{\sec(fx+e)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(1/3),x, algorithm="fricas")
[Out] integral(tan(f*x + e)^2/sec(f*x + e)^(1/3), x)
```

Sympy [F]

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \frac{\tan^2(e+fx)}{\sqrt[3]{\sec(e+fx)}}\,dx$$

```
[In] integrate(tan(f*x+e)**2/sec(f*x+e)**(1/3),x)
[Out] Integral(tan(e + f*x)**2/sec(e + f*x)**(1/3), x)
```

Maxima [F]

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \frac{\tan(fx+e)^2}{\sec(fx+e)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(1/3),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^2/sec(f*x + e)^(1/3), x)
```

Giac [F]

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \frac{\tan(fx+e)^2}{\sec(fx+e)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(1/3),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^2/sec(f*x + e)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \frac{\tan(e+f\,x)^2}{\left(\frac{1}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

```
[In] int(tan(e + f*x)^2/(1/cos(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^2/(1/cos(e + f*x))^(1/3), x)
```

3.275 $\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\cos^2(e+fx)\right)\sqrt[3]{\sec(e+fx)}\sin(e+fx)}{f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3*hypergeom([-1/2, -1/6],[5/6],cos(f*x+e)^2)*sec(f*x+e)^(1/3)*sin(f*x+e)/f/
(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2712, 2656}

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\sin(e+fx)\sqrt[3]{\sec(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\cos^2(e+fx)\right)}{f\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[Sec[e + f*x]^(4/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Hypergeometric2F1[-1/2, -1/6, 5/6, Cos[e + f*x]^2]*Sec[e + f*x]^(1/3)*Si
n[e + f*x])/(f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\right)\int\frac{\sin^2(e+fx)}{\cos^{\frac{4}{3}}(e+fx)}\,dx\\ &= \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\cos^2(e+fx)\right)\sqrt[3]{\sec(e+fx)}\sin(e+fx)}{f\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int\sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = -\frac{3\left(-1+\sqrt[6]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\right)\sqrt[3]{\sec(e+fx)}\sin(e+fx)}{f}$$

```
[In] Integrate[Sec[e + f*x]^(4/3)*Sin[e + f*x]^2,x]
[Out] (-3*(-1 + (Cos[e + f*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, Sin[e + f
*x]^2])*Sec[e + f*x]^(1/3)*Sin[e + f*x])/f
```

Maple [F]

$$\int\frac{\tan^2(fx+e)}{\sec(fx+e)^{\frac{2}{3}}}dx$$

```
[In] int(tan(f*x+e)^2/sec(f*x+e)^(2/3),x)
[Out] int(tan(f*x+e)^2/sec(f*x+e)^(2/3),x)
```

Fricas [F]

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \frac{\tan\left(fx+e\right)^2}{\sec\left(fx+e\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(2/3),x, algorithm="fricas")
[Out] integral(tan(f*x + e)^2/sec(f*x + e)^(2/3), x)
```

Sympy [F]

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \frac{\tan^2\left(e+fx\right)}{\sec^{\frac{2}{3}}\left(e+fx\right)}\,dx$$

```
[In] integrate(tan(f*x+e)**2/sec(f*x+e)**(2/3),x)
[Out] Integral(tan(e + f*x)**2/sec(e + f*x)**(2/3), x)
```

Maxima [F]

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \frac{\tan\left(fx+e\right)^2}{\sec\left(fx+e\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(2/3),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^2/sec(f*x + e)^(2/3), x)
```

Giac [F]

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \frac{\tan\left(fx+e\right)^2}{\sec\left(fx+e\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(2/3),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^2/sec(f*x + e)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \frac{\tan(e+f\,x)^2}{\left(\frac{1}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

```
[In] int(tan(e + f*x)^2/(1/cos(e + f*x))^(2/3),x)
[Out] int(tan(e + f*x)^2/(1/cos(e + f*x))^(2/3), x)
```

3.276 $\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{13}{6},-\frac{3}{2},-\frac{7}{6},\cos^2(e+fx)\right)\sec^{\frac{13}{3}}(e+fx)\sin(e+fx)}{13f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/13*hypergeom([-13/6, -3/2],[-7/6],cos(f*x+e)^2)*sec(f*x+e)^(13/3)*sin(f*x
+e)/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2712, 2656}

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\sin(e+fx)\sec^{\frac{13}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{13}{6},-\frac{3}{2},-\frac{7}{6},\cos^2(e+fx)\right)}{13f\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[Sec[e + f*x]^(16/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Hypergeometric2F1[-13/6, -3/2, -7/6, Cos[e + f*x]^2]*Sec[e + f*x]^(13/3)
*Sin[e + f*x])/(13*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\right)\int\frac{\sin^4(e+fx)}{\cos^{\frac{16}{3}}(e+fx)}\,dx\\ &= \frac{3\,\text{Hypergeometric2F1}\left(-\frac{13}{6},-\frac{3}{2},-\frac{7}{6},\cos^2(e+fx)\right)\sec^{\frac{13}{3}}(e+fx)\sin(e+fx)}{13f\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

$$\int\sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\sqrt[3]{\sec(e+fx)}\left(27\sin(e+fx)-18\sqrt[6]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sin(e+fx)\right.}{91f}$$

```
[In] Integrate[Sec[e + f*x]^(16/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Sec[e + f*x]^(1/3)*(27*Sin[e + f*x] - 18*(Cos[e + f*x]^2)^(1/6)*Hypergeo
metric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*(-16 +
 7*Sec[e + f*x]^2)*Tan[e + f*x]))/(91*f)
```

Maple [F]

$$\int\left(\sec^{\frac{4}{3}}(fx+e)\right)\left(\tan^4(fx+e)\right)dx$$

```
[In] int(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x)
```

```
[Out] int(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x)
```

Fricas [F]

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{4}{3}}\tan\left(fx+e\right)^4\,dx$$

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x, algorithm="fricas")
[Out] integral(sec(f*x + e)^(4/3)*tan(f*x + e)^4, x)
```

Sympy [F]

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \tan^4\left(e+fx\right)\sec^{\frac{4}{3}}\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**(4/3)*tan(f*x+e)**4,x)
[Out] Integral(tan(e + f*x)**4*sec(e + f*x)**(4/3), x)
```

Maxima [F]

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{4}{3}}\tan\left(fx+e\right)^4\,dx$$

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^(4/3)*tan(f*x + e)^4, x)
```

Giac [F]

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{4}{3}}\tan\left(fx+e\right)^4\,dx$$

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate(sec(f*x + e)^(4/3)*tan(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \tan(e+f\,x)^4\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{4/3}\,dx$$

```
[In] int(tan(e + f*x)^4*(1/cos(e + f*x))^(4/3),x)
[Out] int(tan(e + f*x)^4*(1/cos(e + f*x))^(4/3), x)
```

3.277 $\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{11}{6},-\frac{3}{2},-\frac{5}{6},\cos^2(e+fx)\right)\sec^{\frac{11}{3}}(e+fx)\sin(e+fx)}{11f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/11*hypergeom([-11/6, -3/2],[-5/6],cos(f*x+e)^2)*sec(f*x+e)^(11/3)*sin(f*x
+e)/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2712, 2656}

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\sin(e+fx)\sec^{\frac{11}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{11}{6},-\frac{3}{2},-\frac{5}{6},\cos^2(e+fx)\right)}{11f\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[Sec[e + f*x]^(14/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Hypergeometric2F1[-11/6, -3/2, -5/6, Cos[e + f*x]^2]*Sec[e + f*x]^(11/3)
*Sin[e + f*x])/(11*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\right)\int\frac{\sin^4(e+fx)}{\cos^{\frac{14}{3}}(e+fx)}\,dx\\ &= \frac{3\,\text{Hypergeometric2F1}\left(-\frac{11}{6},-\frac{3}{2},-\frac{5}{6},\cos^2(e+fx)\right)\sec^{\frac{11}{3}}(e+fx)\sin(e+fx)}{11f\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int\sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\left(\frac{9\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\sin^2(e+fx)\right)}{\sqrt[6]{\cos^2(e+fx)}}-(2+7\cos(2(e+fx)))\sec^4(e+fx)\right)\sin(e+fx)}{55f\sqrt[3]{\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]^(14/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*((9*Hypergeometric2F1[1/2, 5/6, 3/2, Sin[e + f*x]^2])/(Cos[e + f*x]^2)^(
1/6) - (2 + 7*Cos[2*(e + f*x)])*Sec[e + f*x]^4)*Sin[e + f*x])/(55*f*Sec[e +
 f*x]^(1/3))
```

Maple [F]

$$\int\left(\sec^{\frac{2}{3}}(fx+e)\right)\left(\tan^4(fx+e)\right)dx$$

```
[In] int(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x)
```

```
[Out] int(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x)
```

Fricas [F]

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{2}{3}}\tan\left(fx+e\right)^4\,dx$$

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x, algorithm="fricas")
[Out] integral(sec(f*x + e)^(2/3)*tan(f*x + e)^4, x)
```

Sympy [F]

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \tan^4\left(e+fx\right)\sec^{\frac{2}{3}}\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**(2/3)*tan(f*x+e)**4,x)
[Out] Integral(tan(e + f*x)**4*sec(e + f*x)**(2/3), x)
```

Maxima [F]

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{2}{3}}\tan\left(fx+e\right)^4\,dx$$

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^4, x)
```

Giac [F]

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{2}{3}}\tan\left(fx+e\right)^4\,dx$$

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \tan(e+f\,x)^4\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{2/3}\,dx$$

```
[In] int(tan(e + f*x)^4*(1/cos(e + f*x))^(2/3),x)
[Out] int(tan(e + f*x)^4*(1/cos(e + f*x))^(2/3), x)
```

3.278 $\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{3}{2},-\frac{2}{3},\cos^2(e+fx)\right)\sec^{\frac{10}{3}}(e+fx)\sin(e+fx)}{10f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/10*hypergeom([-5/3, -3/2],[-2/3],cos(f*x+e)^2)*sec(f*x+e)^(10/3)*sin(f*x+
e)/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2712, 2656}

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\sin(e+fx)\sec^{\frac{10}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{3}{2},-\frac{2}{3},\cos^2(e+fx)\right)}{10f\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[Sec[e + f*x]^(13/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Hypergeometric2F1[-5/3, -3/2, -2/3, Cos[e + f*x]^2]*Sec[e + f*x]^(10/3)*
Sin[e + f*x])/(10*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\right)\int \frac{\sin^4(e+fx)}{\cos^{\frac{13}{3}}(e+fx)}\,dx \\
&= \frac{3\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{3}{2},-\frac{2}{3},\cos^2(e+fx)\right)\sec^{\frac{10}{3}}(e+fx)\sin(e+fx)}{10f\sqrt{\sin^2(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$
\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\left(\frac{9\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\sin^2(e+fx)\right)}{\sqrt[3]{\cos^2(e+fx)}}+\sec^2(e+fx)\left(-13+4\sec^2(e+fx)\right)\right)\sin(e+fx)}{40f\sec^{\frac{2}{3}}(e+fx)}
$$

```
[In] Integrate[Sec[e + f*x]^(13/3)*Sin[e + f*x]^4,x]
[Out] (3*((9*Hypergeometric2F1[1/2, 2/3, 3/2, Sin[e + f*x]^2])/(Cos[e + f*x]^2)^(
1/3) + Sec[e + f*x]^2*(-13 + 4*Sec[e + f*x]^2))*Sin[e + f*x])/(40*f*Sec[e +
 f*x]^(2/3))
```

Maple [F]

$$
\int \left(\sec^{\frac{1}{3}}(fx+e)\right)\left(\tan^4(fx+e)\right)dx
$$

```
[In] int(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x)
[Out] int(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x)
```

Fricas [F]

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec(fx+e)^{\frac{1}{3}}\tan(fx+e)^4\,dx$$

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x, algorithm="fricas")
[Out] integral(sec(f*x + e)^(1/3)*tan(f*x + e)^4, x)
```

Sympy [F]

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \tan^4(e+fx)\sqrt[3]{\sec(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)**(1/3)*tan(f*x+e)**4,x)
[Out] Integral(tan(e + f*x)**4*sec(e + f*x)**(1/3), x)
```

Maxima [F]

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec(fx+e)^{\frac{1}{3}}\tan(fx+e)^4\,dx$$

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^4, x)
```

Giac [F]

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec(fx+e)^{\frac{1}{3}}\tan(fx+e)^4\,dx$$

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \tan(e+f\,x)^4\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{1/3}\,dx$$

```
[In] int(tan(e + f*x)^4*(1/cos(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^4*(1/cos(e + f*x))^(1/3), x)
```

3.279 $\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{4}{3},-\frac{1}{3},\cos^2(e+fx)\right)\sec^{\frac{8}{3}}(e+fx)\sin(e+fx)}{8f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/8*hypergeom([-3/2, -4/3],[-1/3],cos(f*x+e)^2)*sec(f*x+e)^(8/3)*sin(f*x+e)
/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2712, 2656}

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\sin(e+fx)\sec^{\frac{8}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{4}{3},-\frac{1}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[Sec[e + f*x]^(11/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Hypergeometric2F1[-3/2, -4/3, -1/3, Cos[e + f*x]^2]*Sec[e + f*x]^(8/3)*S
in[e + f*x])/(8*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\right)\int\frac{\sin^4(e+fx)}{\cos^{\frac{11}{3}}(e+fx)}\,dx\\ &= \frac{3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{4}{3},-\frac{1}{3},\cos^2(e+fx)\right)\sec^{\frac{8}{3}}(e+fx)\sin(e+fx)}{8f\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int\sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx$$
$$=\frac{3\sec^{\frac{2}{3}}(e+fx)\left(-11\sin(e+fx)+9\sqrt[3]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sin(e+fx)\right.}{16f}$$

```
[In] Integrate[Sec[e + f*x]^(11/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Sec[e + f*x]^(2/3)*(-11*Sin[e + f*x] + 9*(Cos[e + f*x]^2)^(1/3)*Hypergeo
metric2F1[1/3, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + 2*Sec[e + f*x]*Tan[
e + f*x]))/(16*f)
```

Maple [F]

$$\int\frac{\tan^4(fx+e)}{\sec(fx+e)^{\frac{1}{3}}}dx$$

```
[In] int(tan(f*x+e)^4/sec(f*x+e)^(1/3),x)
```

```
[Out] int(tan(f*x+e)^4/sec(f*x+e)^(1/3),x)
```

Fricas [F]

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \frac{\tan(fx+e)^4}{\sec(fx+e)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(1/3),x, algorithm="fricas")
[Out] integral(tan(f*x + e)^4/sec(f*x + e)^(1/3), x)
```

Sympy [F]

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \frac{\tan^4(e+fx)}{\sqrt[3]{\sec(e+fx)}}\,dx$$

```
[In] integrate(tan(f*x+e)**4/sec(f*x+e)**(1/3),x)
[Out] Integral(tan(e + f*x)**4/sec(e + f*x)**(1/3), x)
```

Maxima [F]

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \frac{\tan(fx+e)^4}{\sec(fx+e)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(1/3),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^4/sec(f*x + e)^(1/3), x)
```

Giac [F]

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \frac{\tan(fx+e)^4}{\sec(fx+e)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(1/3),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^4/sec(f*x + e)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \frac{\tan(e+f\,x)^4}{\left(\frac{1}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

```
[In] int(tan(e + f*x)^4/(1/cos(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^4/(1/cos(e + f*x))^(1/3), x)
```

3.280 $\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{7}{6},-\frac{1}{6},\cos^2(e+fx)\right)\sec^{\frac{7}{3}}(e+fx)\sin(e+fx)}{7f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/7*hypergeom([-3/2, -7/6],[-1/6],cos(f*x+e)^2)*sec(f*x+e)^(7/3)*sin(f*x+e)
/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2712, 2656}

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\sin(e+fx)\sec^{\frac{7}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{7}{6},-\frac{1}{6},\cos^2(e+fx)\right)}{7f\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[Sec[e + f*x]^(10/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Hypergeometric2F1[-3/2, -7/6, -1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(7/3)*S
in[e + f*x])/(7*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\right)\int \frac{\sin^4(e+fx)}{\cos^{\frac{10}{3}}(e+fx)}\,dx \\ &= \frac{3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{7}{6},-\frac{1}{6},\cos^2(e+fx)\right)\sec^{\frac{7}{3}}(e+fx)\sin(e+fx)}{7f\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\sqrt[3]{\sec(e+fx)}\left(-10\sin(e+fx)+9\sqrt[6]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sin(e+fx)\right.}{7f}$$

```
[In] Integrate[Sec[e + f*x]^(10/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Sec[e + f*x]^(1/3)*(-10*Sin[e + f*x] + 9*(Cos[e + f*x]^2)^(1/6)*Hypergeo
metric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*Tan[e
+ f*x]))/(7*f)
```

Maple [F]

$$\int \frac{\tan^4(fx+e)}{\sec(fx+e)^{\frac{2}{3}}}\,dx$$

```
[In] int(tan(f*x+e)^4/sec(f*x+e)^(2/3),x)
```

```
[Out] int(tan(f*x+e)^4/sec(f*x+e)^(2/3),x)
```

Fricas [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \frac{\tan{(fx+e)}^4}{\sec{(fx+e)}^{\frac{2}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(2/3),x, algorithm="fricas")
[Out] integral(tan(f*x + e)^4/sec(f*x + e)^(2/3), x)
```

Sympy [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \frac{\tan^4{(e+fx)}}{\sec^{\frac{2}{3}}{(e+fx)}}\,dx$$

```
[In] integrate(tan(f*x+e)**4/sec(f*x+e)**(2/3),x)
[Out] Integral(tan(e + f*x)**4/sec(e + f*x)**(2/3), x)
```

Maxima [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \frac{\tan{(fx+e)}^4}{\sec{(fx+e)}^{\frac{2}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(2/3),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^4/sec(f*x + e)^(2/3), x)
```

Giac [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \frac{\tan{(fx+e)}^4}{\sec{(fx+e)}^{\frac{2}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(2/3),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^4/sec(f*x + e)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \frac{\tan(e+f\,x)^4}{\left(\frac{1}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

```
[In] int(tan(e + f*x)^4/(1/cos(e + f*x))^(2/3),x)
[Out] int(tan(e + f*x)^4/(1/cos(e + f*x))^(2/3), x)
```

3.281 $\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx=\frac{\cos^2(e+fx)^{13/6}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{13}{6},\frac{5}{2},\sin^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan^3(e+fx)}{3f}$$

```
[Out] 1/3*(cos(f*x+e)^2)^(13/6)*hypergeom([3/2, 13/6],[5/2],sin(f*x+e)^2)*(d*sec(
f*x+e))^(4/3)*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2697}

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx=\frac{\cos^2(e+fx)^{13/6}\tan^3(e+fx)(d\sec(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{13}{6},\frac{5}{2},\sin^2(e+fx)\right)}{3f}$$

```
[In] Int[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^2,x]
```

```
[Out] ((Cos[e + f*x]^2)^(13/6)*Hypergeometric2F1[3/2, 13/6, 5/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^3)/(3*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\cos^2(e+fx)^{13/6}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{13}{6},\frac{5}{2},\sin^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan^3(e+fx)}{3f}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},\sec^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan(e+fx)}{4f\sqrt{-\tan^2(e+fx)}}$$

```
[In] Integrate[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^2,x]
[Out] (3*Hypergeometric2F1[-1/2, 2/3, 5/3, Sec[e + f*x]^2]*(d*Sec[e + f*x])^(4/3)
*Tan[e + f*x])/(4*f*Sqrt[-Tan[e + f*x]^2])
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{4}{3}}\left(\tan^2(fx+e)\right)dx$$

```
[In] int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x)
[Out] int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{4}{3}}\tan(fx+e)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^(1/3)*d*sec(f*x + e)*tan(f*x + e)^2, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \int (d\sec(e+fx))^{\frac{4}{3}}\tan^2(e+fx)\,dx$$

```
[In] integrate((d*sec(f*x+e))**(4/3)*tan(f*x+e)**2,x)
[Out] Integral((d*sec(e + f*x))**(4/3)*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{4}{3}}\tan(fx+e)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^2, x)
```

Giac [F]

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{4}{3}}\tan(fx+e)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\left(\frac{d}{\cos(e+f\,x)}\right)^{4/3}\,dx$$

```
[In] int(tan(e + f*x)^2*(d/cos(e + f*x))^(4/3),x)
[Out] int(tan(e + f*x)^2*(d/cos(e + f*x))^(4/3), x)
```

3.282 $\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx=\frac{\cos^2(e+fx)^{11/6}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{11}{6},\frac{5}{2},\sin^2(e+fx)\right)(d\sec(e+fx))^{2/3}\tan^3(e+fx)}{3f}$$

```
[Out] 1/3*(cos(f*x+e)^2)^(11/6)*hypergeom([3/2, 11/6],[5/2],sin(f*x+e)^2)*(d*sec(
f*x+e))^(2/3)*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2697}

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx=\frac{\cos^2(e+fx)^{11/6}\tan^3(e+fx)(d\sec(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{11}{6},\frac{5}{2},\sin^2(e+fx)\right)}{3f}$$

```
[In] Int[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^2,x]
```

```
[Out] ((Cos[e + f*x]^2)^(11/6)*Hypergeometric2F1[3/2, 11/6, 5/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^3)/(3*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\cos^2(e+fx)^{11/6}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{11}{6},\frac{5}{2},\sin^2(e+fx)\right)(d\sec(e+fx))^{2/3}\tan^3(e+fx)}{3f}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = -\frac{3\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},\sec^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sqrt{-\tan^2(e+fx)}}{2f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^2,x]
[Out] (-3*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1/3, 4/3, Sec[e + f*x]^2]*(d*Sec[e
 + f*x])^(2/3)*Sqrt[-Tan[e + f*x]^2])/(2*f)
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{2}{3}}\left(\tan^2(fx+e)\right)dx$$

```
[In] int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x)
[Out] int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \int (d\sec(e+fx))^{\frac{2}{3}}\tan^2(e+fx)\,dx$$

```
[In] integrate((d*sec(f*x+e))**(2/3)*tan(f*x+e)**2,x)
[Out] Integral((d*sec(e + f*x))**(2/3)*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2, x)
```

Giac [F]

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}dx$$

```
[In] int(tan(e + f*x)^2*(d/cos(e + f*x))^(2/3),x)
[Out] int(tan(e + f*x)^2*(d/cos(e + f*x))^(2/3), x)
```

3.283 $\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = \frac{\cos^2(e+fx)^{5/3}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{3},\frac{5}{2},\sin^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^3(e+fx)}{3f}$$

```
[Out] 1/3*(cos(f*x+e)^2)^(5/3)*hypergeom([3/2, 5/3],[5/2],sin(f*x+e)^2)*(d*sec(f*
x+e))^(1/3)*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2697}

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = \frac{\cos^2(e+fx)^{5/3}\tan^3(e+fx)\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{3},\frac{5}{2},\sin^2(e+fx)\right)}{3f}$$

```
[In] Int[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

```
[Out] ((Cos[e + f*x]^2)^(5/3)*Hypergeometric2F1[3/2, 5/3, 5/2, Sin[e + f*x]^2]*(d
*Sec[e + f*x])^(1/3)*Tan[e + f*x]^3)/(3*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\cos^2(e+fx)^{5/3}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{3},\frac{5}{2},\sin^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^3(e+fx)}{3f}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = -\frac{3\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},\sec^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
[Out] (-3*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1/6, 7/6, Sec[e + f*x]^2]*(d*Sec[e
 + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])/f
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{1}{3}}\left(\tan^2(fx+e)\right)dx$$

```
[In] int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x)
[Out] int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\tan(fx+e)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2, x)
```

Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx=\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx$$

```
[In] integrate((d*sec(f*x+e))**(1/3)*tan(f*x+e)**2,x)
[Out] Integral((d*sec(e + f*x))**(1/3)*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx=\int (d\sec(fx+e))^{\frac{1}{3}}\tan(fx+e)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2, x)
```

Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx=\int (d\sec(fx+e))^{\frac{1}{3}}\tan(fx+e)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx=\int \tan(e+f\,x)^2\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}dx$$

```
[In] int(tan(e + f*x)^2*(d/cos(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^2*(d/cos(e + f*x))^(1/3), x)
```

3.284 $\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{4/3}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{3}{2},\frac{5}{2},\sin^2(e+fx)\right)\tan^3(e+fx)}{3f\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] 1/3*(cos(f*x+e)^2)^(4/3)*hypergeom([4/3, 3/2],[5/2],sin(f*x+e)^2)*tan(f*x+e
)^3/f/(d*sec(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2697}

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{4/3}\tan^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{3}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3f\sqrt[3]{d\sec(e+fx)}}$$

```
[In] Int[Tan[e + f*x]^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((Cos[e + f*x]^2)^(4/3)*Hypergeometric2F1[4/3, 3/2, 5/2, Sin[e + f*x]^2]*Ta
n[e + f*x]^3)/(3*f*(d*Sec[e + f*x])^(1/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
```

```
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\cos^2(e+fx)^{4/3}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{3}{2},\frac{5}{2},\sin^2(e+fx)\right)\tan^3(e+fx)}{3f\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\sec^2(e+fx)\right)\tan(e+fx)}{f\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

```
[In] Integrate[Tan[e + f*x]^2/(d*Sec[e + f*x])^(1/3),x]
[Out] (-3*Hypergeometric2F1[-1/2, -1/6, 5/6, Sec[e + f*x]^2]*Tan[e + f*x])/(f*(d*
Sec[e + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])
```

Maple [F]

$$\int \frac{\tan^2(fx+e)}{(d\sec(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x)
[Out] int(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan(fx+e)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2/(d*sec(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan^2{(e+fx)}}{\sqrt[3]{d\sec{(e+fx)}}}\,dx$$

```
[In] integrate(tan(f*x+e)**2/(d*sec(f*x+e))**(1/3),x)
[Out] Integral(tan(e + f*x)**2/(d*sec(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan{(fx+e)}^2}{(d\sec{(fx+e)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan{(fx+e)}^2}{(d\sec{(fx+e)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

```
[In] int(tan(e + f*x)^2/(d/cos(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^2/(d/cos(e + f*x))^(1/3), x)
```

3.285 $\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \frac{\cos^2(e+fx)^{7/6}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{3}{2},\frac{5}{2},\sin^2(e+fx)\right)\tan^3(e+fx)}{3f(d\sec(e+fx))^{2/3}}$$

```
[Out] 1/3*(cos(f*x+e)^2)^(7/6)*hypergeom([7/6, 3/2],[5/2],sin(f*x+e)^2)*tan(f*x+e
)^3/f/(d*sec(f*x+e))^(2/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2697}

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \frac{\cos^2(e+fx)^{7/6}\tan^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{3}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3f(d\sec(e+fx))^{2/3}}$$

```
[In] Int[Tan[e + f*x]^2/(d*Sec[e + f*x])^(2/3),x]
```

```
[Out] ((Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 3/2, 5/2, Sin[e + f*x]^2]*Ta
n[e + f*x]^3)/(3*f*(d*Sec[e + f*x])^(2/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\cos^2(e+fx)^{7/6}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{3}{2},\frac{5}{2},\sin^2(e+fx)\right)\tan^3(e+fx)}{3f(d\sec(e+fx))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\sec^2(e+fx)\right)\tan(e+fx)}{2f(d\sec(e+fx))^{2/3}\sqrt{-\tan^2(e+fx)}}$$

```
[In] Integrate[Tan[e + f*x]^2/(d*Sec[e + f*x])^(2/3),x]
[Out] (-3*Hypergeometric2F1[-1/2, -1/3, 2/3, Sec[e + f*x]^2]*Tan[e + f*x])/(2*f*(
d*Sec[e + f*x])^(2/3)*Sqrt[-Tan[e + f*x]^2])
```

Maple [F]

$$\int \frac{\tan^2(fx+e)}{(d\sec(fx+e))^{\frac{2}{3}}}dx$$

```
[In] int(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x)
[Out] int(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x)
```

Fricas [F]

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan(fx+e)^2}{(d\sec(fx+e))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2/(d*sec(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan^2\left(e+fx\right)}{\left(d\sec\left(e+fx\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)**2/(d*sec(f*x+e))**(2/3),x)
[Out] Integral(tan(e + f*x)**2/(d*sec(e + f*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan\left(fx+e\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(2/3), x)
```

Giac [F]

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan\left(fx+e\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

```
[In] int(tan(e + f*x)^2/(d/cos(e + f*x))^(2/3),x)
[Out] int(tan(e + f*x)^2/(d/cos(e + f*x))^(2/3), x)
```

3.286 $\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{19/6}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{19}{6},\frac{7}{2},\sin^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan^5(e+fx)}{5f}$$

```
[Out] 1/5*(cos(f*x+e)^2)^(19/6)*hypergeom([5/2, 19/6],[7/2],sin(f*x+e)^2)*(d*sec(
f*x+e))^(4/3)*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2697}

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{19/6}\tan^5(e+fx)(d\sec(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{19}{6},\frac{7}{2},\sin^2(e+fx)\right)}{5f}$$

```
[In] Int[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^4,x]
```

```
[Out] ((Cos[e + f*x]^2)^(19/6)*Hypergeometric2F1[5/2, 19/6, 7/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^5)/(5*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\cos^2(e+fx)^{19/6}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{19}{6},\frac{7}{2},\sin^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan^5(e+fx)}{5f}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \frac{3d\csc(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},\sec^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{4f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^4,x]
[Out] (3*d*Csc[e + f*x]*Hypergeometric2F1[-3/2, 2/3, 5/3, Sec[e + f*x]^2]*(d*Sec[
e + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])/(4*f)
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{4}{3}}\left(\tan^4(fx+e)\right)dx$$

```
[In] int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x)
[Out] int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{4}{3}}\tan(fx+e)^4\,dx$$

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^(1/3)*d*sec(f*x + e)*tan(f*x + e)^4, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \int (d\sec(e+fx))^{\frac{4}{3}}\tan^4(e+fx)\,dx$$

```
[In] integrate((d*sec(f*x+e))**(4/3)*tan(f*x+e)**4,x)
[Out] Integral((d*sec(e + f*x))**(4/3)*tan(e + f*x)**4, x)
```

Maxima [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{4}{3}}\tan(fx+e)^4\,dx$$

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\left(\frac{d}{\cos(e+f\,x)}\right)^{4/3}\,dx$$

```
[In] int(tan(e + f*x)^4*(d/cos(e + f*x))^(4/3),x)
[Out] int(tan(e + f*x)^4*(d/cos(e + f*x))^(4/3), x)
```

3.287 $\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{17/6}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{17}{6},\frac{7}{2},\sin^2(e+fx)\right)(d\sec(e+fx))^{2/3}\tan^5(e+fx)}{5f}$$

```
[Out] 1/5*(cos(f*x+e)^2)^(17/6)*hypergeom([5/2, 17/6],[7/2],sin(f*x+e)^2)*(d*sec(
f*x+e))^(2/3)*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2697}

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{17/6}\tan^5(e+fx)(d\sec(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{17}{6},\frac{7}{2},\sin^2(e+fx)\right)}{5f}$$

```
[In] Int[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^4,x]
```

```
[Out] ((Cos[e + f*x]^2)^(17/6)*Hypergeometric2F1[5/2, 17/6, 7/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^5)/(5*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\cos^2(e+fx)^{17/6}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{17}{6},\frac{7}{2},\sin^2(e+fx)\right)(d\sec(e+fx))^{2/3}\tan^5(e+fx)}{5f}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \frac{3\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},\sec^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sqrt{-\tan^2(e+fx)}}{2f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^4,x]
[Out] (3*Cot[e + f*x]*Hypergeometric2F1[-3/2, 1/3, 4/3, Sec[e + f*x]^2]*(d*Sec[e
+ f*x])^(2/3)*Sqrt[-Tan[e + f*x]^2])/(2*f)
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{2}{3}}\left(\tan^4(fx+e)\right)dx$$

```
[In] int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x)
[Out] int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^4\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \int (d\sec(e+fx))^{\frac{2}{3}}\tan^4(e+fx)\,dx$$

```
[In] integrate((d*sec(f*x+e))**(2/3)*tan(f*x+e)**4,x)
[Out] Integral((d*sec(e + f*x))**(2/3)*tan(e + f*x)**4, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^4\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4, x)
```

Giac [F]

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^4\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}\,dx$$

```
[In] int(tan(e + f*x)^4*(d/cos(e + f*x))^(2/3),x)
[Out] int(tan(e + f*x)^4*(d/cos(e + f*x))^(2/3), x)
```

3.288 $\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{8/3}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{8}{3},\frac{7}{2},\sin^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^5(e+fx)}{5f}$$

```
[Out] 1/5*(cos(f*x+e)^2)^(8/3)*hypergeom([5/2, 8/3],[7/2],sin(f*x+e)^2)*(d*sec(f*
x+e))^(1/3)*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2697}

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{8/3}\tan^5(e+fx)\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{8}{3},\frac{7}{2},\sin^2(e+fx)\right)}{5f}$$

```
[In] Int[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

```
[Out] ((Cos[e + f*x]^2)^(8/3)*Hypergeometric2F1[5/2, 8/3, 7/2, Sin[e + f*x]^2]*(d
*Sec[e + f*x])^(1/3)*Tan[e + f*x]^5)/(5*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\cos^2(e+fx)^{8/3}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{8}{3},\frac{7}{2},\sin^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^5(e+fx)}{5f}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \frac{3\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{6},\frac{7}{6},\sec^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
[Out] (3*Cot[e + f*x]*Hypergeometric2F1[-3/2, 1/6, 7/6, Sec[e + f*x]^2]*(d*Sec[e
+ f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])/f
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{1}{3}}\left(\tan^4(fx+e)\right)dx$$

```
[In] int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x)
[Out] int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\tan(fx+e)^4\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4, x)
```

Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt[3]{d\sec\left(e+fx\right)}\tan^4\left(e+fx\right)dx$$

```
[In] integrate((d*sec(f*x+e))**(1/3)*tan(f*x+e)**4,x)
[Out] Integral((d*sec(e + f*x))**(1/3)*tan(e + f*x)**4, x)
```

Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \int \left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\tan\left(fx+e\right)^4\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4, x)
```

Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \int \left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\tan\left(fx+e\right)^4\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{1/3}dx$$

```
[In] int(tan(e + f*x)^4*(d/cos(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^4*(d/cos(e + f*x))^(1/3), x)
```

3.289 $\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{7/3}\,\text{Hypergeometric2F1}\left(\frac{7}{3},\frac{5}{2},\frac{7}{2},\sin^2(e+fx)\right)\tan^5(e+fx)}{5f\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] 1/5*(cos(f*x+e)^2)^(7/3)*hypergeom([7/3, 5/2],[7/2],sin(f*x+e)^2)*tan(f*x+e
)^5/f/(d*sec(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2697}

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{7/3}\tan^5(e+fx)\,\text{Hypergeometric2F1}\left(\frac{7}{3},\frac{5}{2},\frac{7}{2},\sin^2(e+fx)\right)}{5f\sqrt[3]{d\sec(e+fx)}}$$

```
[In] Int[Tan[e + f*x]^4/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((Cos[e + f*x]^2)^(7/3)*Hypergeometric2F1[7/3, 5/2, 7/2, Sin[e + f*x]^2]*Ta
n[e + f*x]^5)/(5*f*(d*Sec[e + f*x])^(1/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
```

```
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\cos^2(e+fx)^{7/3}\,\text{Hypergeometric2F1}\left(\frac{7}{3},\frac{5}{2},\frac{7}{2},\sin^2(e+fx)\right)\tan^5(e+fx)}{5f\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{6},\frac{5}{6},\sec^2(e+fx)\right)\tan^3(e+fx)}{f\sqrt[3]{d\sec(e+fx)}\left(-\tan^2(e+fx)\right)^{3/2}}$$

```
[In] Integrate[Tan[e + f*x]^4/(d*Sec[e + f*x])^(1/3),x]
[Out] (-3*Hypergeometric2F1[-3/2, -1/6, 5/6, Sec[e + f*x]^2]*Tan[e + f*x]^3)/(f*(
d*Sec[e + f*x])^(1/3)*(-Tan[e + f*x]^2)^(3/2))
```

Maple [F]

$$\int \frac{\tan^4(fx+e)}{(d\sec(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x)
[Out] int(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan(fx+e)^4}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4/(d*sec(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan^4\left(e+fx\right)}{\sqrt[3]{d\sec\left(e+fx\right)}}\,dx$$

```
[In] integrate(tan(f*x+e)**4/(d*sec(f*x+e))**(1/3),x)
[Out] Integral(tan(e + f*x)**4/(d*sec(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^4}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^4}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^4}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

```
[In] int(tan(e + f*x)^4/(d/cos(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^4/(d/cos(e + f*x))^(1/3), x)
```

3.290 $\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \frac{\cos^2(e+fx)^{13/6}\,\text{Hypergeometric2F1}\left(\frac{13}{6},\frac{5}{2},\frac{7}{2},\sin^2(e+fx)\right)\tan^5(e+fx)}{5f(d\sec(e+fx))^{2/3}}$$

```
[Out] 1/5*(cos(f*x+e)^2)^(13/6)*hypergeom([13/6, 5/2],[7/2],sin(f*x+e)^2)*tan(f*x
+e)^5/f/(d*sec(f*x+e))^(2/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2697}

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \frac{\cos^2(e+fx)^{13/6}\tan^5(e+fx)\,\text{Hypergeometric2F1}\left(\frac{13}{6},\frac{5}{2},\frac{7}{2},\sin^2(e+fx)\right)}{5f(d\sec(e+fx))^{2/3}}$$

```
[In] Int[Tan[e + f*x]^4/(d*Sec[e + f*x])^(2/3),x]
```

```
[Out] ((Cos[e + f*x]^2)^(13/6)*Hypergeometric2F1[13/6, 5/2, 7/2, Sin[e + f*x]^2]*
Tan[e + f*x]^5)/(5*f*(d*Sec[e + f*x])^(2/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\cos^2(e+fx)^{13/6}\,\text{Hypergeometric2F1}\left(\frac{13}{6},\frac{5}{2},\frac{7}{2},\sin^2(e+fx)\right)\tan^5(e+fx)}{5f(d\sec(e+fx))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},\sec^2(e+fx)\right)\tan^3(e+fx)}{2f(d\sec(e+fx))^{2/3}\left(-\tan^2(e+fx)\right)^{3/2}}$$

```
[In] Integrate[Tan[e + f*x]^4/(d*Sec[e + f*x])^(2/3),x]
[Out] (-3*Hypergeometric2F1[-3/2, -1/3, 2/3, Sec[e + f*x]^2]*Tan[e + f*x]^3)/(2*f
*(d*Sec[e + f*x])^(2/3)*(-Tan[e + f*x]^2)^(3/2))
```

Maple [F]

$$\int \frac{\tan^4(fx+e)}{(d\sec(fx+e))^{\frac{2}{3}}}dx$$

```
[In] int(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x)
[Out] int(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x)
```

Fricas [F]

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan(fx+e)^4}{(d\sec(fx+e))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4/(d*sec(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan^4{(e+fx)}}{(d\sec{(e+fx)})^{\frac{2}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)**4/(d*sec(f*x+e))**(2/3),x)
[Out] Integral(tan(e + f*x)**4/(d*sec(e + f*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan{(fx+e)}^4}{(d\sec{(fx+e)})^{\frac{2}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(2/3), x)
```

Giac [F]

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan{(fx+e)}^4}{(d\sec{(fx+e)})^{\frac{2}{3}}}\,dx$$

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan(e+f\,x)^4}{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

```
[In] int(tan(e + f*x)^4/(d/cos(e + f*x))^(2/3),x)
[Out] int(tan(e + f*x)^4/(d/cos(e + f*x))^(2/3), x)
```

3.291 $\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 178

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx=-\frac{\sqrt{b}d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}+\frac{\sqrt{b}d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}+\frac{d^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2bf}$$

```
[Out] -1/4*d^3*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*b^(1/2)*(b*tan(f*x+e))^(1/2)/
f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)+1/4*d^3*arctanh((b*sin(f*x+e))^
(1/2)/b^(1/2))*b^(1/2)*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(f
*x+e))^(1/2)+1/2*d^2*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2)/b/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2693, 2696, 2644, 335, 304, 209, 212}

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx=-\frac{\sqrt{b}d^3\sqrt{b\tan(e+fx)}\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{\sqrt{b}d^3\sqrt{b\tan(e+fx)}\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

```
[In] Int[(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -1/4*(Sqrt[b]*d^3*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]]
)/(f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]]) + (Sqrt[b]*d^3*ArcTanh[Sqrt
```

```
[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(4*f*Sqrt[d*Sec[e + f*x]]*S
qrt[b*Sin[e + f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2))/(2
*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\text{integral} = \frac{d^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2bf}+\frac{1}{4}d^2\int\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx$$

$$=\frac{d^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2bf}+\frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\int\sec(e+fx)\sqrt{b\sin(e+fx)}\,dx}{4\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$=\frac{d^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2bf}+\frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt{x}}{1-\frac{x^2}{b^2}}\,dx,x,b\sin(e+fx)\right)}{4bf\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$=\frac{d^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2bf}+\frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{1-\frac{x^4}{b^2}}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{2bf\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$=\frac{d^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2bf}+\frac{\left(bd^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{b-x^2}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}-\frac{\left(bd^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{b+x^2}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$=-\frac{\sqrt{b}d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}+\frac{\sqrt{b}d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}+\frac{d^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2bf}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.74

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \frac{d^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\left(-\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+\operatorname{arctanh}\left(\frac{\ }{\sqrt[4]{\ }}\right.\right.}{4f\sqrt[4]{\sec^2(e+fx)}\sqrt{\tan(e+f}}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]*(-ArcTan[Sqrt[Tan[e + f*x]]/
(Sec[e + f*x]^2)^(1/4)] + ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)
] + 2*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x]^(3/2)))/(4*f*(Sec[e + f*x]^2)^(1/
4)*Sqrt[Tan[e + f*x]])
```

Maple [A] (verified)

Time = 17.76 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.10

method	result
default	$\frac{\sqrt{b\tan(fx+e)}\,\sqrt{d\sec(fx+e)}\,d^2\left(\operatorname{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)\cos(fx+e)+\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(f\right.\right.}{4f(\cos(fx+e)+1)\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}}$

```
[In] int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*(b*tan(f*x+e))^(1/2)*(d*sec(f*x+e))^(1/2)*d^2/(cos(f*x+e)+1)/(sin(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)*(arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(c
ot(f*x+e)+csc(f*x+e)))*cos(f*x+e)+arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)*(cot(f*x+e)+csc(f*x+e)))*cos(f*x+e)+2*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
*sin(f*x+e)+2*tan(f*x+e)*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(144) = 288.

Time = 0.40 (sec) , antiderivative size = 788, normalized size of antiderivative = 4.43

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \left[-\frac{2\sqrt{-bd}d^2\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-\right.}{4\left(bd\cos(fx+e)^2-bd-(bd\cos(fx+e)+b\right.}\right.}{}\right.$$

$$-\frac{2\sqrt{bd}d^2\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2+\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{4\left(bd\cos(fx+e)^2-bd+(bd\cos(fx+e)+bd)\sin(fx+e)\right)}\right.}{}$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(2*sqrt(-b*d)*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (c
os(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt
(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x
+ e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) - sqrt(
-b*d)*d^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 - 8*
(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(
f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))
 + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 -
 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*d^2*sqrt
(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x
 + e)), -1/32*(2*sqrt(b*d)*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^
2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4
)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos
(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) -
sqrt(b*d)*d^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2
- 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*
cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x +
e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^
4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*d^2*s
qrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(
f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(5/2)*(b*tan(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

Giac [F]

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan(e+f\,x)}\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}\,dx$$

```
[In] int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(5/2),x)
[Out] int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(5/2), x)
```

3.292 $\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}$$

```
[Out] d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE
(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(
1/2)/sin(f*x+e)^(1/2)+d^2*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2693, 2696, 2721, 2719}

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}} - \frac{d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[In] Int[(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -((d^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec
[e + f*x]]*Sqrt[Sin[e + f*x]])) + (d^2*(b*Tan[e + f*x])^(3/2))/(b*f*Sqrt[d*
Sec[e + f*x]])
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}} - \frac{1}{2}d^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx \\
&= \frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}} - \frac{\left(d^2\sqrt{b\tan(e+fx)}\right)\int \sqrt{b\sin(e+fx)}\,dx}{2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} \\
&= \frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}} - \frac{\left(d^2\sqrt{b\tan(e+fx)}\right)\int \sqrt{\sin(e+fx)}\,dx}{2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} \\
&= -\frac{d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} + \frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.55 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx =$$
$$-\frac{d\sqrt{d\sec(e+fx)}\left(-3+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\right)\sin(e+fx)\sqrt{b\tan}}{3f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -1/3*(d*Sqrt[d*Sec[e + f*x]]*(-3 + Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e
+ f*x]^2]*(Sec[e + f*x]^2)^(1/4))*Sin[e + f*x]*Sqrt[b*Tan[e + f*x]])/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 454, normalized size of antiderivative = 4.88

method	result
default	$-\frac{\csc(fx+e)\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e}\right.\right.}{}$

```
[In] int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*csc(f*x+e)*((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+c
sc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f
*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)^2-2*(-I*(I-cot(f*x+e)+csc(
f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x
+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos
(f*x+e)^2+(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)
))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc
(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)-2*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1
/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)
*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+2^(
1/2)*cos(f*x+e)-2^(1/2))*(b*tan(f*x+e))^(1/2)*(d*sec(f*x+e))^(1/2)*d*2^(1/2
)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \frac{2\,d\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin(fx+e) - i\sqrt{-2i\,bd}d\text{weierstrassZeta}(4,0,\text{weiers}}{}$$

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e
) - I*sqrt(-2*I*b*d)*d*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(
f*x + e) + I*sin(f*x + e))) + I*sqrt(2*I*b*d)*d*weierstrassZeta(4, 0, weier
strassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/f
```

Sympy [F]

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan(e+fx)}(d\sec(e+fx))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(3/2)*(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

Giac [F]

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan(e+f\,x)}\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

```
[In] int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2),x)
[Out] int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2), x)
```

3.293 $\int \sqrt{d \sec(e+fx)}\sqrt{b \tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \sqrt{d \sec(e+fx)}\sqrt{b \tan(e+fx)}\,dx = -\frac{\sqrt{b}d \arctan\left(\frac{\sqrt{b \sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b \tan(e+fx)}}{f\sqrt{d \sec(e+fx)}\sqrt{b \sin(e+fx)}} + \frac{\sqrt{b}d \text{arctanh}\left(\frac{\sqrt{b \sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b \tan(e+fx)}}{f\sqrt{d \sec(e+fx)}\sqrt{b \sin(e+fx)}}$$

```
[Out] -d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*b^(1/2)*(b*tan(f*x+e))^(1/2)/f/(d*s
ec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)+d*arctanh((b*sin(f*x+e))^(1/2)/b^(1/2
))*b^(1/2)*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2696, 2644, 335, 304, 209, 212}

$$\int \sqrt{d \sec(e+fx)}\sqrt{b \tan(e+fx)}\,dx = \frac{\sqrt{b}d\sqrt{b \tan(e+fx)}\text{arctanh}\left(\frac{\sqrt{b \sin(e+fx)}}{\sqrt{b}}\right)}{f\sqrt{b \sin(e+fx)}\sqrt{d \sec(e+fx)}} - \frac{\sqrt{b}d\sqrt{b \tan(e+fx)}\arctan\left(\frac{\sqrt{b \sin(e+fx)}}{\sqrt{b}}\right)}{f\sqrt{b \sin(e+fx)}\sqrt{d \sec(e+fx)}}$$

```
[In] Int[Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[b]*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(f*
Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])) + (Sqrt[b]*d*ArcTanh[Sqrt[b*Sin
```

```
[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Si
n[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\text{integral} = \frac{\left(d\sqrt{b\tan(e+fx)}\right)\int\sec(e+fx)\sqrt{b\sin(e+fx)}\,dx}{\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= \frac{\left(d\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{b^2}}\,dx, x, b\sin(e+fx)\right)}{bf\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= \frac{\left(2d\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{bf\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= \frac{\left(bd\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{\left(bd\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{\sqrt{b}d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{\sqrt{b}d\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.77

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \frac{\left(-\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right) + \text{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\right)\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{f\sqrt[4]{\sec^2(e+fx)}\sqrt{\tan(e+fx)}}$$

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] ((-ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + ArcTanh[Sqrt[Tan[e +
 f*x]]/(Sec[e + f*x]^2)^(1/4)])*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])/
(f*(Sec[e + f*x]^2)^(1/4)*Sqrt[Tan[e + f*x]])
```

Maple [A] (verified)

Time = 17.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

method	result
default	$\frac{\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}\left(\text{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)+\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx\right.\right.}{f(\cos(fx+e)+1)\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}}$

```
[In] int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)*(arctanh((sin(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))+arctan((sin(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)*(cot(f*x+e)+csc(f*x+e))))*cos(f*x+e)/(cos(f*x+e)+1)/(sin(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(108) = 216$.

Time = 0.38 (sec) , antiderivative size = 654, normalized size of antiderivative = 4.95

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx$$

$$= \left[-\frac{2\sqrt{-bd}\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{-bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\ldots}}{4\left(bd\cos(fx+e)^2-bd-(bd\cos(fx+e)+bd)\sin(fx+e)\right)}\ldots\right.}{\ldots}\right.$$

$$-\frac{2\sqrt{bd}\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2+\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+\ldots}}}{4\left(bd\cos(fx+e)^2-bd+(bd\cos(fx+e)+bd)\sin(fx+e)\right)}\ldots\right.}{\ldots}$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*sqrt(-b*d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (cos(f*
x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(-b*d
)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x + e)^
2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e))) - sqrt(-b*d)*log((b*d*cos
(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3
 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x
 + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)) + 72*b*d + 28*(b*d*cos(f*x + e)^2
- 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)
^2 - 2)*sin(f*x + e) + 8)))/f, -1/8*(2*sqrt(b*d)*arctan(1/4*(cos(f*x + e)^3
 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) -
2*cos(f*x + e) + 4)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(
f*x + e))/(b*d*cos(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)
)) - sqrt(b*d)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 - 8*(7*cos(f
*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e)
)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)) + 72*b*d
 - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 - 8*cos(f*
x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))/f]
```

Sympy [F]

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(1/2)*(b*tan(f*x+e))**(1/2),x)
[Out] Integral(sqrt(b*tan(e + f*x))*sqrt(d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{d\sec(fx+e)}\sqrt{b\tan(fx+e)}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

Giac [F]

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{d\sec(fx+e)}\sqrt{b\tan(fx+e)}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan(e+f\,x)}\sqrt{\frac{d}{\cos(e+f\,x)}}\,dx$$

```
[In] int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(1/2),x)
[Out] int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(1/2), x)
```

3.294 $\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 55

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1
/2)/sin(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2696, 2721, 2719}

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[In] Int[Sqrt[b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec[e +
 f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}\,dx}{\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\ &= \frac{\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}\,dx}{\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}\\ &= \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.52 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}(b\tan(e+fx))^{3/2}}{3bf\sqrt{d\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]],x]
[Out] (2*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1/4)
*(b*Tan[e + f*x])^(3/2))/(3*b*f*Sqrt[d*Sec[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.74 (sec) , antiderivative size = 342, normalized size of antiderivative = 6.22

method	result
default	$-\frac{\sqrt{-\frac{b(\csc(fx+e)-\cot(fx+e))}{\left(\csc^2(fx+e)\right)(1-\cos(fx+e))^2-1}}\left(2\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{2}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{i(\csc(fx+e)-\cot(f}\right.}{}$
risch	$-\frac{i\sqrt{2}\sqrt{-\frac{ib\left(e^{2i(fx+e)}-1\right)}{e^{2i(fx+e)}+1}}}{f\sqrt{\frac{d\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}}}+\frac{i\left(\frac{2i\left(-ibd\,e^{2i(fx+e)}+idb\right)}{bd\sqrt{e^{i(fx+e)}\left(-ibd\,e^{2i(fx+e)}+idb\right)}}-\frac{\sqrt{e^{i(fx+e)}+1}\,\sqrt{-2\,e^{i(fx+e)}+2}\,\sqrt{-e^{i(fx+e)}}\left(-2E\left(\sqrt{e^{i(fx+e)}+1},\right.\right.}{\sqrt{-ibd\,e^{3i(fx+e)}+idb\,e^{i(fx+e)}}}\right.}{f\sqrt{\frac{d\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}}\left(e^{2i(f\right.}}$

```
[In] int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(-b/(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(csc(f*x+e)-cot(f*x+e)))^(1/2)*(
2*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*2^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I
))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(
f*x+e)))^(1/2),1/2*2^(1/2))-(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*2^(1/2)*(-
I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*Ellipt
icF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-2*csc(f*x+e)^2*(1-cos
(f*x+e))^2)/(-d*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)/(csc(f*x+e)^2*(1-cos(f*x+
e))^2-1))^(1/2)/(1-cos(f*x+e))*sin(f*x+e)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx$$
$$=\frac{i\sqrt{-2i\,bd}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e)))-i\sqrt{2i\,bd}\,\mathrm{weierstr}}{df}$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] (I*sqrt(-2*I*b*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x +
 e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*weierstrassZeta(4, 0, weierstrassP
Inverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d*f)
```

Sympy [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\tan\left(e+fx\right)}}{\sqrt{d\sec\left(e+fx\right)}}\,dx$$

```
[In] integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(b*tan(e + f*x))/sqrt(d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(f*x + e))/sqrt(d*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*tan(f*x + e))/sqrt(d*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(1/2),x)
[Out] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(1/2), x)
```

3.295 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 34

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

```
[Out] 2/3*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2685}

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(b*Tan[e + f*x])^(3/2))/(3*b*f*(d*Sec[e + f*x])^(3/2))
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(3/2),x]
[Out] (2*(b*Tan[e + f*x])^(3/2))/(3*b*f*(d*Sec[e + f*x])^(3/2))
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2\sin(fx+e)\sqrt{b\tan(fx+e)}}{3fd\sqrt{d\sec(fx+e)}}$	35

```
[In] int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3/f*sin(f*x+e)*(b*tan(f*x+e))^(1/2)/d/(d*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{3\,d^2f}$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e)/(d^2*f)
```

Sympy [A] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \begin{cases} \frac{2\sqrt{b\tan{(e+fx)}}\tan{(e+fx)}}{3f(d\sec{(e+fx)})^{\frac{3}{2}}} & \text{for } f\neq 0 \\ \frac{x\sqrt{b\tan{(e)}}}{(d\sec{(e)})^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(3/2),x)
```

```
[Out] Piecewise((2*sqrt(b*tan(e + f*x))*tan(e + f*x)/(3*f*(d*sec(e + f*x))**(3/2)
), Ne(f, 0)), (x*sqrt(b*tan(e))/(d*sec(e))**(3/2), True))
```

Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\tan{(fx+e)}}}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\tan{(fx+e)}}}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sin{(2\,e+2\,f\,x)}\,\sqrt{\frac{d}{\cos(e+f\,x)}}\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{3\,d^2\,f}$$

```
[In] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(3/2),x)
```

```
[Out] (sin(2*e + 2*f*x)*(d/cos(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(cos(2*e + 2
*f*x) + 1))^(1/2))/(3*d^2*f)
```

3.296 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{4E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

```
[Out] -4/5*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
E(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/d^2/f/(d*sec(f*x+
e))^(1/2)/sin(f*x+e)^(1/2)+2/5*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(5/2
)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2692, 2696, 2721, 2719}

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (4*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt[d*S
ec[e + f*x]]*Sqrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(5*b*f*(d*Sec
[e + f*x])^(5/2))
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
```

```
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}} + \frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{5d^2} \\
&= \frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}} + \frac{\left(2\sqrt{b\tan(e+fx)}\right)\int \sqrt{b\sin(e+fx)}\,dx}{5d^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} \\
&= \frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}} + \frac{\left(2\sqrt{b\tan(e+fx)}\right)\int \sqrt{\sin(e+fx)}\,dx}{5d^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} \\
&= \frac{4E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} + \frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.92 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{2\left(3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/4}\right)(b\tan(e+f}{15bf(d\sec(e+fx))^{5/2}}$$

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*(3 + 2*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^2
)^(5/4))*(b*Tan[e + f*x])^(3/2))/(15*b*f*(d*Sec[e + f*x])^(5/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.90 (sec) , antiderivative size = 454, normalized size of antiderivative = 4.78

method	result
default	$-\frac{\csc(fx+e)\left(-2\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e}\right.\right.}{}$

```
[In] int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/f*csc(f*x+e)*(-2*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-
csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-co
t(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+4*(-I*(I-cot(f*x+e)+csc
(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*
x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*co
s(f*x+e)+2^(1/2)*cos(f*x+e)^3-2*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(c
ot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF
((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+4*(-I*(I-cot(f*x+e)+csc(
f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x
+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+2^(
1/2)*cos(f*x+e)-2*2^(1/2))*(b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2)/d^2*2^
(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{2\left(\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)^2\sin(fx+e)+i\sqrt{-2i\,bd}\text{weierstrassZeta}(4\right.}{}$$

[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] 2/5*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2*
sin(f*x + e) + I*sqrt(-2*I*b*d)*weierstrassZeta(4, 0, weierstrassPInverse(4
, 0, cos(f*x + e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*weierstrassZeta(4, 0
, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d^3*f)

Sympy [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{\frac{5}{2}}}\,dx$$

[In] integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(5/2),x)

[Out] Integral(sqrt(b*tan(e + f*x))/(d*sec(e + f*x))**(5/2), x)

Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan(fx+e)}}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(5/2), x)

Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan(fx+e)}}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan(e+f\,x)}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(5/2),x)
[Out] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(5/2), x)
```

3.297 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}} + \frac{8(b\tan(e+fx))^{3/2}}{21bd^2f(d\sec(e+fx))^{3/2}}$$

```
[Out] 2/7*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(7/2)+8/21*(b*tan(f*x+e))^(3/2)
/b/d^2/f/(d*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2692, 2685}

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{8(b\tan(e+fx))^{3/2}}{21bd^2f(d\sec(e+fx))^{3/2}} + \frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}}$$

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*(b*Tan[e + f*x])^(3/2))/(7*b*f*(d*Sec[e + f*x])^(7/2)) + (8*(b*Tan[e + f
*x])^(3/2))/(21*b*d^2*f*(d*Sec[e + f*x])^(3/2))
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}} + \frac{4\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx}{7d^2} \\ &= \frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}} + \frac{8(b\tan(e+fx))^{3/2}}{21bd^2f(d\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{(19\sin(e+fx)+3\sin(3(e+fx)))\sqrt{b\tan(e+fx)}}{42d^3f\sqrt{d\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(7/2),x]
[Out] ((19*Sin[e + f*x] + 3*Sin[3*(e + f*x)])*Sqrt[b*Tan[e + f*x]])/(42*d^3*f*Sqr
t[d*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{2\sin(fx+e)\sqrt{b\tan(fx+e)}\,(3(\cos^2(fx+e))+4)}{21f\sqrt{d\sec(fx+e)}\,d^3}$	47

```
[In] int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/21/f*sin(f*x+e)*(b*tan(f*x+e))^(1/2)*(3*cos(f*x+e)^2+4)/(d*sec(f*x+e))^(1
/2)/d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2\left(3\cos\left(fx+e\right)^{3}+4\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin\left(fx+e\right)}{21\,d^{4}f}$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
[Out] 2/21*(3*cos(f*x + e)^3 + 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*
sqrt(d/cos(f*x + e))*sin(f*x + e)/(d^4*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}\,\left(22\,\sin\left(2\,e+2\,f\,x\right)+3\,\sin\left(4\,e+4\,f\,x\right)\right)\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{84\,d^4\,f}$$

```
[In] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(7/2),x)
[Out] ((d/cos(e + f*x))^(1/2)*(22*sin(2*e + 2*f*x) + 3*sin(4*e + 4*f*x))*((b*sin(
2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(84*d^4*f)
```

3.298 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{8E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} + \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}} + \frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}}$$

```
[Out] -8/15*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ellipti
cE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/d^4/f/(d*sec(f*x
+e))^(1/2)/sin(f*x+e)^(1/2)+2/9*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(9/
2)+4/15*(b*tan(f*x+e))^(3/2)/b/d^2/f/(d*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2692, 2696, 2721, 2719}

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{8E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}} + \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (8*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(15*d^4*f*Sqrt[d*
Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(9*b*f*(d*Se
```

```
c[e + f*x])^(9/2)) + (4*(b*Tan[e + f*x])^(3/2))/(15*b*d^2*f*(d*Sec[e + f*x]
)^(5/2))
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}+\frac{2\int\frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx}{3d^2}\\
&= \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}+\frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}}+\frac{4\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{15d^4}\\
&= \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}+\frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}}+\frac{\left(4\sqrt{b\tan(e+fx)}\right)\int\sqrt{b\sin(e+fx)}\,dx}{15d^4\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}+\frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}}+\frac{\left(4\sqrt{b\tan(e+fx)}\right)\int\sqrt{\sin(e+fx)}\,dx}{15d^4\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}\\
&= \frac{8E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}+\frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{\left(17+5\cos(2(e+fx))+8\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sec^2(e+$$
$$45d^3f(d\sec(e+fx))^{3/2}$$

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] ((17 + 5*Cos[2*(e + f*x)] + 8*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x
]^2]*(Sec[e + f*x]^2)^(5/4))*Sin[e + f*x]*Sqrt[b*Tan[e + f*x]])/(45*d^3*f*(
d*Sec[e + f*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.60 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.55

method	result
default	$-\frac{\csc(fx+e)\left(5\left(\cos^5(fx+e)\right)\sqrt{2}-12\sqrt{i(-i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\right.}{}$

```
[In] int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/45/f*csc(f*x+e)*(5*cos(f*x+e)^5*2^(1/2)-12*(I*(-I+cot(f*x+e)-csc(f*x+e))
)^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(
1/2)*EllipticF((I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)
+24*(I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/
2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticE((I*(-I+cot(f*x+e)-csc(f*x+e
)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+2^(1/2)*cos(f*x+e)^3-12*(I*(-I+cot(f*x+e)
-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-cs
c(f*x+e)))^(1/2)*EllipticF((I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2),1/2*2^(1/2)
)+24*(I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1
/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticE((I*(-I+cot(f*x+e)-csc(f*x+
e)))^(1/2),1/2*2^(1/2))+6*2^(1/2)*cos(f*x+e)-12*2^(1/2))*(b*tan(f*x+e))^(1/
2)/(d*sec(f*x+e))^(1/2)/d^4*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{2\left(\left(5\cos\left(fx+e\right)^4+6\cos\left(fx+e\right)^2\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin\left(fx+e\right)+6i\sqrt{-}\right.}{}$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] 2/45*((5*cos(f*x + e)^4 + 6*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x + e
))*sqrt(d/cos(f*x + e))*sin(f*x + e) + 6*I*sqrt(-2*I*b*d)*weierstrassZeta(4
, 0, weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) - 6*I*sqrt(2
*I*b*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*si
n(f*x + e))))/(d^5*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="giac")

[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(9/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{9/2}}\,dx$$

[In] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(9/2),x)

[Out] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(9/2), x)

3.299 $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = -\frac{b^2d^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{6f\sqrt{b\tan(e+fx)}} - \frac{bd^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{6f} + \frac{b(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{3f}$$

```
[Out] 1/6*b^2*d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*E
llipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)
^(1/2)/f/(b*tan(f*x+e))^(1/2)+1/3*b*(d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/
2)/f-1/6*b*d^2*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2691, 2693, 2696, 2721, 2720}

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = -\frac{b^2d^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{6f\sqrt{b\tan(e+fx)}} - \frac{bd^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{6f} + \frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}$$

```
[In] Int[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] -1/6*(b^2*d^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Si
n[e + f*x]])/(f*Sqrt[b*Tan[e + f*x]]) - (b*d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*
Tan[e + f*x]])/(6*f) + (b*(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]])/(3*f
)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{b(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{3f} - \frac{1}{6}b^2\int\frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$$

$$= -\frac{bd^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{6f} + \frac{b(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{3f} - \frac{1}{12}(b^2d^2)\int\frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$$

$$= -\frac{bd^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{6f} + \frac{b(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{3f} - \frac{\left(b^2d^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int\frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{12\sqrt{b\tan(e+fx)}}$$

$$= -\frac{bd^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{6f} + \frac{b(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{3f} - \frac{\left(b^2d^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int\frac{1}{\sqrt{\sin(e+fx)}}\,dx}{12\sqrt{b\tan(e+fx)}}$$

$$= -\frac{b^2d^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{6f\sqrt{b\tan(e+fx)}} - \frac{bd^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{6f} + \frac{b(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{3f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.62 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.63

$$\int(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = -\frac{b(d\sec(e+fx))^{5/2}\left(-2+\cos^2(e+fx)+\cos^4(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e\right.}{6f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] -1/6*(b*(d*Sec[e + f*x])^(5/2)*(-2 + Cos[e + f*x]^2 + Cos[e + f*x]^4*Hyperg
eometric2F1[1/4, 3/4, 5/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(3/4))*Sqrt[b*
Tan[e + f*x]])/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.60 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.25

method	result
default	$\frac{\sqrt{b\tan(fx+e)}\,\sqrt{d\sec(fx+e)}\,b\,d^2\left(i(\cos^2(fx+e))\sin(fx+e)\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,F\left(\sqrt{-i(i\right.\right.}{}$

```
[In] int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/f*(b*tan(f*x+e))^(1/2)*(d*sec(f*x+e))^(1/2)*b*d^2/(cos(f*x+e)-1)/(cos(
f*x+e)+1)*(I*cos(f*x+e)^2*sin(f*x+e)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(
I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(
1/2),1/2*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)+I*cos(f*x+e)*sin(f*x
+e)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*
EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*(-I*(I-cot(f*x+
e)+csc(f*x+e)))^(1/2)+sin(f*x+e)^2*2^(1/2)-2*tan(f*x+e)^2*2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx =$$

$$-\frac{\sqrt{-2i\,bd}bd^2\cos{(fx+e)}^2\operatorname{weierstrassPInverse}(4,0,\cos{(fx+e)}+i\,\sin{(fx+e)})+\sqrt{2i\,bd}bd^2\cos{(fx+e)}}{12\,f\,\mathrm{co}}$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(-2*I*b*d)*b*d^2*cos(f*x + e)^2*weierstrassPInverse(4, 0, cos(f*
x + e) + I*sin(f*x + e)) + sqrt(2*I*b*d)*b*d^2*cos(f*x + e)^2*weierstrassPI
nverse(4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(b*d^2*cos(f*x + e)^2 - 2*b
*d^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x +
e)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(5/2)*(b*tan(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)
```

Giac [F]

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(e+f\,x))^{3/2}\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}\,dx$$

```
[In] int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2),x)
[Out] int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2), x)
```

3.300 $\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 169

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = -\frac{b^{3/2}d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4f\sqrt{b\tan(e+fx)}} - \frac{b^{3/2}d\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4f\sqrt{b\tan(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f}$$

```
[Out] -1/4*b^(3/2)*d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b
*sin(f*x+e))^(1/2)/f/(b*tan(f*x+e))^(1/2)-1/4*b^(3/2)*d*arctanh((b*sin(f*x+
e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/f/(b*tan(f*x+e
))^(1/2)+1/2*b*(d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

$= \{2691, 2696, 2644, 335, 218, 212, 209\}$

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = -\frac{b^{3/2}d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\tan(e+fx)}} - \frac{b^{3/2}d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\operatorname{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\tan(e+fx)}} + \frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}$$

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] -1/4*(b^(3/2)*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*S
qrt[b*Sin[e + f*x]])/(f*Sqrt[b*Tan[e + f*x]]) - (b^(3/2)*d*ArcTanh[Sqrt[b*S
in[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(4*f*Sqrt[
b*Tan[e + f*x]]) + (b*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f}-\frac{1}{4}b^2\int\frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx\\
&= \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f}-\frac{\left(b^2d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int\frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}\,dx}{4\sqrt{b\tan(e+fx)}}\\
&= \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f}\\
&\quad-\frac{\left(bd\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1-\frac{x^2}{b^2}\right)}\,dx,x,b\sin(e+fx)\right)}{4f\sqrt{b\tan(e+fx)}}\\
&= \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f}\\
&\quad-\frac{\left(bd\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int\frac{1}{1-\frac{x^4}{b^2}}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{2f\sqrt{b\tan(e+fx)}}
\end{aligned}$$

$$= \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f} - \frac{\left(b^2d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{4f\sqrt{b\tan(e+fx)}} - \frac{\left(b^2d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{4f\sqrt{b\tan(e+fx)}}$$

$$= -\frac{b^{3/2}d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4f\sqrt{b\tan(e+fx)}} - \frac{b^{3/2}d\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4f\sqrt{b\tan(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.77

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \frac{(d\sec(e+fx))^{3/2}\left(-\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right) - \text{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\right.}{4f\sec^2(e+fx)^{3/4}\tan^{\frac{3}{2}}(e+}$$

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((d*Sec[e + f*x])^(3/2)*(-ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)]
 - ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + 2*(Sec[e + f*x]^2)^
(3/4)*Sqrt[Tan[e + f*x]])*(b*Tan[e + f*x])^(3/2))/(4*f*(Sec[e + f*x]^2)^(3/
4)*Tan[e + f*x]^(3/2))
```

Maple [A] (verified)

Time = 20.76 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.12

method	result
default	$\frac{\sqrt{b\tan(fx+e)}\,\sqrt{d\sec(fx+e)}\,bd\left(\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)\cos(fx+e)-\text{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(f\right.\right.}{4f(\cos(fx+e)+1)\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}}$

```
[In] int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*(b*tan(f*x+e))^(1/2)*(d*sec(f*x+e))^(1/2)*b*d/(cos(f*x+e)+1)/(sin(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)*(arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(co
t(f*x+e)+csc(f*x+e)))*cos(f*x+e)-arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)*(cot(f*x+e)+csc(f*x+e)))*cos(f*x+e)+2*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
+2*sec(f*x+e)*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(135) = 270$.

Time = 0.37 (sec) , antiderivative size = 769, normalized size of antiderivative = 4.55

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \left[\frac{2\sqrt{-bd}bd\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2 \cdots}{4\left(bd\cos(fx+e)^2-bd-(bd\cos(fx+e)+bd \cdots}\right.}{\cdots}\right.$$

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(2*sqrt(-b*d)*b*d*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (co
s(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(
-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x +
 e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) + sqrt(-
b*d)*b*d*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 - 8*(
7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f
*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))
+ 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 -
8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 16*b*d*sqrt(
b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)), -1/32*
(2*sqrt(b*d)*b*d*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x +
 e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*d)*sq
rt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x + e)^2 -
b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) - sqrt(b*d)*b*d*
cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 + 8*(7*cos(f*x
 + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*
sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)) + 72*b*d -
 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 - 8*cos(f*x
+ e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*b*d*sqrt(b*sin(f*x
+ e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(3/2)*(b*tan(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

Giac [F]

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(e+f\,x))^{3/2}\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

```
[In] int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(3/2),x)
[Out] int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(3/2), x)
```

3.301 $\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = -\frac{b^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}} + \frac{b\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{f}$$

```
[Out] b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF
(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/f
/(b*tan(f*x+e))^(1/2)+b*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2691, 2696, 2721, 2720}

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f} - \frac{b^2\sqrt{\sin(e+fx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] -((b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f
*x]])/(f*Sqrt[b*Tan[e + f*x]])) + (b*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*
x]])/f
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{f} - \frac{1}{2}b^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx \\
&= \frac{b\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{f} - \frac{\left(b^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{2\sqrt{b\tan(e+fx)}} \\
&= \frac{b\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{f} - \frac{\left(b^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{2\sqrt{b\tan(e+fx)}} \\
&= -\frac{b^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}} \\
&\quad + \frac{b\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{f}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.57 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx =$$
$$-\frac{b\sqrt{d\sec(e+fx)}\left(-1+\frac{\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)}{\sqrt[4]{\sec^2(e+fx)}}\right)\sqrt{b\tan(e+fx)}}{f}$$

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] -((b*Sqrt[d*Sec[e + f*x]]*(-1 + Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[e + f
*x]^2]/(Sec[e + f*x]^2)^(1/4))*Sqrt[b*Tan[e + f*x]])/f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.09

method	result
default	$\frac{\sin(fx+e)\left(i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)+\right.\right.}{}$

```
[In] int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*sin(f*x+e)*(I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-cs
c(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(
f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)^2+I*(-I*(I-cot(f*x+e)+csc
(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*
x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*co
s(f*x+e)-2^(1/2)*sin(f*x+e))*(b*tan(f*x+e))^(1/2)*(d*sec(f*x+e))^(1/2)*b/(c
os(f*x+e)-1)/(cos(f*x+e)+1)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx =$$

$$-\frac{\sqrt{-2i\,bd}b\text{weierstrassPInverse}(4,0,\cos{(fx+e)}+i\,\sin{(fx+e)})+\sqrt{2i\,bd}b\text{weierstrassPInverse}(4,0,\cos{(fx}}{2\,f}$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] -1/2*(sqrt(-2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x +
 e)) + sqrt(2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x +
 e)) - 2*b*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/f
```

Sympy [F]

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int (b\tan{(e+fx)})^{\frac{3}{2}}\sqrt{d\sec{(e+fx)}}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(1/2)*(b*tan(f*x+e))**(3/2),x)
[Out] Integral((b*tan(e + f*x))**(3/2)*sqrt(d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int \sqrt{d\sec{(fx+e)}}(b\tan{(fx+e)})^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int \sqrt{d\sec\left(fx+e\right)}(b\tan\left(fx+e\right))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(e+f\,x))^{3/2}\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}\,dx$$

```
[In] int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2),x)
[Out] int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2), x)
```

3.302 $\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = -\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}} + \frac{b^{3/2}d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}} + \frac{b^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}$$

```
[Out] -2*d*csc(f*x+e)*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/2)+b^(3/2)*d*arcta
n((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/2)
/(b*sin(f*x+e))^(3/2)+b^(3/2)*d*arctanh((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*ta
n(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/2)/(b*sin(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2696, 2644, 327, 335, 218, 212, 209}

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{b^{3/2}d(b\tan(e+fx))^{3/2}\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}} + \frac{b^{3/2}d(b\tan(e+fx))^{3/2}\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}} - \frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}}$$

```
[In] Int[(b*Tan[e + f*x])^(3/2)/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-2*d*Csc[e + f*x]*(b*Tan[e + f*x])^(3/2))/(f*(d*Sec[e + f*x])^(3/2)) + (b^
(3/2)*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*(b*Tan[e + f*x])^(3/2))/(f*(d*
Sec[e + f*x])^(3/2)*(b*Sin[e + f*x])^(3/2)) + (b^(3/2)*d*ArcTanh[Sqrt[b*Sin
[e + f*x]]/Sqrt[b]]*(b*Tan[e + f*x])^(3/2))/(f*(d*Sec[e + f*x])^(3/2)*(b*Si
n[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(d(b\tan(e+fx))^{3/2}\right)\int\sec(e+fx)(b\sin(e+fx))^{3/2}\,dx}{(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}\\
&= \frac{\left(d(b\tan(e+fx))^{3/2}\right)\operatorname{Subst}\left(\int\frac{x^{3/2}}{1-\frac{x^2}{b^2}}\,dx,x,b\sin(e+fx)\right)}{bf(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}\\
&= -\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}}\\
&\quad+\frac{\left(bd(b\tan(e+fx))^{3/2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{x}\left(1-\frac{x^2}{b^2}\right)}\,dx,x,b\sin(e+fx)\right)}{f(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}\\
&= -\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}}\\
&\quad+\frac{\left(2bd(b\tan(e+fx))^{3/2}\right)\operatorname{Subst}\left(\int\frac{1}{1-\frac{x^4}{b^2}}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{f(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}\\
&= -\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}}\\
&\quad+\frac{\left(b^2d(b\tan(e+fx))^{3/2}\right)\operatorname{Subst}\left(\int\frac{1}{b-x^2}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{f(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}\\
&\quad+\frac{\left(b^2d(b\tan(e+fx))^{3/2}\right)\operatorname{Subst}\left(\int\frac{1}{b+x^2}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{f(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}\\
&= -\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}}+\frac{b^{3/2}d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}\\
&\quad+\frac{b^{3/2}d\operatorname{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.70 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.40

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{4},\frac{9}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}(b\tan(e+fx))^{5/2}}{5bf\sqrt{d\sec(e+fx)}}$$

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/Sqrt[d*Sec[e + f*x]],x]
[Out] (2*Hypergeometric2F1[5/4, 5/4, 9/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1/4)
*(b*Tan[e + f*x])^(5/2))/(5*b*f*Sqrt[d*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(139) = 278.

Time = 19.50 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.75

method	result
default	$-\frac{\sin(fx+e)\left(\operatorname{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,\cos(fx+e)-\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\right.\right.}{}$

```
[In] int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/f*sin(f*x+e)*(arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+cs
c(f*x+e)))*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-(sin(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)*arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+c
sc(f*x+e)))*cos(f*x+e)+(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*arctanh((sin(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))-(sin(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)*arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*
x+e)))-2*sin(f*x+e))*(b*tan(f*x+e))^(1/2)*b/(cos(f*x+e)-1)/(d*sec(f*x+e))^(
1/2)/(cos(f*x+e)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(139) = 278.

Time = 0.55 (sec) , antiderivative size = 741, normalized size of antiderivative = 4.44

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \left[-\frac{2\,bd\sqrt{-\frac{b}{d}}\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos\right.}{4\left(b\cos(fx+e)^2-(b\cos(fx+e)+b)\sin(fx+\right.}\right.}{}\right.$$

[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [-1/8*(2*b*d*sqrt(-b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x + e))/(b*cos(f*x + e)^2 - (b*cos(f*x + e) + b)*sin(f*x + e) - b)) - b*d*sqrt(-b/d)*log((b*cos(f*x + e)^4 - 72*b*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x + e)) + 28*(b*cos(f*x + e)^2 - 2*b)*sin(f*x + e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 16*b*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e))/(d*f), 1/8*(2*b*d*sqrt(b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/d)*sqrt(d/cos(f*x + e))/(b*cos(f*x + e)^2 + (b*cos(f*x + e) + b)*sin(f*x + e) - b)) + b*d*sqrt(b/d)*log((b*cos(f*x + e)^4 - 72*b*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/d)*sqrt(d/cos(f*x + e)) - 28*(b*cos(f*x + e)^2 - 2*b)*sin(f*x + e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*b*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e))/(d*f)]

Sympy [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan{(e+fx)})^{\frac{3}{2}}}{\sqrt{d\sec{(e+fx)}}}\,dx$$

[In] integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(1/2),x)

[Out] Integral((b*tan(e + f*x))**(3/2)/sqrt(d*sec(e + f*x)), x)

Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan{(fx+e)})^{\frac{3}{2}}}{\sqrt{d\sec{(fx+e)}}}\,dx$$

[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((b*tan(f*x + e))^(3/2)/sqrt(d*sec(f*x + e)), x)

Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{\sqrt{d\sec(fx+e)}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(3/2)/sqrt(d*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(e+f\,x))^{3/2}}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(1/2),x)
[Out] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(1/2), x)
```

3.303 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2b^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}} - \frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

```
[Out] -2/3*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1
/2)/d^2/f/(b*tan(f*x+e))^(1/2)-2/3*b*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^
(3/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2690, 2696, 2721, 2720}

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2b^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}} - \frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f
*x]])/(3*d^2*f*Sqrt[b*Tan[e + f*x]]) - (2*b*Sqrt[b*Tan[e + f*x]])/(3*f*(d*S
ec[e + f*x])^(3/2))
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}+\frac{b^2\int\frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{3d^2}\\
&= -\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}+\frac{\left(b^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int\frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{3d^2\sqrt{b\tan(e+fx)}}\\
&= -\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}+\frac{\left(b^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int\frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d^2\sqrt{b\tan(e+fx)}}\\
&= \frac{2b^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.64 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2b\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)\sqrt{b\tan(e+}}{3f(d\sec(e+fx))^{3/2}}$$

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*b*(-1 + Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^
2)^(3/4))*Sqrt[b*Tan[e + f*x]])/(3*f*(d*Sec[e + f*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.55 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.83

method	result
default	$\frac{\sin(fx+e)\left(-i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)+}\right.\right.}{}$

```
[In] int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*sin(f*x+e)*(-I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-c
sc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot
(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)-I*(-I*(I-cot(f*x+e)+csc(
f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x
+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+2^(
1/2)*cos(f*x+e)*sin(f*x+e))*b*(b*tan(f*x+e))^(1/2)/(cos(f*x+e)-1)/(d*sec(f*
x+e))^(1/2)/d/(cos(f*x+e)+1)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{2\,b\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)^2-\sqrt{-2i\,bd}b\text{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))-}{3\,d^2 f}$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x +
e)^2 - sqrt(-2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x
+ e)) - sqrt(2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x
+ e)))/(d^2*f)
```

Sympy [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan{(e+fx)})^{\frac{3}{2}}}{(d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(3/2),x)
[Out] Integral((b*tan(e + f*x))**(3/2)/(d*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan{(fx+e)})^{\frac{3}{2}}}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan{(fx+e)})^{\frac{3}{2}}}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{3/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(3/2),x)
[Out] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(3/2), x)
```

3.304 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}} dx$

Optimal result

Integrand size = 25, antiderivative size = 34

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}} dx = \frac{2(b\tan(e+fx))^{5/2}}{5bf(d\sec(e+fx))^{5/2}}$$

```
[Out] 2/5*(b*tan(f*x+e))^(5/2)/b/f/(d*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2685}

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}} dx = \frac{2(b\tan(e+fx))^{5/2}}{5bf(d\sec(e+fx))^{5/2}}$$

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*(b*Tan[e + f*x])^(5/2))/(5*b*f*(d*Sec[e + f*x])^(5/2))
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2(b\tan(e+fx))^{5/2}}{5bf(d\sec(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{2b\sin^2(e+fx)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}}$$

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(5/2),x]
[Out] (2*b*Sin[e + f*x]^2*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{2\left(\sin^2(fx+e)\right)\sqrt{b\tan(fx+e)}\,b}{5f\,d^2\sqrt{d\sec(fx+e)}}$	38

```
[In] int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/5/f*sin(f*x+e)^2*(b*tan(f*x+e))^(1/2)*b/d^2/(d*sec(f*x+e))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(28) = 56.

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = -\frac{2\left(b\cos\left(fx+e\right)^3 - b\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{5\,d^3f}$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
[Out] -2/5*(b*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*
sqrt(d/cos(f*x + e))/(d^3*f)
```

Sympy [A] (verification not implemented)

Time = 45.79 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = \begin{cases} \frac{2(b\tan(e+fx))^{\frac{3}{2}}\tan(e+fx)}{5f(d\sec(e+fx))^{\frac{5}{2}}} & \text{for } f\neq 0 \\ \frac{x(b\tan(e))^{\frac{3}{2}}}{(d\sec(e))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(5/2),x)
[Out] Piecewise((2*(b*tan(e + f*x))**(3/2)*tan(e + f*x)/(5*f*(d*sec(e + f*x))**(5
/2)), Ne(f, 0)), (x*(b*tan(e))**(3/2)/(d*sec(e))**(5/2), True))
```

Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{b\sqrt{\frac{d}{\cos(e+f\,x)}}\,\left(\cos\left(e+f\,x\right)-\cos\left(3\,e+3\,f\,x\right)\right)\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{10\,d^3\,f}$$

```
[In] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(5/2),x)
[Out] (b*(d/cos(e + f*x))^(1/2)*(cos(e + f*x) - cos(3*e + 3*f*x))*((b*sin(2*e + 2
*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(10*d^3*f)
```

3.305 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{4b^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{21d^4f\sqrt{b\tan(e+fx)}} - \frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}}$$

```
[Out] -4/21*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ell
ipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(
1/2)/d^4/f/(b*tan(f*x+e))^(1/2)-2/7*b*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))
^(7/2)+2/21*b*(b*tan(f*x+e))^(1/2)/d^2/f/(d*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2690, 2692, 2696, 2721, 2720}

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{4b^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{21d^4f\sqrt{b\tan(e+fx)}} + \frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}} - \frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (4*b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f
*x]])/(21*d^4*f*Sqrt[b*Tan[e + f*x]]) - (2*b*Sqrt[b*Tan[e + f*x]])/(7*f*(d*
```

```
Sec[e + f*x])^(7/2)) + (2*b*Sqrt[b*Tan[e + f*x]])/(21*d^2*f*(d*Sec[e + f*x]
)^(3/2))
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{b^2\int\frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx}{7d^2}$$

$$= -\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}} + \frac{(2b^2)\int\frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{21d^4}$$

$$= -\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}} + \frac{\left(2b^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int\frac{1}{\sqrt{b\sin(e+fx)}}dx}{21d^4\sqrt{b\tan(e+fx)}}$$

$$= -\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}} + \frac{\left(2b^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{21d^4\sqrt{b\tan(e+fx)}}$$

$$= \frac{4b^2\operatorname{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{21d^4f\sqrt{b\tan(e+fx)}} - \frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.93 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

$$\int\frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}dx = -\frac{b\left(1+3\cos(2(e+fx))-4\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}}$$

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] -1/21*(b*(1 + 3*Cos[2*(e + f*x)] - 4*Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[
e + f*x]^2]*(Sec[e + f*x]^2)^(3/4))*Sqrt[b*Tan[e + f*x]])/(d^2*f*(d*Sec[e +
 f*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.12 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.23

method	result
default	$\frac{\sin(fx+e)\left(-2i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)\right.\right.}{}$

```
[In] int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/f*sin(f*x+e)*(-2*I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e
)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-
cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+3*2^(1/2)*cos(f*x+e)^
3*sin(f*x+e)-2*I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f
*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x
+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-2^(1/2)*cos(f*x+e)*sin(f*x+e))*(b*tan(f
*x+e))^(1/2)*b/(cos(f*x+e)-1)/d^3/(d*sec(f*x+e))^(1/2)/(cos(f*x+e)+1)*2^(1/
2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.89

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2\left(\sqrt{-2i\,bd}b\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e))+\sqrt{2i\,bd}b\mathrm{we}\right.}{}$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/21*(sqrt(-2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x +
 e)) + sqrt(2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x +
 e)) - (3*b*cos(f*x + e)^4 - b*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x
+ e))*sqrt(d/cos(f*x + e)))/(d^4*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(7/2), x)
```

Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{3/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

```
[In] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(7/2),x)
[Out] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(7/2), x)
```

3.306 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = -\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}} + \frac{2b\sqrt{b\tan(e+fx)}}{45d^2f(d\sec(e+fx))^{5/2}} + \frac{8b\sqrt{b\tan(e+fx)}}{45d^4f\sqrt{d\sec(e+fx)}}$$

```
[Out] -2/9*b*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(9/2)+2/45*b*(b*tan(f*x+e))^(1
/2)/d^2/f/(d*sec(f*x+e))^(5/2)+8/45*b*(b*tan(f*x+e))^(1/2)/d^4/f/(d*sec(f*x
+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2690, 2692, 2685}

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{8b\sqrt{b\tan(e+fx)}}{45d^4f\sqrt{d\sec(e+fx)}} + \frac{2b\sqrt{b\tan(e+fx)}}{45d^2f(d\sec(e+fx))^{5/2}} - \frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}$$

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (-2*b*Sqrt[b*Tan[e + f*x]])/(9*f*(d*Sec[e + f*x])^(9/2)) + (2*b*Sqrt[b*Tan[
e + f*x]])/(45*d^2*f*(d*Sec[e + f*x])^(5/2)) + (8*b*Sqrt[b*Tan[e + f*x]])/(
45*d^4*f*Sqrt[d*Sec[e + f*x]])
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}+\frac{b^2\int\frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx}{9d^2}\\
&= -\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b\sqrt{b\tan(e+fx)}}{45d^2f(d\sec(e+fx))^{5/2}}+\frac{(4b^2)\int\frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx}{45d^4}\\
&= -\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b\sqrt{b\tan(e+fx)}}{45d^2f(d\sec(e+fx))^{5/2}}+\frac{8b\sqrt{b\tan(e+fx)}}{45d^4f\sqrt{d\sec(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.53

$$\int\frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx=\frac{b(13+5\cos(2(e+fx)))\sin^2(e+fx)\sqrt{b\tan(e+fx)}}{45d^4f\sqrt{d\sec(e+fx)}}$$

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(9/2),x]
[Out] (b*(13 + 5*Cos[2*(e + f*x)])*Sin[e + f*x]^2*Sqrt[b*Tan[e + f*x]])/(45*d^4*f
*Sqrt[d*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{2\left(\sin^{2}\left(fx+e\right)\right)b\sqrt{b\tan\left(fx+e\right)}\,\left(5\left(\cos^{2}\left(fx+e\right)\right)+4\right)}{45f\sqrt{d\sec\left(fx+e\right)}\,d^{4}}$	50

```
[In] int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45/f*sin(f*x+e)^2*b*(b*tan(f*x+e))^(1/2)*(5*cos(f*x+e)^2+4)/(d*sec(f*x+e)
)^(1/2)/d^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = -\frac{2\left(5\,b\cos\left(fx+e\right)^{5}-b\cos\left(fx+e\right)^{3}-4\,b\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{45\,d^{5}f}$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] -2/45*(5*b*cos(f*x + e)^5 - b*cos(f*x + e)^3 - 4*b*cos(f*x + e))*sqrt(b*sin
(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(d^5*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan{(fx+e)})^{\frac{3}{2}}}{(d\sec{(fx+e)})^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(9/2), x)
```

Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan{(fx+e)})^{\frac{3}{2}}}{(d\sec{(fx+e)})^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = -\frac{b\sqrt{\frac{d}{\cos(e+f\,x)}}\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}\,(21\,\cos{(3\,e+3\,f\,x)}-26\,\cos{(e+f\,x)}+5\,\cos{(5\,e+5\,f\,x)})}{360\,d^5\,f}$$

```
[In] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(9/2),x)
[Out] -(b*(d/cos(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1
/2)*(21*cos(3*e + 3*f*x) - 26*cos(e + f*x) + 5*cos(5*e + 5*f*x)))/(360*d^5*
f)
```

3.307 $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 208

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \frac{3b^{5/2}d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{32f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{3b^{5/2}d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{32f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{3bd^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f}$$

```
[Out] 3/32*b^(5/2)*d^3*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/
f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)-3/32*b^(5/2)*d^3*arctanh((b*sin
(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(
f*x+e))^(1/2)+1/4*b*(d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2)/f-3/16*b*d^2*
(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2)/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used

$= \{2691, 2693, 2696, 2644, 335, 304, 209, 212\}$

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \frac{3b^{5/2}d^3\sqrt{b\tan(e+fx)}\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{32f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}} - \frac{3b^{5/2}d^3\sqrt{b\tan(e+fx)}\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{32f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}} - \frac{3bd^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{16f} + \frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}$$

```
[In] Int[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (3*b^(5/2)*d^3*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(
32*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]]) - (3*b^(5/2)*d^3*ArcTanh[Sq
rt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(32*f*Sqrt[d*Sec[e + f*x]
]*Sqrt[b*Sin[e + f*x]]) - (3*b*d^2*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3
/2))/(16*f) + (b*(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2))/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f} - \frac{1}{8}\left(3b^2\right)\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx \\
&= -\frac{3bd^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f} \\
&\quad - \frac{1}{32}\left(3b^2d^2\right)\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx \\
&= -\frac{3bd^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f} \\
&\quad - \frac{\left(3b^2d^3\sqrt{b\tan(e+fx)}\right)\int \sec(e+fx)\sqrt{b\sin(e+fx)}\,dx}{32\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}
\end{aligned}
$$

$$= -\frac{3bd^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f} - \frac{\left(3bd^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{b^2}}\,dx, x, b\sin(e+fx)\right)}{32f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{3bd^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f} - \frac{\left(3bd^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{16f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{3bd^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f} - \frac{\left(3b^3d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{32f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{\left(3b^3d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{32f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= \frac{3b^{5/2}d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{32f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{3b^{5/2}d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{32f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{3bd^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f}$$

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.74

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \frac{d^4(b\tan(e+fx))^{5/2}\left(3\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\sec^2(e+fx)^{3/4} - 3\text{arctanh}\ldots\right.}{32f(d\sec(e\ldots}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (d^4*(b*Tan[e + f*x])^(5/2)*(3*ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(
1/4)]*(Sec[e + f*x]^2)^(3/4) - 3*ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2
)^(1/4)]*(Sec[e + f*x]^2)^(3/4) + 2*Sec[e + f*x]^2*(-3 + 4*Sec[e + f*x]^2)*
Tan[e + f*x]^(3/2)))/(32*f*(d*Sec[e + f*x])^(3/2)*Tan[e + f*x]^(5/2))
```

Maple [A] (verified)

Time = 169.05 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.44

method	result
default	$\frac{\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}\,b^2d^2\left(3\cos(fx+e)\operatorname{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)(\sin^2(fx+e))+3\cos(fx+e)\text{ a}}{}$

[In] int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/32/f*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)*b^2*d^2/(cos(f*x+e)-1)/(cos(f*x+e)+1)^2/(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(3*cos(f*x+e)*arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))*sin(f*x+e)^2+3*cos(f*x+e)*arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))*sin(f*x+e)^2+6*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)^3+6*sin(f*x+e)^2*tan(f*x+e)*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-8*sin(f*x+e)*tan(f*x+e)^2*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-8*tan(f*x+e)^3*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(168) = 336.

Time = 0.45 (sec) , antiderivative size = 852, normalized size of antiderivative = 4.10

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \text{Too large to display}$$

[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")

[Out] [1/256*(6*sqrt(-b*d)*b^2*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x + e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e)^3 + 3*sqrt(-b*d)*b^2*d^2*cos(f*x + e)^3*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)) + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*(3*b^2*d^2*cos(f*x + e)^2 - 4*b^2*d^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3), 1/256*(6*sqrt(b*d)*b^2*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e)^3 + 3*sqrt(b*d)*b^2*d^

```
2*cos(f*x + e)^3*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 + 8*(7*cos
(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x +
e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)) + 72*b
*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 - 8*cos(
f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*(3*b^2*d^2*cos(
f*x + e)^2 - 4*b^2*d^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x +
e))*sin(f*x + e))/(f*cos(f*x + e)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(5/2)*(b*tan(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \int (b\tan(e+f\,x))^{5/2}\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}\,dx$$

```
[In] int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(5/2),x)
[Out] int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(5/2), x)
```

3.308 $\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \frac{b^2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} - \frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f}$$

```
[Out] -1/2*b^2*d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*
EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(
f*x+e))^(1/2)/sin(f*x+e)^(1/2)+1/3*b*(d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3
/2)/f-1/2*b*d^2*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2691, 2693, 2696, 2721, 2719}

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \frac{b^2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}} - \frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}} + \frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}$$

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (b^2*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(2*f*Sqrt[d
*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) - (b*d^2*(b*Tan[e + f*x])^(3/2))/(2*f*Sq
```

```
rt[d*Sec[e + f*x]]) + (b*(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2))/(3*
f)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f} - \frac{1}{2}b^2\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx \\ &= -\frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f} \\ &\quad + \frac{1}{4}\left(b^2d^2\right)\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx\end{aligned}$$

$$= -\frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f} + \frac{\left(b^2d^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{b\sin(e+fx)}\,dx}{4\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f} + \frac{\left(b^2d^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{\sin(e+fx)}\,dx}{4\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

$$= \frac{b^2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} - \frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.74 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \frac{bd^2\left(-3+2\sec^2(e+fx)+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\text{se}}\right.}{6f\sqrt{d\sec(e+fx)}}$$

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2),x]
[Out] (b*d^2*(-3 + 2*Sec[e + f*x]^2 + Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f
*x]^2]*(Sec[e + f*x]^2)^(1/4))*(b*Tan[e + f*x])^(3/2))/(6*f*Sqrt[d*Sec[e +
f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.72

method	result
default	$\frac{(\sec^2(fx+e))\csc(fx+e)\left(-6\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\sqrt{i(\csc(fx+e)-\cot(fx+e))}E\left(\sqrt{-i(i-\right.}{}$

```
[In] int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/f*sec(f*x+e)^2*csc(f*x+e)*(-6*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I
*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*Ellipti
cE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)^4+3*(-I*(I-
cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(
f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/
2*2^(1/2))*cos(f*x+e)^4-6*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x
+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((-I*(I
-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)^3+3*(-I*(I-cot(f*x+e
)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-co
t(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2)
)*cos(f*x+e)^3+3*2^(1/2)*cos(f*x+e)^3-5*2^(1/2)*cos(f*x+e)^2+2*2^(1/2))*d*(
d*sec(f*x+e))^(1/2)*b^2*(b*tan(f*x+e))^(1/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.18

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \frac{3i\sqrt{-2i\,bd}b^2d\cos{(fx+e)}^2\operatorname{weierstrassZeta}(4,0,\operatorname{weierstrassPInverse}(4,0,\cos$$

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*I*sqrt(-2*I*b*d)*b^2*d*cos(f*x + e)^2*weierstrassZeta(4, 0, weierst
rassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) - 3*I*sqrt(2*I*b*d)*b^2*
d*cos(f*x + e)^2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x +
e) - I*sin(f*x + e))) - 2*(3*b^2*d*cos(f*x + e)^2 - 2*b^2*d)*sqrt(b*sin(f*x
 + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(3/2)*(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \int (b\tan(e+f\,x))^{5/2}\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}dx$$

```
[In] int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2),x)
[Out] int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2), x)
```

3.309 $\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 169

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \frac{3b^{5/2}d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{3b^{5/2}d\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{b\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2f}$$

```
[Out] 3/4*b^(5/2)*d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/f/(
d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)-3/4*b^(5/2)*d*arctanh((b*sin(f*x+e
))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e)
)^(1/2)+1/2*b*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2)/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2691, 2696, 2644, 335, 304, 209, 212}

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \frac{3b^{5/2}d\sqrt{b\tan(e+fx)}\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}} - \frac{3b^{5/2}d\sqrt{b\tan(e+fx)}\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}$$

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (3*b^(5/2)*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(4*
f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]]) - (3*b^(5/2)*d*ArcTanh[Sqrt[b*
```

```
Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(4*f*Sqrt[d*Sec[e + f*x]]*Sqrt
[b*Sin[e + f*x]]) + (b*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2))/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
```

```
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\text{integral} = \frac{b\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2f} - \frac{1}{4}\left(3b^2\right)\int\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx$$

$$= \frac{b\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2f} - \frac{\left(3b^2d\sqrt{b\tan(e+fx)}\right)\int\sec(e+fx)\sqrt{b\sin(e+fx)}\,dx}{4\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= \frac{b\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2f} - \frac{\left(3bd\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt{x}}{1-\frac{x^2}{b^2}}\,dx, x, b\sin(e+fx)\right)}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= \frac{b\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2f} - \frac{\left(3bd\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{1-\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{2f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= \frac{b\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2f} - \frac{\left(3b^3d\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{\left(3b^3d\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{b+x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= \frac{3b^{5/2}d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{3b^{5/2}d\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{b\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2f}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.77

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \frac{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\left(3\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)-3\text{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\right.}{4f\sqrt[4]{\sec^2(e+fx)}\tan^{\frac{5}{2}}(e+fx)}$$

[In] Integrate[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2),x]

[Out] (Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)*(3*ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] - 3*ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + 2*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x]^(3/2)))/(4*f*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x]^(5/2))

Maple [A] (verified)

Time = 12.08 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}\,b^2\left(2\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\sin^3(fx+e))-3\cos(fx+e)\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e)\right.\right.}{4f(\cos(fx+e)-1)(}$

[In] int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/4/f*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)*b^2/(cos(f*x+e)-1)/(cos(f*x+e)+1)^2/(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(2*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)^3-3*cos(f*x+e)*arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))*sin(f*x+e)^2-3*cos(f*x+e)*arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))*sin(f*x+e)^2+2*sin(f*x+e)^2*tan(f*x+e)*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(135) = 270$.

Time = 0.39 (sec) , antiderivative size = 788, normalized size of antiderivative = 4.66

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \left[\frac{6\sqrt{-bd}b^2\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{-bd}\sqrt{}}{4\left(bd\cos(fx+e)^2-bd-(bd\cos(fx+e)+bd)\sin(fx+e)\right)}\right.}{}\right.$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/32*(6*sqrt(-b*d)*b^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (co
s(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(
-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x +
 e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) + 3*sqrt
(-b*d)*b^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 + 8
*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos
(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)
) + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4
 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 16*b^2*sqr
t(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(f*
x + e)), 1/32*(6*sqrt(b*d)*b^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^
2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4
)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos
(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) +
3*sqrt(b*d)*b^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^
2 + 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) -
8*cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x
+ e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e
)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 16*b^2
*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*co
s(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(1/2)*(b*tan(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx=\int \sqrt{d\sec(fx+e)}(b\tan(fx+e))^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx=\int (b\tan(e+f\,x))^{5/2}\,\sqrt{\frac{d}{\cos(e+f\,x)}}\,dx$$

```
[In] int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(1/2),x)
[Out] int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(1/2), x)
```

3.310 $\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = -\frac{3b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}$$

```
[Out] 3*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ellipti
cE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))
^(1/2)/sin(f*x+e)^(1/2)+b*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2691, 2696, 2721, 2719}

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[In] Int[(b*Tan[e + f*x])^(5/2)/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-3*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Se
c[e + f*x]]*Sqrt[Sin[e + f*x]]) + (b*(b*Tan[e + f*x])^(3/2))/(f*Sqrt[d*Sec[
e + f*x]])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}} - \frac{1}{2}\left(3b^2\right)\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx \\
&= \frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}} - \frac{\left(3b^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{b\sin(e+fx)}\,dx}{2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} \\
&= \frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}} - \frac{\left(3b^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{\sin(e+fx)}\,dx}{2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} \\
&= -\frac{3b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} + \frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.63 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$
\int\frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = -\frac{b\left(-1+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\right)(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}
$$

```
[In] Integrate[(b*Tan[e + f*x])^(5/2)/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] -((b*(-1 + Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^
2)^(1/4))*(b*Tan[e + f*x])^(3/2))/(f*Sqrt[d*Sec[e + f*x]]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 471, normalized size of antiderivative = 5.35

method	result
default	$\frac{b^2\sqrt{b\tan(fx+e)}\left(6\cot(fx+e)\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,E\left(\sqrt{\right.}\right.}{}$

```
[In] int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*b^2*(b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2)*(6*cot(f*x+e)*(-I*(I-co
t(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f
*x+e)-csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2
*2^(1/2))-3*cot(f*x+e)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)
-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-c
ot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+6*csc(f*x+e)*(-I*(I-cot(f*x+e)+cs
c(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f
*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-3
*csc(f*x+e)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)
+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+c
sc(f*x+e)))^(1/2),1/2*2^(1/2))+2*2^(1/2)*cot(f*x+e)-3*csc(f*x+e)*2^(1/2)+se
c(f*x+e)*csc(f*x+e)*2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2\,b^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin\left(fx+e\right) - 3i\,\sqrt{-2i\,bd}b^2\text{weierstrassZeta}(4,0,\text{weiers}}{}$$

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x +
 e) - 3*I*sqrt(-2*I*b*d)*b^2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0
, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2*I*b*d)*b^2*weierstrassZeta(4
, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{\sqrt{d\sec(fx+e)}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^(5/2)/sqrt(d*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{\sqrt{d\sec(fx+e)}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(5/2)/sqrt(d*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{\frac{d}{\cos(e+fx)}}}\,dx$$

```
[In] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(1/2),x)
[Out] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(1/2), x)
```

3.311 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 168

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

```
[Out] -b^(5/2)*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/d/f/(d*s
ec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)+b^(5/2)*arctanh((b*sin(f*x+e))^(1/2)/
b^(1/2))*(b*tan(f*x+e))^(1/2)/d/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)
-2/3*b*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2690, 2696, 2644, 335, 304, 209, 212}

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{b^{5/2}\sqrt{b\tan(e+fx)}\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{b^{5/2}\sqrt{b\tan(e+fx)}\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}} - \frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

```
[In] Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((b^(5/2)*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(d*f*
Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])) + (b^(5/2)*ArcTanh[Sqrt[b*Sin[e
 + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(d*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Si
n[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(3*f*(d*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}+\frac{b^2\int\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx}{d^2}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}+\frac{\left(b^2\sqrt{b\tan(e+fx)}\right)\int\sec(e+fx)\sqrt{b\sin(e+fx)}\,dx}{d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}+\frac{\left(b\sqrt{b\tan(e+fx)}\right)\operatorname{Subst}\left(\int\frac{\sqrt{x}}{1-\frac{x^2}{b^2}}\,dx,x,b\sin(e+fx)\right)}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}+\frac{\left(2b\sqrt{b\tan(e+fx)}\right)\operatorname{Subst}\left(\int\frac{x^2}{1-\frac{x^4}{b^2}}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}+\frac{\left(b^3\sqrt{b\tan(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{b-x^2}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&\quad-\frac{\left(b^3\sqrt{b\tan(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{b+x^2}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&\quad+\frac{b^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.83

$$\int\frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx=$$
$$-\frac{\sqrt{d\sec(e+fx)}\left(3\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)-3\operatorname{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+\sqrt[4]{\sec^2(e+fx)}\sin(2(e+\ldots\right.}{3d^2f\sqrt[4]{\sec^2(e+fx)}\tan^{\frac{5}{2}}(e+fx)}$$

```
[In] Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/3*(Sqrt[d*Sec[e + f*x]]*(3*ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1
/4)] - 3*ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + (Sec[e + f*x]
^2)^(1/4)*Sin[2*(e + f*x)]*Sqrt[Tan[e + f*x]])*(b*Tan[e + f*x])^(5/2))/(d^2
*f*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x]^(5/2))
```

Maple [A] (verified)

Time = 10.71 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00

method	result
default	$-\frac{\sin(fx+e)\sqrt{b\tan(fx+e)}\left(3\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)+3\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,\operatorname{arctanh}\left(\right.\right.}{3f(\cos(fx+e)-1)\sqrt{d\sec(fx+e)}\,d}$

```
[In] int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*sin(f*x+e)*(b*tan(f*x+e))^(1/2)*(3*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))+3*(s
in(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*(cot(f*x+e)+csc(f*x+e)))+2*cos(f*x+e)-2)*b^2/(cos(f*x+e)-1)/(d*sec(f*x+e
))^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(138) = 276$.

Time = 0.59 (sec) , antiderivative size = 766, normalized size of antiderivative = 4.56

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \left[-\frac{16\,b^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+6\,b^2d\sqrt{-\frac{b}{d}}\arctan\left(\frac{(\cos(}{}\right.}{}\right.$$

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(16*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f
*x + e)*sin(f*x + e) + 6*b^2*d*sqrt(-b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*co
s(f*x + e)^2 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f
*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x +
e))/(b*cos(f*x + e)^2 - (b*cos(f*x + e) + b)*sin(f*x + e) - b)) - 3*b^2*d*s
qrt(-b/d)*log((b*cos(f*x + e)^4 - 72*b*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3
 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*
sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x + e)) + 28*(b*cos(f*x
+ e)^2 - 2*b)*sin(f*x + e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(
```

```
cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))/(d^2*f), -1/24*(16*b^2*sqrt(b*sin(f
*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 6*b^
2*d*sqrt(b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)
^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x
+ e)/cos(f*x + e))*sqrt(b/d)*sqrt(d/cos(f*x + e))/(b*cos(f*x + e)^2 + (b*co
s(f*x + e) + b)*sin(f*x + e) - b)) - 3*b^2*d*sqrt(b/d)*log((b*cos(f*x + e)^
4 - 72*b*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x
 + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqr
t(b/d)*sqrt(d/cos(f*x + e)) - 28*(b*cos(f*x + e)^2 - 2*b)*sin(f*x + e) + 72
*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e
) + 8)))/(d^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{5/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(3/2),x)
[Out] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(3/2), x)
```

3.312 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx=\frac{6b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

```
[Out] -6/5*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/d^2/f/(d*sec(
f*x+e))^(1/2)/sin(f*x+e)^(1/2)-2/5*b*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^
(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2690, 2696, 2721, 2719}

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx=\frac{6b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

```
[In] Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (6*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt
[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(5*f*(d
*Sec[e + f*x])^(5/2))
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
```

```
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}+\frac{(3b^2)\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{5d^2}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}+\frac{\left(3b^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{b\sin(e+fx)}\,dx}{5d^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}+\frac{\left(3b^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{\sin(e+fx)}\,dx}{5d^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}\\
&= \frac{6b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.84 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx = -\frac{b\left(1+\cos(2(e+fx))-2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\right)(b\tan(e+fx))^{3/}}{5d^2f\sqrt{d\sec(e+fx)}}$$

```
[In] Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/5*(b*(1 + Cos[2*(e + f*x)] - 2*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e +
 f*x]^2]*(Sec[e + f*x]^2)^(1/4))*(b*Tan[e + f*x])^(3/2))/(d^2*f*Sqrt[d*Sec[
e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 458, normalized size of antiderivative = 4.77

method	result
default	$\frac{\csc(fx+e)b^2\sqrt{b\tan(fx+e)}\left(-6\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\sqrt{i(\csc(fx+e)-\cot(fx+e))}E\left(\sqrt{\cdot}\right.\right.}{}$

```
[In] int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/f*csc(f*x+e)*b^2*(b*tan(f*x+e))^(1/2)*(-6*(-I*(I-cot(f*x+e)+csc(f*x+e))
)^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1
/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+
3*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)
*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))
^(1/2),1/2*2^(1/2))*cos(f*x+e)+2^(1/2)*cos(f*x+e)^3-6*(-I*(I-cot(f*x+e)+csc
(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(csc(f*x+e)-cot(f*x
+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+3*(
-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I
*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1
/2),1/2*2^(1/2))-4*2^(1/2)*cos(f*x+e)+3*2^(1/2))/(d*sec(f*x+e))^(1/2)/d^2*2
^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx = -\frac{2\,b^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)^2\sin\left(fx+e\right) - 3i\,\sqrt{-2i\,bd}b^2\text{weierstrassZeta}(4,0,\text{weierstrassPInvers}$$

[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] -1/5*(2*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2*sin(f*x + e) - 3*I*sqrt(-2*I*b*d)*b^2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2*I*b*d)*b^2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d^3*f)

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

[In] integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan\left(fx+e\right))^{\frac{5}{2}}}{(d\sec\left(fx+e\right))^{\frac{5}{2}}}\,dx$$

[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(5/2), x)

Giac [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{5/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(5/2),x)
[Out] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(5/2), x)
```

3.313 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 34

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2(b\tan(e+fx))^{7/2}}{7bf(d\sec(e+fx))^{7/2}}$$

```
[Out] 2/7*(b*tan(f*x+e))^(7/2)/b/f/(d*sec(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2685}

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2(b\tan(e+fx))^{7/2}}{7bf(d\sec(e+fx))^{7/2}}$$

```
[In] Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*(b*Tan[e + f*x])^(7/2))/(7*b*f*(d*Sec[e + f*x])^(7/2))
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2(b\tan(e+fx))^{7/2}}{7bf(d\sec(e+fx))^{7/2}}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2b^2\sin^3(e+fx)\sqrt{b\tan(e+fx)}}{7d^3f\sqrt{d\sec(e+fx)}}$$

```
[In] Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(7/2),x]
[Out] (2*b^2*Sin[e + f*x]^3*Sqrt[b*Tan[e + f*x]])/(7*d^3*f*Sqrt[d*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{2(\sin^3(fx+e))\sqrt{b\tan(fx+e)}\,b^2}{7f\,d^3\sqrt{d\sec(fx+e)}}$	40

```
[In] int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/7/f*sin(f*x+e)^3*(b*tan(f*x+e))^(1/2)*b^2/d^3/(d*sec(f*x+e))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(28) = 56.

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = -\frac{2\left(b^2\cos\left(fx+e\right)^3-b^2\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin\left(fx+e\right)}{7\,d^4f}$$

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
[Out] -2/7*(b^2*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x +
e))*sqrt(d/cos(f*x + e))*sin(f*x + e)/(d^4*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(7/2), x)
```

Giac [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.12

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{b^2\sqrt{\frac{d}{\cos(e+f\,x)}}\,\left(2\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{28\,d^4\,f}$$

```
[In] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(7/2),x)
[Out] (b^2*(d/cos(e + f*x))^(1/2)*(2*sin(2*e + 2*f*x) - sin(4*e + 4*f*x))*((b*sin
(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(28*d^4*f)
```

3.314 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{4b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} - \frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}} + \frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}}$$

```
[Out] -4/15*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ell
ipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/d^4/f/(d*sec
(f*x+e))^(1/2)/sin(f*x+e)^(1/2)-2/9*b*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))
^(9/2)+2/15*b*(b*tan(f*x+e))^(3/2)/d^2/f/(d*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2690, 2692, 2696, 2721, 2719}

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{4b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}} - \frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

```
[In] Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (4*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(15*d^4*f*Sqr
t[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(9*f*(
```

```
d*Sec[e + f*x])^(9/2)) + (2*b*(b*Tan[e + f*x])^(3/2))/(15*d^2*f*(d*Sec[e +
f*x])^(5/2))
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}} + \frac{b^2\int\frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx}{3d^2}$$

$$= -\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}} + \frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}} + \frac{(2b^2)\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{15d^4}$$

$$= -\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}} + \frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}} + \frac{\left(2b^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{b\sin(e+fx)}\,dx}{15d^4\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}} + \frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}} + \frac{\left(2b^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{\sin(e+fx)}\,dx}{15d^4\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

$$= \frac{4b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} - \frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}} + \frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.69

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{b^3\left(1-5\cos(2(e+fx))+4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)\right)}{45d^4f\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (b^3*(1 - 5*Cos[2*(e + f*x)] + 4*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e +
f*x]^2]*(Sec[e + f*x]^2)^(5/4))*Sin[e + f*x]^2)/(45*d^4*f*Sqrt[d*Sec[e + f*
x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.60

method	result
default	$\frac{\csc(fx+e)\left(5\left(\cos^5(fx+e)\right)\sqrt{2}-12\sqrt{i(-i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,E\right.}{}$

```
[In] int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/45/f*csc(f*x+e)*(5*cos(f*x+e)^5*2^(1/2)-12*(I*(-I+cot(f*x+e)-csc(f*x+e)))
^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1
/2)*EllipticE((I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+
6*(I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)
*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((I*(-I+cot(f*x+e)-csc(f*x+e))
)^(1/2),1/2*2^(1/2))*cos(f*x+e)-8*2^(1/2)*cos(f*x+e)^3-12*(I*(-I+cot(f*x+e)
```

```
-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-cs
c(f*x+e)))^(1/2)*EllipticE((I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2),1/2*2^(1/2)
)+6*(I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/
2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((I*(-I+cot(f*x+e)-csc(f*x+e
)))^(1/2),1/2*2^(1/2))-3*2^(1/2)*cos(f*x+e)+6*2^(1/2))*(b*tan(f*x+e))^(1/2)
*b^2/(d*sec(f*x+e))^(1/2)/d^4*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx =$$

$$-\frac{2\left(-3i\sqrt{-2i\,bd}b^2\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))+3i\sqrt{2i\,bd}\right.}{}$$

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] -2/45*(-3*I*sqrt(-2*I*b*d)*b^2*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2*I*b*d)*b^2*weierstrassZeta
(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))) + (5*b^2*c
os(f*x + e)^4 - 3*b^2*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqr
t(d/cos(f*x + e))*sin(f*x + e))/(d^5*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(9/2), x)
```

Giac [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{5/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

```
[In] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(9/2),x)
[Out] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(9/2), x)
```

3.315 $\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 178

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{3d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{3d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf}$$

```
[Out] 3/4*d^3*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*
x+e))^(1/2)/f/b^(1/2)/(b*tan(f*x+e))^(1/2)+3/4*d^3*arctanh((b*sin(f*x+e))^(
1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/f/b^(1/2)/(b*tan(f*
x+e))^(1/2)+1/2*d^2*(d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2)/b/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2693, 2696, 2644, 335, 218, 212, 209}

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{3d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{3d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

```
[In] Int[(d*Sec[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (3*d^3*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin
[e + f*x]])/(4*Sqrt[b]*f*Sqrt[b*Tan[e + f*x]]) + (3*d^3*ArcTanh[Sqrt[b*Sin[
e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(4*Sqrt[b]*f*
Sqrt[b*Tan[e + f*x]]) + (d^2*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(
2*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
```

```
*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\text{integral} = \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf} + \frac{1}{4}(3d^2)\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$$

$$= \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf} + \frac{\left(3d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}\,dx}{4\sqrt{b\tan(e+fx)}}$$

$$= \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf} + \frac{\left(3d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{b^2}\right)}\,dx, x, b\sin(e+fx)\right)}{4bf\sqrt{b\tan(e+fx)}}$$

$$= \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf} + \frac{\left(3d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{2bf\sqrt{b\tan(e+fx)}}$$

$$= \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf} + \frac{\left(3d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{4f\sqrt{b\tan(e+fx)}} + \frac{\left(3d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{4f\sqrt{b\tan(e+fx)}}$$

$$= \frac{3d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{3d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.75

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{d^2(d\sec(e+fx))^{3/2}\left(3\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+3\text{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\right.}{4f\sec^2(e+fx)^{3/4}\sqrt{b\tan(e+f}}$$

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (d^2*(d*Sec[e + f*x])^(3/2)*(3*ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(
1/4)] + 3*ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + 2*(Sec[e + f
*x]^2)^(3/4)*Sqrt[Tan[e + f*x]])*Sqrt[Tan[e + f*x]])/(4*f*(Sec[e + f*x]^2)^
(3/4)*Sqrt[b*Tan[e + f*x]])
```

Maple [A] (verified)

Time = 12.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.14

method	result
default	$-\frac{\sqrt{d\sec(fx+e)}\,d^3\left(3\sin(fx+e)\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)-3\sin(fx+e)\,\text{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot\right.\right.}{4f(\cos(fx+e)+1)\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,\sqrt{b\,t}}$

```
[In] int((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*(d*sec(f*x+e))^(1/2)*d^3/(cos(f*x+e)+1)/(sin(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)/(b*tan(f*x+e))^(1/2)*(3*sin(f*x+e)*arctan((sin(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))-3*sin(f*x+e)*arctanh((sin(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))-2*tan(f*x+e)*(sin(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-2*tan(f*x+e)*sec(f*x+e)*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(144) = 288$.

Time = 0.42 (sec) , antiderivative size = 782, normalized size of antiderivative = 4.39

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \left[-\frac{6\,bd^3\sqrt{-\frac{d}{b}}\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos\right.}{4\left(d\cos(fx+e)^2-(d\cos(fx+e)+d)\sin(fx\right.}\right.}{}\right.$$

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(6*b*d^3*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 -
(cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sq
rt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x
+ e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d))*cos(f*x + e) - 3*b*d^3*sqr
t(-d/b)*cos(f*x + e)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 + 8*(7*cos
(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x +
e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e)) + 28*
(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x +
 e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*d^3*sqrt(b*sin(f*x +
 e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(b*f*cos(f*x + e)), 1/32*(6*b*d^3*s
qrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 +
6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/
cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^2 + (d*cos(f*x
 + e) + d)*sin(f*x + e) - d))*cos(f*x + e) + 3*b*d^3*sqrt(d/b)*cos(f*x + e)
*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f
*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x +
 e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(f*x + e)^2 - 2
*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x +
e)^2 - 2)*sin(f*x + e) + 8)) + 16*d^3*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqr
t(d/cos(f*x + e)))/(b*f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(7/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{\sqrt{b\tan(e+f\,x)}}\,dx$$

```
[In] int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(1/2),x)
[Out] int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(1/2), x)
```

3.316 $\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 92

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{d^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}} + \frac{d^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{bf}$$

```
[Out] -d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
F(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/
f/(b*tan(f*x+e))^(1/2)+d^2*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/b/f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2693, 2696, 2721, 2720}

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf} + \frac{d^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

```
[In] Int[(d*Sec[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (d^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x
]])/(f*Sqrt[b*Tan[e + f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x
]])/(b*f)
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{d^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{bf} + \frac{1}{2}d^2\int\frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$$

$$= \frac{d^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{bf} + \frac{\left(d^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int\frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{2\sqrt{b\tan(e+fx)}}$$

$$= \frac{d^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{bf} + \frac{\left(d^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int\frac{1}{\sqrt{\sin(e+fx)}}\,dx}{2\sqrt{b\tan(e+fx)}}$$

$$= \frac{d^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}} + \frac{d^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{bf}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.66 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{d^2\sqrt{d\sec(e+fx)}\left(\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e\right.}{f\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (d^2*Sqrt[d*Sec[e + f*x]]*(Cos[e + f*x]*Hypergeometric2F1[1/4, 3/4, 5/4, -T
an[e + f*x]^2]*(Sec[e + f*x]^2)^(3/4)*Sin[e + f*x] + Tan[e + f*x]))/(f*Sqrt
[b*Tan[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.60

method	result
default	$\frac{\sqrt{d\sec(fx+e)}\,d^2\left(i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e\right.\right.}{}$

```
[In] int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(d*sec(f*x+e))^(1/2)*d^2/(b*tan(f*x+e))^(1/2)*(I*(-I*(I-cot(f*x+e)+cs
c(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f*
x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*co
s(f*x+e)+I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+
I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc
(f*x+e)))^(1/2),1/2*2^(1/2))+tan(f*x+e)*2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.09

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{-2i\,bd}d^2\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e))+\sqrt{2i\,bd}d^2\text{weier}}{2\,bf}$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-2*I*b*d)*d^2*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x
+ e)) + sqrt(2*I*b*d)*d^2*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*
x + e)) + 2*d^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(b*
f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

```
[In] int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2),x)
[Out] int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2), x)
```

3.317 $\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{d\,\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{\sqrt{b}f\sqrt{b\tan(e+fx)}}$$

```
[Out] d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^
(1/2)/f/b^(1/2)/(b*tan(f*x+e))^(1/2)+d*arctanh((b*sin(f*x+e))^(1/2)/b^(1/2)
)*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/f/b^(1/2)/(b*tan(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2696, 2644, 335, 218, 212, 209}

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f\sqrt{b\tan(e+fx)}}$$

```
[In] Int[(d*Sec[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e +
 f*x]])/(Sqrt[b]*f*Sqrt[b*Tan[e + f*x]]) + (d*ArcTanh[Sqrt[b*Sin[e + f*x]]/
```

```
Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(Sqrt[b]*f*Sqrt[b*Tan[e
 + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\text{integral} = \frac{\left(d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int\frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}\,dx}{\sqrt{b\tan(e+fx)}}$$

$$= \frac{\left(d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{b^2}\right)}\,dx, x, b\sin(e+fx)\right)}{bf\sqrt{b\tan(e+fx)}}$$

$$= \frac{\left(2d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{bf\sqrt{b\tan(e+fx)}}$$

$$= \frac{\left(d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{f\sqrt{b\tan(e+fx)}} + \frac{\left(d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{f\sqrt{b\tan(e+fx)}}$$

$$= \frac{d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{d\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{\sqrt{b}f\sqrt{b\tan(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\left(\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+\text{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\right)(d\sec(e+fx))^{3/2}\sqrt{\cdots}}{f\sec^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
[Out] ((ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + ArcTanh[Sqrt[Tan[e +
f*x]]/(Sec[e + f*x]^2)^(1/4)])*(d*Sec[e + f*x])^(3/2)*Sqrt[Tan[e + f*x]])/(
f*(Sec[e + f*x]^2)^(3/4)*Sqrt[b*Tan[e + f*x]])
```

Maple [A] (verified)

Time = 10.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

method	result
default	$-\frac{\sin(fx+e)\left(\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)-\text{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)\right)\sqrt{d\sec\cdots}}{f(\cos(fx+e)+1)\sqrt{b\tan(fx+e)}\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}}$

```
[In] int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*sin(f*x+e)*(arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc
(f*x+e)))-arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e
))))*(d*sec(f*x+e))^(1/2)*d/(cos(f*x+e)+1)/(b*tan(f*x+e))^(1/2)/(sin(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(107) = 214.

Time = 0.39 (sec) , antiderivative size = 653, normalized size of antiderivative = 4.98

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \left[-\frac{2\,d\sqrt{-\frac{d}{b}}\arctan\left(\frac{\left(\cos(fx+e)^3-5\,\cos(fx+e)^2-\left(\cos(fx+e)^2+6\,\cos(fx+e)+4\right)\sin(fx+e)-2\,\cos(fx\cdots}{4\left(d\cos(fx+e)^2-(d\cos(fx+e)+d)\sin(fx+e)-\cdots\right.}\right.}{\cdots}\right.$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*d*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (cos(
f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*
sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^
2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d)) - d*sqrt(-d/b)*log((d*cos(f*x +
 e)^4 - 72*d*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos
(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))
*sqrt(-d/b)*sqrt(d/cos(f*x + e)) + 28*(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e)
 + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*
x + e) + 8)))/f, 1/8*(2*d*sqrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x
+ e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e
) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*
cos(f*x + e)^2 + (d*cos(f*x + e) + d)*sin(f*x + e) - d)) + d*sqrt(d/b)*log(
(d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*x +
e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/c
os(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(f*x + e)^2 - 2*d)*s
in(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2
- 2)*sin(f*x + e) + 8)))/f]
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec{(e+fx)})^{\frac{3}{2}}}{\sqrt{b\tan{(e+fx)}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
[Out] Integral((d*sec(e + f*x))**(3/2)/sqrt(b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec{(fx+e)})^{\frac{3}{2}}}{\sqrt{b\tan{(fx+e)}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec{(fx+e)})^{\frac{3}{2}}}{\sqrt{b\tan{(fx+e)}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

```
[In] int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(1/2),x)
[Out] int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(1/2), x)
```

3.318 $\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 55

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx=\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/f/
(b*tan(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2696, 2721, 2720}

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx=\frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

```
[In] Int[Sqrt[d*Sec[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]
)/(f*Sqrt[b*Tan[e + f*x]])
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int\frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{\sqrt{b\tan(e+fx)}}$$

$$= \frac{\left(\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int\frac{1}{\sqrt{\sin(e+fx)}}\,dx}{\sqrt{b\tan(e+fx)}}$$

$$= \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.54 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int\frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\sin(e+fx)}{f\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
[Out] (2*d*Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(3/
4)*Sin[e + f*x])/(f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.36

method	result
default	$\frac{i(\cos(fx+e)+1)F\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))},\frac{\sqrt{2}}{2}\right)\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(i-\cot(f}{f\sqrt{b\tan(fx+e)}}$

[In] int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] I/f*(cos(f*x+e)+1)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/
2))*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)
(-I(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(d*sec(f*x+e))^(1/2)*2^(1/2)/(b*tan(f
*x+e))^(1/2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{-2i\,bd}\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))+\sqrt{2i\,bd}\text{weierstrassPInverse}(4,0,\cos(fx}{bf}$$

[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")

[Out] (sqrt(-2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) +
sqrt(2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(b*
f)

Sympy [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{d\sec{(e+fx)}}}{\sqrt{b\tan{(e+fx)}}}\,dx$$

[In] integrate((d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)

[Out] Integral(sqrt(d*sec(e + f*x))/sqrt(b*tan(e + f*x)), x)

Maxima [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

```
[In] int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2),x)
[Out] int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2), x)
```

3.319 $\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

```
[Out] 2*(b*tan(f*x+e))^(1/2)/b/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2685}

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
[Out] (2*Sqrt[b*Tan[e + f*x]])/(b*f*Sqrt[d*Sec[e + f*x]])
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx=\frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
[Out] (2*Sqrt[b*Tan[e + f*x]])/(b*f*Sqrt[d*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2\tan(fx+e)}{f\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}}$	32
risch	$-\frac{i\sqrt{2}\,(\mathrm{e}^{2i(fx+e)}-1)}{\sqrt{\frac{d\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}\,(\mathrm{e}^{2i(fx+e)}+1)\sqrt{-\frac{ib\left(\mathrm{e}^{2i(fx+e)}-1\right)}{\mathrm{e}^{2i(fx+e)}+1}}\,f}$	90

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/f*tan(f*x+e)/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx=\frac{2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)}{bdf}$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
[Out] 2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)/(b*d*
f)
```

Sympy [A] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \begin{cases} \frac{2\tan(e+fx)}{f\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}} & \text{for } f\neq 0 \\ \frac{x}{\sqrt{b\tan(e)}\sqrt{d\sec(e)}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
[Out] Piecewise((2*tan(e + f*x)/(f*sqrt(b*tan(e + f*x))*sqrt(d*sec(e + f*x))), Ne
(f, 0)), (x/(sqrt(b*tan(e))*sqrt(d*sec(e))), True))
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\sec(fx+e)}\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\sec(fx+e)}\sqrt{b\tan(fx+e)}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sin(e+fx)\sqrt{\frac{d}{\cos(e+fx)}}}{d\,f\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}$$

```
[In] int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(1/2)),x)
[Out] (2*sin(e + f*x)*(d/cos(e + f*x))^(1/2))/(d*f*((b*sin(2*e + 2*f*x))/(cos(2*e
 + 2*f*x) + 1))^(1/2))
```

3.320 $\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}} + \frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

```
[Out] -4/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
F(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/
d^2/f/(b*tan(f*x+e))^(1/2)+2/3*(b*tan(f*x+e))^(1/2)/b/f/(d*sec(f*x+e))^(3/2
)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2692, 2696, 2721, 2720}

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{4\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}} + \frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (4*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]
)/(3*d^2*f*Sqrt[b*Tan[e + f*x]]) + (2*Sqrt[b*Tan[e + f*x]])/(3*b*f*(d*Sec[e
 + f*x])^(3/2))
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}} + \frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{3d^2} \\
&= \frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}} + \frac{\left(2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{3d^2\sqrt{b\tan(e+fx)}} \\
&= \frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}} + \frac{\left(2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d^2\sqrt{b\tan(e+fx)}} \\
&= \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}} + \frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.74 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\left(1+2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e+fx\right.}{3bf(d\sec(e+fx))^{3/2}}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
[Out] (2*(1 + 2*Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^2
)^(3/4))*Sqrt[b*Tan[e + f*x]])/(3*b*f*(d*Sec[e + f*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.52

method	result
default	$\frac{\left(2i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e)}\right.\right.}{}$

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3/f/(b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2)/d*(2*I*(-I*(I-cot(f*x+e)+cs
c(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(csc(f*x+e)-cot(f*
x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+2*
I*sec(f*x+e)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x
+e)))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+c
sc(f*x+e)))^(1/2),1/2*2^(1/2))+2^(1/2)*sin(f*x+e))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\left(\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)^2+\sqrt{-2i\,bd}\text{weierstrassPIn}\right.}{}$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2
+ sqrt(-2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) +
 sqrt(2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(b
*d^2*f)
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(e+fx\right)}\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
[Out] Integral(1/(sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx=\int \frac{1}{\sqrt{b\tan\left(e+f\,x\right)}\,{\left(\frac{d}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

```
[In] int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2)),x)
[Out] int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2)), x)
```

3.321 $\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}+\frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}$$

```
[Out] 2/5*(b*tan(f*x+e))^(1/2)/b/f/(d*sec(f*x+e))^(5/2)+8/5*(b*tan(f*x+e))^(1/2)/
b/d^2/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2692, 2685}

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (2*Sqrt[b*Tan[e + f*x]])/(5*b*f*(d*Sec[e + f*x])^(5/2)) + (8*Sqrt[b*Tan[e +
 f*x]])/(5*b*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}+\frac{4\int\frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx}{5d^2}\\ &= \frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}+\frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int\frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx=\frac{(9+\cos(2(e+fx)))\sqrt{d\sec(e+fx)}\sin(e+fx)}{5d^3f\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
[Out] ((9 + Cos[2*(e + f*x)])*Sqrt[d*Sec[e + f*x]]*Sin[e + f*x])/(5*d^3*f*Sqrt[b*
Tan[e + f*x]])
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{2\tan(fx+e)(\cos^2(fx+e)+4)}{5f\sqrt{b\tan(fx+e)}\,\sqrt{d\sec(fx+e)}\,d^2}$	45

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5/f*tan(f*x+e)*(cos(f*x+e)^2+4)/(b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2)
/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\left(\cos\left(fx+e\right)^{3}+4\,\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{5\,bd^{3}f}$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
[Out] 2/5*(cos(f*x + e)^3 + 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqr
t(d/cos(f*x + e))/(b*d^3*f)
```

Sympy [A] (verification not implemented)

Time = 57.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \begin{cases} \frac{8\tan^{3}\left(e+fx\right)}{5f\sqrt{b\tan\left(e+fx\right)}\left(d\sec\left(e+fx\right)\right)^{\frac{5}{2}}}+\frac{2\tan\left(e+fx\right)}{f\sqrt{b\tan\left(e+fx\right)}\left(d\sec\left(e+fx\right)\right)^{\frac{5}{2}}} & \text{for}\ f\neq 0 \\ \frac{x}{\sqrt{b\tan\left(e\right)}\left(d\sec\left(e\right)\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
[Out] Piecewise((8*tan(e + f*x)**3/(5*f*sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(5
/2)) + 2*tan(e + f*x)/(f*sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(5/2)), Ne(
f, 0)), (x/(sqrt(b*tan(e))*(d*sec(e))**(5/2)), True))
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(1/((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)

Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{(17\sin(e+fx)+\sin(3e+3fx))\sqrt{\frac{d}{\cos(e+fx)}}}{10\,d^3 f\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}$$

[In] int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(5/2)),x)

[Out] ((17*sin(e + f*x) + sin(3*e + 3*f*x))*(d/cos(e + f*x))^(1/2))/(10*d^3*f*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))

3.322 $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{b^{3/2}f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{b^{3/2}f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

```
[Out] -2*d^2*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)-d^3*arctan((b*sin(f*x+
e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/b^(3/2)/f/(d*sec(f*x+e))^(1/2)/(b*s
in(f*x+e))^(1/2)+d^3*arctanh((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(
1/2)/b^(3/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2688, 2696, 2644, 335, 304, 209, 212}

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{d^3\sqrt{b\tan(e+fx)}\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{d^3\sqrt{b\tan(e+fx)}\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}} - \frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

```
[In] Int[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*d^2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]]) - (d^3*ArcTan[Sqrt
[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(b^(3/2)*f*Sqrt[d*Sec[e + f
*x]]*Sqrt[b*Sin[e + f*x]]) + (d^3*ArcTanh[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqr
t[b*Tan[e + f*x]])/(b^(3/2)*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}+\frac{d^2\int\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx}{b^2}\\
&= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}+\frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\int\sec(e+fx)\sqrt{b\sin(e+fx)}\,dx}{b^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}+\frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt{x}}{1-\frac{x^2}{b^2}}\,dx,x,b\sin(e+fx)\right)}{b^3f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}+\frac{\left(2d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{1-\frac{x^4}{b^2}}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{b^3f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}+\frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{b-x^2}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{bf\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&\quad-\frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{b+x^2}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{bf\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{b^{3/2}f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&\quad+\frac{d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{b^{3/2}f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.69

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{d(d\sec(e+fx))^{3/2}\left(-2\sin(e+fx)+\frac{\left(-\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\right)\cos(e+fx)\tan^{3/2}(e+fx)}{\sqrt[4]{\sec^2(e+fx)}}\right)}{f(b\tan(e+fx))^{3/2}}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (d*(d*Sec[e + f*x])^(3/2)*(-2*Sin[e + f*x] + ((-ArcTan[Sqrt[Tan[e + f*x]]/(
Sec[e + f*x]^2)^(1/4)] + ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)]
)*Cos[e + f*x]*Tan[e + f*x]^(3/2))/(Sec[e + f*x]^2)^(1/4)))/(f*(b*Tan[e + f
*x])^(3/2))
```

Maple [A] (verified)

Time = 12.63 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.40

method	result
default	$-\frac{d^2\sqrt{d\sec(fx+e)}\left(\cot(fx+e)\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)+\cot(fx+e)\operatorname{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(f\ldots\right.\right.}{\ldots}$

```
[In] int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*d^2*(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/(sin(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)/b*(cot(f*x+e)*arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f
*x+e)+csc(f*x+e)))+cot(f*x+e)*arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(
cot(f*x+e)+csc(f*x+e)))+2*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-csc(f*x+e)*ar
ctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))-csc(f*x+e
)*arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(143) = 286.

Time = 0.41 (sec) , antiderivative size = 794, normalized size of antiderivative = 4.64

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \left[-\frac{2\,bd^2\sqrt{-\frac{d}{b}}\arctan\left(\frac{\left(\cos(fx+e)^3-5\,\cos(fx+e)^2-\left(\cos(fx+e)^2+6\,\cos(fx+e)+4\right)\sin(fx+e)-2\,\cos\right.}{4\left(d\cos(fx+e)^2-(d\cos(fx+e)+d)\sin(fx+\right.}\right.}{}\right.$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*b*d^2*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (
cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqr
t(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x +
 e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d))*sin(f*x + e) - b*d^2*sqrt(-
d/b)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 - (c
os(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f
*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e)) + 28*(d*cos(f*x + e)^
2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f
*x + e)^2 - 2)*sin(f*x + e) + 8))*sin(f*x + e) + 16*d^2*sqrt(b*sin(f*x + e)
/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin(f*x + e)), -1/
8*(2*b*d^2*sqrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f
*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*s
in(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^2
+ (d*cos(f*x + e) + d)*sin(f*x + e) - d))*sin(f*x + e) - b*d^2*sqrt(d/b)*lo
g((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*x
+ e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)
/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(f*x + e)^2 - 2*d)
*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^
2 - 2)*sin(f*x + e) + 8))*sin(f*x + e) + 16*d^2*sqrt(b*sin(f*x + e)/cos(f*x
 + e))*sqrt(d/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{(b\tan(e+f\,x))^{3/2}}\,dx$$

```
[In] int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(3/2),x)
[Out] int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(3/2), x)
```

3.323 $\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2*d^2/b/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)+2*d^2*(sin(1/2*e+1/4*P
i+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/
2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/b^2/f/(d*sec(f*x+e))^(1/2)/sin(f*x+e)^
(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2688, 2696, 2721, 2719}

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}} - \frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[In] Int[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*d^2)/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (2*d^2*EllipticE
[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[d*Sec[e + f*x]]*S
qrt[Sin[e + f*x]])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{d^2\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{b^2} \\
&= -\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\left(d^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{b\sin(e+fx)}\,dx}{b^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} \\
&= -\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\left(d^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{\sin(e+fx)}\,dx}{b^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} \\
&= -\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.82 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2d^2\left(3+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\tan^2(e+fx)\right)}{3bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*d^2*(3 + Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^2]*(Sec[e + f*x
]^2)^(1/4)*Tan[e + f*x]^2))/(3*b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]
])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 365, normalized size of antiderivative = 3.76

method	result
default	$\frac{\csc(fx+e)\left(2\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{2}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,E\left(\sqrt{-i(i-\cot(fx+e)+\right.\right.}{}$

```
[In] int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*csc(f*x+e)*(2*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*2^(1/2)*(-I*(cot(f*x
+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((-I*(I
-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-(-I*(I-cot(f*x+e)+csc(f*x+e)))^
(1/2)*2^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e
)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-csc(f
*x+e)^2*(1-cos(f*x+e))^2-1)*(-d*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)/(csc(f*x+
e)^2*(1-cos(f*x+e))^2-1))^(3/2)*(1-cos(f*x+e))/(csc(f*x+e)^2*(1-cos(f*x+e))
^2+1)^2/(-b/(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(csc(f*x+e)-cot(f*x+e)))^(3/2
)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.34

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\,d\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)^2 + i\sqrt{-2i\,bd}d\sin(fx+e)\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}\ldots}{\ldots}$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2
 + I*sqrt(-2*I*b*d)*d*sin(f*x + e)*weierstrassZeta(4, 0, weierstrassPInvers
e(4, 0, cos(f*x + e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*d*sin(f*x + e)*we
ierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)
)))/(b^2*f*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(e+fx))^{\frac{3}{2}}}{(b\tan(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**(3/2)/(b*tan(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{(b\tan(e+f\,x))^{3/2}}\,dx$$

```
[In] int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2),x)
[Out] int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2), x)
```

3.324 $\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

```
[Out] -2*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2685}

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

```
[In] Int[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
[Out] (-2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]])
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
[Out] (-2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]])
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{2\sqrt{d\sec(fx+e)}}{bf\sqrt{b\tan(fx+e)}}$	29

```
[In] int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)}{b^2f\sin(fx+e)}$$

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)/(b^2
*f*sin(f*x + e))
```

Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \begin{cases} -\frac{2\sqrt{d\sec(e+fx)}\tan(e+fx)}{f(b\tan(e+fx))^{\frac{3}{2}}} & \text{for } f\neq 0 \\ \frac{x\sqrt{d\sec(e)}}{(b\tan(e))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
[Out] Piecewise((-2*sqrt(d*sec(e + f*x))*tan(e + f*x)/(f*(b*tan(e + f*x))**(3/2))
, Ne(f, 0)), (x*sqrt(d*sec(e))/(b*tan(e))**(3/2), True))
```

Maxima [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{d\sec(fx+e)}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{d\sec(fx+e)}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{\frac{d}{\cos(e+f\,x)}}}{b\,f\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}$$

```
[In] int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(3/2),x)
[Out] -(2*(d/cos(e + f*x))^(1/2))/(b*f*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) +
1))^(1/2))
```

3.325 $\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = -\frac{2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{4E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2/b/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)+4*(sin(1/2*e+1/4*Pi+1/2*f*
x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2
^(1/2))*(b*tan(f*x+e))^(1/2)/b^2/f/(d*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2689, 2696, 2721, 2719}

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = -\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}} - \frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -2/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (4*EllipticE[(e - Pi/2
 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e
+ f*x]])
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{b^2}\\
&= -\frac{2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\left(2\sqrt{b\tan(e+fx)}\right)\int\sqrt{b\sin(e+fx)}\,dx}{b^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= -\frac{2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\left(2\sqrt{b\tan(e+fx)}\right)\int\sqrt{\sin(e+fx)}\,dx}{b^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}\\
&= -\frac{2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{4E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.64 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/4}\sin^2(e+fx)\right)}{3bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (-2*(3 + 2*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^
2)^(5/4)*Sin[e + f*x]^2))/(3*b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 365, normalized size of antiderivative = 4.01

method	result
default	$\frac{\csc(fx+e)(1-\cos(fx+e))\left(4\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{2}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,E\left(\sqrt{-i}\right.\right.}{f\left(-\frac{b(\csc(fx}{\left(\csc^2(fx+e)\right.}\right.}$

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*csc(f*x+e)*(1-cos(f*x+e))*(4*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*2^(1/
2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*E
llipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-2*(-I*(I-cot(f*x
+e)+csc(f*x+e)))^(1/2)*2^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc
(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1
/2*2^(1/2))-3*csc(f*x+e)^2*(1-cos(f*x+e))^2-1)/(-b/(csc(f*x+e)^2*(1-cos(f*x
+e))^2-1)*(csc(f*x+e)-cot(f*x+e)))^(3/2)/(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)^
2/(-d*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)/(csc(f*x+e)^2*(1-cos(f*x+e))^2-1))^
(1/2)*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)^{2}+i\sqrt{-2i\,bd}\sin\left(fx+e\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(\right.}{}$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -2*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2 +
 I*sqrt(-2*I*b*d)*sin(f*x + e)*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cos(f*x + e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*sin(f*x + e)*weierstr
assZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b
^2*d*f*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\tan(e+fx))^{\frac{3}{2}}\sqrt{d\sec(e+fx)}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((b*tan(e + f*x))**(3/2)*sqrt(d*sec(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{d\sec(fx+e)}\,(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(e+f\,x\right)\right)^{3/2}\,\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2)),x)
[Out] int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2)), x)
```

3.326 $\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{2}{3bf(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} - \frac{8\sqrt{d\sec(e+fx)}}{3bd^2f\sqrt{b\tan(e+fx)}}$$

```
[Out] 2/3/b/f/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2)-8/3*(d*sec(f*x+e))^(1/2)/
b/d^2/f/(b*tan(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2689, 2685}

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{8(b\tan(e+fx))^{3/2}}{3b^3f(d\sec(e+fx))^{3/2}} - \frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -2/(b*f*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]) - (8*(b*Tan[e + f*x])^
(3/2))/(3*b^3*f*(d*Sec[e + f*x])^(3/2))
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
```

```
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2}{bf(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} - \frac{4\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx}{b^2}\\ &= -\frac{2}{bf(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} - \frac{8(b\tan(e+fx))^{3/2}}{3b^3f(d\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.72

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{(-7+\cos(2(e+fx)))\sec^2(e+fx)}{3bf(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
[Out] ((-7 + Cos[2*(e + f*x)])*Sec[e + f*x]^2)/(3*b*f*(d*Sec[e + f*x])^(3/2)*Sqrt
[b*Tan[e + f*x]])
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\frac{2\cos(fx+e)}{3}-\frac{8\sec(fx+e)}{3}}{f\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}\,bd}$	47

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/b/d*(cos(f*x+e)-4*sec(f*x+e
))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{2\left(\cos\left(fx+e\right)^{3}-4\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{3\,b^{2}d^{2}f\sin\left(fx+e\right)}$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*(cos(f*x + e)^3 - 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqr
t(d/cos(f*x + e))/(b^2*d^2*f*sin(f*x + e))
```

Sympy [A] (verification not implemented)

Time = 26.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \begin{cases} -\frac{8\tan^{3}(e+fx)}{3f(b\tan(e+fx))^{\frac{3}{2}}(d\sec(e+fx))^{\frac{3}{2}}}-\frac{2\tan(e+fx)}{f(b\tan(e+fx))^{\frac{3}{2}}(d\sec(e+fx))^{\frac{3}{2}}} & \text{for} \\ \frac{x}{(b\tan(e))^{\frac{3}{2}}(d\sec(e))^{\frac{3}{2}}} & \text{oth} \end{cases}$$

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Piecewise((-8*tan(e + f*x)**3/(3*f*(b*tan(e + f*x))**(3/2)*(d*sec(e + f*x))
**(3/2)) - 2*tan(e + f*x)/(f*(b*tan(e + f*x))**(3/2)*(d*sec(e + f*x))**(3/2
)), Ne(f, 0)), (x/((b*tan(e))**(3/2)*(d*sec(e))**(3/2)), True))
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx=\int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}\,(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx=\frac{(\cos(2e+2fx)-7)\sqrt{\frac{d}{\cos(e+fx)}}}{3bd^2f\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}$$

```
[In] int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(3/2)),x)
[Out] ((cos(2*e + 2*f*x) - 7)*(d/cos(e + f*x))^(1/2))/(3*b*d^2*f*((b*sin(2*e + 2*
f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.327 $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{24E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5b^2d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} - \frac{12(b\tan(e+fx))^{3/2}}{5b^3f(d\sec(e+fx))^{5/2}}$$

```
[Out] -2/b/f/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2)+24/5*(sin(1/2*e+1/4*Pi+1/2
*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x
),2^(1/2))*(b*tan(f*x+e))^(1/2)/b^2/d^2/f/(d*sec(f*x+e))^(1/2)/sin(f*x+e)^(
1/2)-12/5*(b*tan(f*x+e))^(3/2)/b^3/f/(d*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2689, 2692, 2696, 2721, 2719}

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{12(b\tan(e+fx))^{3/2}}{5b^3f(d\sec(e+fx))^{5/2}} - \frac{24E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5b^2d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}} - \frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -2/(b*f*(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) - (24*EllipticE[(e - P
i/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*b^2*d^2*f*Sqrt[d*Sec[e + f*x]]*Sq
```

```
rt[Sin[e + f*x]]) - (12*(b*Tan[e + f*x])^(3/2))/(5*b^3*f*(d*Sec[e + f*x])^(
5/2))
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{6\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx}{b^2}$$

$$= -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{12(b\tan(e+fx))^{3/2}}{5b^3f(d\sec(e+fx))^{5/2}} - \frac{12\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{5b^2d^2}$$

$$= -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{12(b\tan(e+fx))^{3/2}}{5b^3f(d\sec(e+fx))^{5/2}} - \frac{\left(12\sqrt{b\tan(e+fx)}\right)\int\sqrt{b\sin(e+fx)}\,dx}{5b^2d^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{12(b\tan(e+fx))^{3/2}}{5b^3f(d\sec(e+fx))^{5/2}} - \frac{\left(12\sqrt{b\tan(e+fx)}\right)\int\sqrt{\sin(e+fx)}\,dx}{5b^2d^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

$$= -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{24E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5b^2d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} - \frac{12(b\tan(e+fx))^{3/2}}{5b^3f(d\sec(e+fx))^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.89 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.68

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{-11+\cos(2(e+fx))-8\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(\right.}{5bd^2f\sqrt{d\sec(e+fx)}\sqrt{b\tan(e}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (-11 + Cos[2*(e + f*x)] - 8*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^
2]*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x]^2)/(5*b*d^2*f*Sqrt[d*Sec[e + f*x]]*S
qrt[b*Tan[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.45 (sec) , antiderivative size = 452, normalized size of antiderivative = 3.48

method	result
default	$\frac{\left(24\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,E\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e)}\right.\right.}{}$

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/b/d^2*(24*(-I*(I-cot(f*x+e)
+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(csc(f*x+e)-cot
```

```
(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))
-12*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/
2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)
))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(f*x+e)^2+24*sec(f*x+e)*(-I*(I-cot(f*x+e)+
csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(csc(f*x+e)-cot(
f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-
12*sec(f*x+e)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*
x+e)))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+
csc(f*x+e)))^(1/2),1/2*2^(1/2))+6*2^(1/2)-12*sec(f*x+e)*2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(6i\sqrt{-2i\,bd}\sin\left(fx+e\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))-6\right.}{}$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -2/5*(6*I*sqrt(-2*I*b*d)*sin(f*x + e)*weierstrassZeta(4, 0, weierstrassPInv
erse(4, 0, cos(f*x + e) + I*sin(f*x + e))) - 6*I*sqrt(2*I*b*d)*sin(f*x + e)
*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x +
 e))) - (cos(f*x + e)^4 - 6*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x + e
))*sqrt(d/cos(f*x + e)))/(b^2*d^3*f*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\tan(e+fx))^{3/2}\left(\frac{d}{\cos(e+fx)}\right)^{5/2}}\,dx$$

```
[In] int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2)), x)
```

3.328 $\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}} + \frac{d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{b^{5/2}f\sqrt{b\tan(e+fx)}} + \frac{d^3\operatorname{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{b^{5/2}f\sqrt{b\tan(e+fx)}}$$

```
[Out] d^3*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e)
)^(1/2)/b^(5/2)/f/(b*tan(f*x+e))^(1/2)+d^3*arctanh((b*sin(f*x+e))^(1/2)/b^(
1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/b^(5/2)/f/(b*tan(f*x+e))^(1
/2)-2/3*d^2*(d*sec(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2688, 2696, 2644, 335, 218, 212, 209}

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f\sqrt{b\tan(e+fx)}} + \frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\operatorname{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f\sqrt{b\tan(e+fx)}} - \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

```
[In] Int[(d*Sec[e + f*x])^(7/2)/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*d^2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2)) + (d^3*ArcTa
n[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/
(b^(5/2)*f*Sqrt[b*Tan[e + f*x]]) + (d^3*ArcTanh[Sqrt[b*Sin[e + f*x]]/Sqrt[b
]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(b^(5/2)*f*Sqrt[b*Tan[e + f*x
]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
```

```
*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}+\frac{d^2\int\frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx}{b^2}\\
&= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}+\frac{\left(d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int\frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}\,dx}{b^2\sqrt{b\tan(e+fx)}}\\
&= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}\\
&\quad+\frac{\left(d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1-\frac{x^2}{b^2}\right)}\,dx,x,b\sin(e+fx)\right)}{b^3f\sqrt{b\tan(e+fx)}}\\
&= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}\\
&\quad+\frac{\left(2d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int\frac{1}{1-\frac{x^4}{b^2}}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{b^3f\sqrt{b\tan(e+fx)}}\\
&= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}\\
&\quad+\frac{\left(d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int\frac{1}{b-x^2}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{b^2f\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{\left(d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int\frac{1}{b+x^2}\,dx,x,\sqrt{b\sin(e+fx)}\right)}{b^2f\sqrt{b\tan(e+fx)}}\\
&= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}+\frac{d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{b^{5/2}f\sqrt{b\tan(e+fx)}}\\
&\quad+\frac{d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{b^{5/2}f\sqrt{b\tan(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\cos(e+fx)(d\sec(e+fx))^{7/2}\sin(e+fx)}{3f(b\tan(e+fx))^{5/2}} + \frac{\left(\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\right)\cos^2(e+fx)(d\sec(e+fx))^{7/2}\tan^{\frac{5}{2}}(e+fx)}{f\sec^2(e+fx)^{3/4}(b\tan(e+fx))^{5/2}}$$

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Sec[e + f*x])^(7/2)*Sin[e + f*x])/(3*f*(b*Tan[e + f*x])
^(5/2)) + ((ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + ArcTanh[Sqr
t[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)])*Cos[e + f*x]^2*(d*Sec[e + f*x])^(7
/2)*Tan[e + f*x]^(5/2))/(f*(Sec[e + f*x]^2)^(3/4)*(b*Tan[e + f*x])^(5/2))
```

Maple [A] (verified)

Time = 17.89 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.37

method	result
default	$-\frac{\csc(fx+e)d^3\sqrt{d\sec(fx+e)}\left(-3\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)\cos(fx+e)+3\ \operatorname{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\mathrm{c}\right.\right.}{}$

```
[In] int((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*csc(f*x+e)*d^3*(d*sec(f*x+e))^(1/2)*(-3*arctan((sin(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))*cos(f*x+e)+3*arctanh((sin(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))*cos(f*x+e)+2*(sin(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)+3*arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+
e)+csc(f*x+e)))-3*arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+c
sc(f*x+e))))/(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*tan(f*x+e))^(1/2)/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(142) = 284$.

Time = 0.47 (sec) , antiderivative size = 850, normalized size of antiderivative = 4.94

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(16*d^3*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*
x + e) - 6*(b*d^3*cos(f*x + e)^2 - b*d^3)*sqrt(-d/b)*arctan(1/4*(cos(f*x +
e)^3 - 5*cos(f*x + e)^2 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e
) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d
/cos(f*x + e))/(d*cos(f*x + e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d))
+ 3*(b*d^3*cos(f*x + e)^2 - b*d^3)*sqrt(-d/b)*log((d*cos(f*x + e)^4 - 72*d*
cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*si
n(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*s
qrt(d/cos(f*x + e)) + 28*(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos
(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))
/(b^3*f*cos(f*x + e)^2 - b^3*f), 1/24*(16*d^3*sqrt(b*sin(f*x + e)/cos(f*x +
 e))*sqrt(d/cos(f*x + e))*cos(f*x + e) + 6*(b*d^3*cos(f*x + e)^2 - b*d^3)*s
qrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 +
6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/
cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^2 + (d*cos(f*x
 + e) + d)*sin(f*x + e) - d)) + 3*(b*d^3*cos(f*x + e)^2 - b*d^3)*sqrt(d/b)*
log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*
x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x +
e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(f*x + e)^2 - 2*
d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e
)^2 - 2)*sin(f*x + e) + 8)))/(b^3*f*cos(f*x + e)^2 - b^3*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(7/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{(b\tan(e+f\,x))^{5/2}}\,dx$$

```
[In] int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(5/2),x)
[Out] int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(5/2), x)
```

3.329 $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 101

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} + \frac{2d^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}$$

```
[Out] -2/3*d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1
/2)/b^2/f/(b*tan(f*x+e))^(1/2)-2/3*d^2*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+
e))^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2688, 2696, 2721, 2720}

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{2d^2\sqrt{\sin(e+fx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}} - \frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

```
[In] Int[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*d^2*Sqrt[d*Sec[e + f*x]])/(3*b*f*(b*Tan[e + f*x])^(3/2)) + (2*d^2*Ellip
ticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b^2
*f*Sqrt[b*Tan[e + f*x]])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}+\frac{d^2\int\frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{3b^2}\\
&= -\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}+\frac{\left(d^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int\frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{3b^2\sqrt{b\tan(e+fx)}}\\
&= -\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}+\frac{\left(d^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int\frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3b^2\sqrt{b\tan(e+fx)}}\\
&= -\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}+\frac{2d^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.92 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{2d^2\sqrt{d\sec(e+fx)}\left(-\cot^2(e+fx)+\frac{\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)}{\sqrt[4]{\sec^2(e+fx)}}\right)\sqrt{b\tan(e}}{3b^3f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*d^2*Sqrt[d*Sec[e + f*x]]*(-Cot[e + f*x]^2 + Hypergeometric2F1[1/4, 3/4,
5/4, -Tan[e + f*x]^2]/(Sec[e + f*x]^2)^(1/4))*Sqrt[b*Tan[e + f*x]])/(3*b^3*
f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.50

method	result
default	$-\frac{d^2\sqrt{d\sec(fx+e)}\left(i(-\cos(fx+e)-1)\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\sqrt{i(\csc(fx+e)-\cot(fx+e))}\right.}{3f\,b^2\sqrt{b\tan(fx+e)}}$

```
[In] int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*d^2*(d*sec(f*x+e))^(1/2)/b^2/(b*tan(f*x+e))^(1/2)*(I*(-cos(f*x+e)-1)
*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*
(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^
(1/2),1/2*2^(1/2))+2^(1/2)*cot(f*x+e))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.52

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{2\,d^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)^2+\left(d^2\cos(fx+e)^2-d^2\right)\sqrt{-2i\,bd}\text{weierstr}}{}$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*d^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x +
 e)^2 + (d^2*cos(f*x + e)^2 - d^2)*sqrt(-2*I*b*d)*weierstrassPInverse(4, 0,
```

```
 cos(f*x + e) + I*sin(f*x + e)) + (d^2*cos(f*x + e)^2 - d^2)*sqrt(2*I*b*d)*
weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(b^3*f*cos(f*x +
e)^2 - b^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{(b\tan(e+f\,x))^{5/2}}\,dx$$

```
[In] int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(5/2),x)
[Out] int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(5/2), x)
```

3.330 $\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 34

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx=-\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

```
[Out] -2/3*(d*sec(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2685}

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx=-\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

```
[In] Int[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2))
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\text{integral}=-\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(5/2),x]
[Out] (-2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2))
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{2\csc(fx+e)\sqrt{d\sec(fx+e)}\,d}{3f\,b^2\sqrt{b\tan(fx+e)}}$	36

```
[In] int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3/f*csc(f*x+e)*(d*sec(f*x+e))^(1/2)*d/b^2/(b*tan(f*x+e))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{2\,d\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)}{3\left(b^3f\cos(fx+e)^2-b^3f\right)}$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
[Out] 2/3*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)/(
b^3*f*cos(f*x + e)^2 - b^3*f)
```

Sympy [A] (verification not implemented)

Time = 44.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = \begin{cases} -\frac{2(d\sec(e+fx))^{\frac{3}{2}}\tan(e+fx)}{3f(b\tan(e+fx))^{\frac{5}{2}}} & \text{for } f\neq 0 \\ \frac{x(d\sec(e))^{\frac{3}{2}}}{(b\tan(e))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(5/2),x)
[Out] Piecewise((-2*(d*sec(e + f*x))**(3/2)*tan(e + f*x)/(3*f*(b*tan(e + f*x))**(
5/2)), Ne(f, 0)), (x*(d*sec(e))**(3/2)/(b*tan(e))**(5/2), True))
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\,d\,\sqrt{\frac{d}{\cos(e+f\,x)}}}{3\,b^2\,f\,\sin(e+f\,x)\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}$$

```
[In] int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(5/2),x)
[Out] -(2*d*(d/cos(e + f*x))^(1/2))/(3*b^2*f*sin(e + f*x)*((b*sin(2*e + 2*f*x))/(
cos(2*e + 2*f*x) + 1))^(1/2))
```

3.331 $\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}$$

```
[Out] 4/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF
(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/b
^2/f/(b*tan(f*x+e))^(1/2)-2/3*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2689, 2696, 2721, 2720}

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{4\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}} - \frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

```
[In] Int[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[d*Sec[e + f*x]])/(3*b*f*(b*Tan[e + f*x])^(3/2)) - (4*EllipticF[(e
- Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b^2*f*Sqrt[
b*Tan[e + f*x]])
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} - \frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{3b^2} \\
&= -\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} - \frac{\left(2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{3b^2\sqrt{b\tan(e+fx)}} \\
&= -\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} - \frac{\left(2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3b^2\sqrt{b\tan(e+fx)}} \\
&= -\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.82 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2d^2\left(\csc^2(e+fx)+2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}}$$

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*d^2*(Csc[e + f*x]^2 + 2*Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[e + f*x]^
2]*(Sec[e + f*x]^2)^(3/4))*Sqrt[b*Tan[e + f*x]])/(3*b^3*f*(d*Sec[e + f*x])^
(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.52

method	result
default	$-\frac{\sqrt{d\sec(fx+e)}\left(2i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(\right.\right.}{}$

```
[In] int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/b^2*(2*I*cos(f*x+e)*(-I*(I
-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(co
t(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),
1/2*2^(1/2))+2*I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f
*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x
+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+2^(1/2)*cot(f*x+e))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{2\left(\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)^2 - \sqrt{-2i\,bd}\left(\cos(fx+e)^2-1\right)\text{weierstrassP}\right.}{}$$

[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")

[Out] 2/3*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2 - sqrt(-2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) - sqrt(2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(b^3*f*cos(f*x + e)^2 - b^3*f)

Sympy [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{\frac{5}{2}}}\,dx$$

[In] integrate((d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(5/2),x)

[Out] Integral(sqrt(d*sec(e + f*x))/(b*tan(e + f*x))**(5/2), x)

Maxima [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{d\sec(fx+e)}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(5/2), x)

Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)}}{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{\left(b\tan\left(e+f\,x\right)\right)^{5/2}}\,dx$$

```
[In] int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(5/2),x)
[Out] int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(5/2), x)
```

3.332 $\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2}{3bf\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}-\frac{8\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}$$

```
[Out] -8/3*(b*tan(f*x+e))^(1/2)/b^3/f/(d*sec(f*x+e))^(1/2)-2/3/b/f/(d*sec(f*x+e))
^(1/2)/(b*tan(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2689, 2685}

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = -\frac{8\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}-\frac{2}{3bf(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}$$

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)),x]
```

```
[Out] -2/(3*b*f*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)) - (8*Sqrt[b*Tan[e +
f*x]])/(3*b^3*f*Sqrt[d*Sec[e + f*x]])
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
```

```
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rubi steps

$$\text{integral} = -\frac{2}{3bf\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}} - \frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx}{3b^2}$$

$$= -\frac{2}{3bf\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}} - \frac{8\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2(3+\csc^2(e+fx))\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}$$

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)),x]
[Out] (-2*(3 + Csc[e + f*x]^2)*Sqrt[b*Tan[e + f*x]])/(3*b^3*f*Sqrt[d*Sec[e + f*x]
])
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{2\cot(fx+e)-\frac{8\sec(fx+e)\csc(fx+e)}{3}}{f\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}\,b^2}$	52

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/3/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/b^2*(3*cot(f*x+e)-4*sec(f*x
+e)*csc(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(3\,\cos\left(fx+e\right)^{3}-4\,\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{3\left(b^{3}df\cos\left(fx+e\right)^{2}-b^{3}df\right)}$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -2/3*(3*cos(f*x + e)^3 - 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*
sqrt(d/cos(f*x + e))/(b^3*d*f*cos(f*x + e)^2 - b^3*d*f)
```

Sympy [A] (verification not implemented)

Time = 56.80 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = \begin{cases} -\frac{8\tan^{3}\left(e+fx\right)}{3f\left(b\tan\left(e+fx\right)\right)^{\frac{5}{2}}\sqrt{d\sec\left(e+fx\right)}} - \frac{2\tan\left(e+fx\right)}{3f\left(b\tan\left(e+fx\right)\right)^{\frac{5}{2}}\sqrt{d\sec\left(e+fx\right)}} & \text{for}\ f \neq \\ \frac{x}{\left(b\tan\left(e\right)\right)^{\frac{5}{2}}\sqrt{d\sec\left(e\right)}} & \text{otherw} \end{cases}$$

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Piecewise((-8*tan(e + f*x)**3/(3*f*(b*tan(e + f*x))**(5/2)*sqrt(d*sec(e + f
*x))) - 2*tan(e + f*x)/(3*f*(b*tan(e + f*x))**(5/2)*sqrt(d*sec(e + f*x))),
Ne(f, 0)), (x/((b*tan(e))**(5/2)*sqrt(d*sec(e))), True))
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2)), x)

Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = \frac{\left(\frac{13\,\sin(e+f\,x)}{3}-\sin\left(3\,e+3\,f\,x\right)\right)\,\sqrt{\frac{d}{\cos(e+f\,x)}}}{b^2\,d\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-1\right)\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}$$

[In] int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(1/2)),x)

[Out] (((13*sin(e + f*x))/3 - sin(3*e + 3*f*x))*(d/cos(e + f*x))^(1/2))/(b^2*d*f*(cos(2*e + 2*f*x) - 1)*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))

3.333 $\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{8\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2d^2f\sqrt{b\tan(e+fx)}} - \frac{4\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}}$$

```
[Out] 8/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF
(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/b
^2/d^2/f/(b*tan(f*x+e))^(1/2)-4/3*(b*tan(f*x+e))^(1/2)/b^3/f/(d*sec(f*x+e))
^(3/2)-2/3/b/f/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2689, 2692, 2696, 2721, 2720}

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{4\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}} - \frac{8\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3b^2d^2f\sqrt{b\tan(e+fx)}} - \frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2)),x]
```

```
[Out] -2/(3*b*f*(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)) - (8*EllipticF[(e
- Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b^2*d^2*f*S
```

```
qrt[b*Tan[e + f*x]]) - (4*Sqrt[b*Tan[e + f*x]])/(3*b^3*f*(d*Sec[e + f*x])^(
3/2))
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{2\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx}{b^2}$$

$$= -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{4\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}} - \frac{4\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{3b^2d^2}$$

$$= -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{4\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}} - \frac{\left(4\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int\frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{3b^2d^2\sqrt{b\tan(e+fx)}}$$

$$= -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{4\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}} - \frac{\left(4\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int\frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3b^2d^2\sqrt{b\tan(e+fx)}}$$

$$= -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{8\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2d^2f\sqrt{b\tan(e+fx)}} - \frac{4\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.72

$$\int\frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = \frac{(-3+\cos(2(e+fx)))\csc(e+fx) - 8\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4}\right.}{3b^2df\sqrt{d\sec(e+fx)}\sqrt{b\,\text{ta}}}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2)),x]
```

```
[Out] ((-3 + Cos[2*(e + f*x)])*Csc[e + f*x] - 8*Hypergeometric2F1[1/4, 3/4, 5/4,
-Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(3/4)*Sin[e + f*x])/(3*b^2*d*f*Sqrt[d*Sec
[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.09 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.96

method	result
default	$\frac{\left(-4i\sqrt{i(-i+\cot(fx+e)-\csc(fx+e))}\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{i(-i+\cot(fx+e)-\csc(fx+e}\right.\right.}{}$

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/b^2/d*(-4*I*(I*(-I+cot(f*x+
e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-
csc(f*x+e)))^(1/2)*EllipticF((I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2),1/2*2^(1/
2))-4*I*sec(f*x+e)*(I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc
(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((I*(-I+cot(f
*x+e)-csc(f*x+e)))^(1/2),1/2*2^(1/2))+cos(f*x+e)*cot(f*x+e)*2^(1/2)-2*csc(f
*x+e)*2^(1/2))*2^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.16

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(2\sqrt{-2i\,bd}\left(\cos\left(fx+e\right)^2-1\right)\text{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+2\sqrt{2i\,bd}\left(\cos\left(f\right.\right.\right.}{3\left(b^3\right.}$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -2/3*(2*sqrt(-2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstrassPInverse(4, 0, cos(f
*x + e) + I*sin(f*x + e)) + 2*sqrt(2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstras
sPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)) + (cos(f*x + e)^4 - 2*cos(f*
x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(b^3*d^2*
f*cos(f*x + e)^2 - b^3*d^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}\,(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}\,(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\tan(e+f\,x))^{5/2}\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2)),x)
[Out] int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2)), x)
```

3.334 $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2}{3bf(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}} - \frac{16\sqrt{b\tan(e+fx)}}{15b^3f(d\sec(e+fx))^{5/2}} - \frac{64\sqrt{b\tan(e+fx)}}{15b^3d^2f\sqrt{d\sec(e+fx)}}$$

```
[Out] -16/15*(b*tan(f*x+e))^(1/2)/b^3/f/(d*sec(f*x+e))^(5/2)-64/15*(b*tan(f*x+e))
^(1/2)/b^3/d^2/f/(d*sec(f*x+e))^(1/2)-2/3/b/f/(d*sec(f*x+e))^(5/2)/(b*tan(f
*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2689, 2692, 2685}

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{64\sqrt{b\tan(e+fx)}}{15b^3d^2f\sqrt{d\sec(e+fx)}} - \frac{16\sqrt{b\tan(e+fx)}}{15b^3f(d\sec(e+fx))^{5/2}} - \frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2)),x]
```

```
[Out] -2/(3*b*f*(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)) - (16*Sqrt[b*Tan[e
 + f*x]])/(15*b^3*f*(d*Sec[e + f*x])^(5/2)) - (64*Sqrt[b*Tan[e + f*x]])/(15
*b^3*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2}{3bf(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}} - \frac{8\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx}{3b^2} \\
&= -\frac{2}{3bf(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}} \\
&\quad - \frac{16\sqrt{b\tan(e+fx)}}{15b^3f(d\sec(e+fx))^{5/2}} - \frac{32\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx}{15b^2d^2} \\
&= -\frac{2}{3bf(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}} \\
&\quad - \frac{16\sqrt{b\tan(e+fx)}}{15b^3f(d\sec(e+fx))^{5/2}} - \frac{64\sqrt{b\tan(e+fx)}}{15b^3d^2f\sqrt{d\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$
\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = \frac{(-151+108\cos(2(e+fx))+3\cos(4(e+fx)))\csc(e+fx)\sqrt{d}}{60b^2d^3f\sqrt{b\tan(e+fx)}}
$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2)),x]
[Out] ((-151 + 108*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)])*Csc[e + f*x]*Sqrt[d*Sec
[e + f*x]])/(60*b^2*d^3*f*Sqrt[b*Tan[e + f*x]])
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{2\sec(fx+e)\csc(fx+e)\left(3\left(\cos^4(fx+e)\right)+24\left(\cos^2(fx+e)\right)-32\right)}{15f\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}\,b^2d^2}$	66

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/15/f*sec(f*x+e)*csc(f*x+e)*(3*cos(f*x+e)^4+24*cos(f*x+e)^2-32)/(d*sec(f*x
+e))^(1/2)/(b*tan(f*x+e))^(1/2)/b^2/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.84

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(3\cos\left(fx+e\right)^5+24\cos\left(fx+e\right)^3-32\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{15\left(b^3d^3f\cos\left(fx+e\right)^2-b^3d^3f\right)}$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
[Out] -2/15*(3*cos(f*x + e)^5 + 24*cos(f*x + e)^3 - 32*cos(f*x + e))*sqrt(b*sin(f
*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b^3*d^3*f*cos(f*x + e)^2 - b^3*
d^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{\sqrt{\frac{d}{\cos(e+f\,x)}}\,\left(105\,\sin\left(3\,e+3\,f\,x\right)-410\,\sin\left(e+f\,x\right)+3\,\sin\left(5\,e+5\,f\,x\right)\right)}{60\,b^2\,d^3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-1\right)\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}$$

```
[In] int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(5/2)),x)
[Out] -((d/cos(e + f*x))^(1/2)*(105*sin(3*e + 3*f*x) - 410*sin(e + f*x) + 3*sin(5
*e + 5*f*x)))/(60*b^2*d^3*f*(cos(2*e + 2*f*x) - 1)*((b*sin(2*e + 2*f*x))/(c
os(2*e + 2*f*x) + 1))^(1/2))
```

3.335 $\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx=\frac{2\cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{12},\frac{7}{4},\sin^2(e+fx)\right)(b\sec(e+fx)$$
$$3df$$

```
[Out] 2/3*(cos(f*x+e)^2)^(17/12)*hypergeom([3/4, 17/12],[7/4],sin(f*x+e)^2)*(b*se
c(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2697}

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx=\frac{2\cos^2(e+fx)^{17/12}(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\right.$$
$$3df$$

```
[In] Int[(b*Sec[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[3/4, 17/12, 7/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{2\cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{12},\frac{7}{4},\sin^2(e+fx)\right)(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}}{3df}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \frac{3d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{2}{3},\frac{5}{3},\sec^2(e+fx)\right)(b\sec(e+fx))^{4/3}\sqrt[4]{-\tan^2(e+f}}{4f\sqrt{d\tan(e+fx)}}$$

```
[In] Integrate[(b*Sec[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
[Out] (3*d*Hypergeometric2F1[1/4, 2/3, 5/3, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(4/3
)*(-Tan[e + f*x]^2)^(1/4))/(4*f*Sqrt[d*Tan[e + f*x]])
```

Maple [F]

$$\int (b\sec(fx+e))^{\frac{4}{3}}\sqrt{d\tan(fx+e)}dx$$

```
[In] int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
[Out] int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{4}{3}}\sqrt{d\tan(fx+e)}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*sec(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(4/3)*(d*tan(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{4}{3}}\sqrt{d\tan(fx+e)}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(4/3)*sqrt(d*tan(f*x + e)), x)
```

Giac [F]

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{4}{3}}\sqrt{d\tan(fx+e)}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(4/3)*sqrt(d*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(e+f\,x)}\left(\frac{b}{\cos(e+f\,x)}\right)^{4/3}\,dx$$

```
[In] int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(4/3),x)
[Out] int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(4/3), x)
```

3.336 $\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{7}{4},\sin^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}}{3df}$$

```
[Out] 2/3*(cos(f*x+e)^2)^(11/12)*hypergeom([3/4, 11/12],[7/4],sin(f*x+e)^2)*(b*se
c(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{11/12}\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{7}{4},\sin^2(e+fx)\right)}{3df}$$

```
[In] Int[(b*Sec[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(11/12)*Hypergeometric2F1[3/4, 11/12, 7/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

integral
$$= \frac{2\cos^2(e+fx)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{7}{4},\sin^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}}{3df}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \frac{3d\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{4},\frac{7}{6},\sec^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}\sqrt[4]{-\tan^2(e+fx)}}{f\sqrt{d\tan(e+fx)}}$$

```
[In] Integrate[(b*Sec[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
[Out] (3*d*Hypergeometric2F1[1/6, 1/4, 7/6, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(1/3
)*(-Tan[e + f*x]^2)^(1/4))/(f*Sqrt[d*Tan[e + f*x]])
```

Maple [F]

$$\int (b\sec(fx+e))^{\frac{1}{3}}\sqrt{d\tan(fx+e)}dx$$

```
[In] int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
[Out] int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{1}{3}}\sqrt{d\tan(fx+e)}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

Sympy [F]

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int \sqrt[3]{b\sec{(e+fx)}}\sqrt{d\tan{(e+fx)}}\,dx$$

```
[In] integrate((b*sec(f*x+e))**(1/3)*(d*tan(f*x+e))**(1/2),x)
[Out] Integral((b*sec(e + f*x))**(1/3)*sqrt(d*tan(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec{(fx+e)})^{\frac{1}{3}}\sqrt{d\tan{(fx+e)}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

Giac [F]

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec{(fx+e)})^{\frac{1}{3}}\sqrt{d\tan{(fx+e)}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan{(e+f\,x)}}\left(\frac{b}{\cos{(e+f\,x)}}\right)^{1/3}\,dx$$

```
[In] int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(1/3),x)
[Out] int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(1/3), x)
```

3.337 $\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \frac{2\cos^2(e+fx)^{7/12}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{7}{4},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{3df\sqrt[3]{b\sec(e+fx)}}$$

```
[Out] 2/3*(cos(f*x+e)^2)^(7/12)*hypergeom([7/12, 3/4],[7/4],sin(f*x+e)^2)*(d*tan(
f*x+e))^(3/2)/d/f/(b*sec(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2697}

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \frac{2\cos^2(e+fx)^{7/12}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{7}{4},\sin^2(e+fx)\right)}{3df\sqrt[3]{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(1/3),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[7/12, 3/4, 7/4, Sin[e + f*x]^2
]*(d*Tan[e + f*x])^(3/2))/(3*d*f*(b*Sec[e + f*x])^(1/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{2\cos^2(e+fx)^{7/12}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{7}{4},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{3df\sqrt[3]{b\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int\frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = -\frac{3d\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{4},\frac{5}{6},\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}}{f\sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}}$$

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(1/3),x]
[Out] (-3*d*Hypergeometric2F1[-1/6, 1/4, 5/6, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(
1/4))/(f*(b*Sec[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]])
```

Maple [F]

$$\int\frac{\sqrt{d\tan(fx+e)}}{(b\sec(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x)
[Out] int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int\frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int\frac{\sqrt{d\tan(fx+e)}}{(b\sec(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b*sec(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\tan{(e+fx)}}}{\sqrt[3]{b\sec{(e+fx)}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))**(1/2)/(b*sec(f*x+e))**(1/3),x)
[Out] Integral(sqrt(d*tan(e + f*x))/(b*sec(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\tan{(fx+e)}}}{(b\sec{(fx+e)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\tan{(fx+e)}}}{(b\sec{(fx+e)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\tan{(e+f\,x)}}}{{\left(\frac{b}{\cos(e+f\,x)}\right)}^{1/3}}\,dx$$

```
[In] int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(1/3),x)
[Out] int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(1/3), x)
```

3.338 $\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{2\sqrt[12]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{7}{4},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{3df(b\sec(e+fx))^{4/3}}$$

```
[Out] 2/3*(cos(f*x+e)^2)^(1/12)*hypergeom([1/12, 3/4],[7/4],sin(f*x+e)^2)*(d*tan(
f*x+e))^(3/2)/d/f/(b*sec(f*x+e))^(4/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{2\sqrt[12]{\cos^2(e+fx)}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{7}{4},\sin^2(e+fx)\right)}{3df(b\sec(e+fx))^{4/3}}$$

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(4/3),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(1/12)*Hypergeometric2F1[1/12, 3/4, 7/4, Sin[e + f*x]^2
]*(d*Tan[e + f*x])^(3/2))/(3*d*f*(b*Sec[e + f*x])^(4/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt[12]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{7}{4},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{3df(b\sec(e+fx))^{4/3}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = -\frac{3d\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{4},\frac{1}{3},\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}}{4f(b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}}$$

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(4/3),x]
[Out] (-3*d*Hypergeometric2F1[-2/3, 1/4, 1/3, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(
1/4))/(4*f*(b*Sec[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]])
```

Maple [F]

$$\int \frac{\sqrt{d\tan(fx+e)}}{(b\sec(fx+e))^{\frac{4}{3}}}dx$$

```
[In] int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x)
[Out] int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x)
```

Fricas [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan(fx+e)}}{(b\sec(fx+e))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b^2*sec(f*x + e)^2),
x)
```

Sympy [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan{(e+fx)}}}{(b\sec{(e+fx)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))**(1/2)/(b*sec(f*x+e))**(4/3),x)
[Out] Integral(sqrt(d*tan(e + f*x))/(b*sec(e + f*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan{(fx+e)}}}{(b\sec{(fx+e)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(4/3), x)
```

Giac [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan{(fx+e)}}}{(b\sec{(fx+e)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x, algorithm="giac")
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan{(e+f\,x)}}}{{\left(\frac{b}{\cos(e+f\,x)}\right)}^{4/3}}\,dx$$

```
[In] int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(4/3),x)
[Out] int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(4/3), x)
```

3.339 $\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \frac{2\cos^2(e+fx)^{23/12}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{23}{12},\frac{9}{4},\sin^2(e+fx)\right)(b\sec(e+fx}{5df}$$

```
[Out] 2/5*(cos(f*x+e)^2)^(23/12)*hypergeom([5/4, 23/12],[9/4],sin(f*x+e)^2)*(b*se
c(f*x+e))^(4/3)*(d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \frac{2\cos^2(e+fx)^{23/12}(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{5/2}\,\text{Hypergeometric2F1}}{5df}$$

```
[In] Int[(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(23/12)*Hypergeometric2F1[5/4, 23/12, 9/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(5/2))/(5*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{2\cos^2(e+fx)^{23/12}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{23}{12},\frac{9}{4},\sin^2(e+fx)\right)(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{5/2}}{5df}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \frac{3d\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{2}{3},\frac{5}{3},\sec^2(e+fx)\right)(b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}}{4f\sqrt[4]{-\tan^2(e+fx)}}$$

```
[In] Integrate[(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
[Out] (3*d*Hypergeometric2F1[-1/4, 2/3, 5/3, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(4/
3)*Sqrt[d*Tan[e + f*x]])/(4*f*(-Tan[e + f*x]^2)^(1/4))
```

Maple [F]

$$\int (b\sec(fx+e))^{\frac{4}{3}}(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
[Out] int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{4}{3}}(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*d*sec(f*x + e)*tan(f
*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(4/3)*(d*tan(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{4}{3}}(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(4/3)*(d*tan(f*x + e))^(3/2), x)
```

Giac [F]

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{4}{3}}(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(4/3)*(d*tan(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (d\tan(e+f\,x))^{3/2}\left(\frac{b}{\cos(e+f\,x)}\right)^{4/3}\,dx$$

```
[In] int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(4/3),x)
[Out] int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(4/3), x)
```

3.340 $\int \sqrt[3]{b \sec(e+fx)}(d \tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt[3]{b \sec(e+fx)}(d \tan(e+fx))^{3/2}\,dx=\frac{2 \cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{17}{12},\frac{9}{4},\sin^2(e+fx)\right)\sqrt[3]{b \sec(e+fx)}(d \tan(e+fx))^{5/2}}{5df}$$

```
[Out] 2/5*(cos(f*x+e)^2)^(17/12)*hypergeom([5/4, 17/12],[9/4],sin(f*x+e)^2)*(b*se
c(f*x+e))^(1/3)*(d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2697}

$$\int \sqrt[3]{b \sec(e+fx)}(d \tan(e+fx))^{3/2}\,dx=\frac{2 \cos^2(e+fx)^{17/12}\sqrt[3]{b \sec(e+fx)}(d \tan(e+fx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{17}{12},\frac{9}{4},\sin^2(e+fx)\right)}{5df}$$

```
[In] Int[(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[5/4, 17/12, 9/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(5/2))/(5*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{2\cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{17}{12},\frac{9}{4},\sin^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{5/2}}{5df}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \frac{3d\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{6},\frac{7}{6},\sec^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}}{f\sqrt[4]{-\tan^2(e+fx)}}$$

```
[In] Integrate[(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
[Out] (3*d*Hypergeometric2F1[-1/4, 1/6, 7/6, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(1/
3)*Sqrt[d*Tan[e + f*x]])/(f*(-Tan[e + f*x]^2)^(1/4))
```

Maple [F]

$$\int (b\sec(fx+e))^{\frac{1}{3}}(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
[Out] int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{1}{3}}(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e), x)
```

Sympy [F]

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int \sqrt[3]{b\sec{(e+fx)}}(d\tan{(e+fx)})^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sec(f*x+e))**(1/3)*(d*tan(f*x+e))**(3/2),x)
[Out] Integral((b*sec(e + f*x))**(1/3)*(d*tan(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int (b\sec{(fx+e)})^{\frac{1}{3}}\,(d\tan{(fx+e)})^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int (b\sec{(fx+e)})^{\frac{1}{3}}\,(d\tan{(fx+e)})^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int (d\tan(e+f\,x))^{3/2}\left(\frac{b}{\cos{(e+f\,x)}}\right)^{1/3}\,dx$$

```
[In] int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(1/3),x)
[Out] int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(1/3), x)
```

3.341 $\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \frac{2\cos^2(e+fx)^{13/12}\,\text{Hypergeometric2F1}\left(\frac{13}{12},\frac{5}{4},\frac{9}{4},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{5df\sqrt[3]{b\sec(e+fx)}}$$

```
[Out] 2/5*(cos(f*x+e)^2)^(13/12)*hypergeom([13/12, 5/4],[9/4],sin(f*x+e)^2)*(d*ta
n(f*x+e))^(5/2)/d/f/(b*sec(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2697}

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \frac{2\cos^2(e+fx)^{13/12}(d\tan(e+fx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{13}{12},\frac{5}{4},\frac{9}{4},\sin^2(e+fx)\right)}{5df\sqrt[3]{b\sec(e+fx)}}$$

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(1/3),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(13/12)*Hypergeometric2F1[13/12, 5/4, 9/4, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(5/2))/(5*d*f*(b*Sec[e + f*x])^(1/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{2\cos^2(e+fx)^{13/12}\,\text{Hypergeometric2F1}\left(\frac{13}{12},\frac{5}{4},\frac{9}{4},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{5df\sqrt[3]{b\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \frac{3\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},-\frac{1}{6},\frac{5}{6},\sec^2(e+fx)\right)(d\tan(e+fx))^{3/2}\,(-}{f\sqrt[3]{b\sec(e+fx)}}$$

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(1/3),x]
[Out] (3*Cot[e + f*x]^3*Hypergeometric2F1[-1/4, -1/6, 5/6, Sec[e + f*x]^2]*(d*Tan
[e + f*x])^(3/2)*(-Tan[e + f*x]^2)^(3/4))/(f*(b*Sec[e + f*x])^(1/3))
```

Maple [F]

$$\int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x)
[Out] int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b*sec(
f*x + e)), x)
```

Sympy [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{(d\tan{(e+fx)})^{\frac{3}{2}}}{\sqrt[3]{b\sec{(e+fx)}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))**(3/2)/(b*sec(f*x+e))**(1/3),x)
[Out] Integral((d*tan(e + f*x))**(3/2)/(b*sec(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{(d\tan{(fx+e)})^{\frac{3}{2}}}{(b\sec{(fx+e)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{(d\tan{(fx+e)})^{\frac{3}{2}}}{(b\sec{(fx+e)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{(d\tan(e+f\,x))^{3/2}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

```
[In] int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(1/3),x)
[Out] int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(1/3), x)
```

3.342 $\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{2\cos^2(e+fx)^{7/12}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{9}{4},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{5df(b\sec(e+fx))^{4/3}}$$

```
[Out] 2/5*(cos(f*x+e)^2)^(7/12)*hypergeom([7/12, 5/4],[9/4],sin(f*x+e)^2)*(d*tan(
f*x+e))^(5/2)/d/f/(b*sec(f*x+e))^(4/3)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{2697\}$

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{2\cos^2(e+fx)^{7/12}(d\tan(e+fx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{9}{4},\sin^2(e+fx)\right)}{5df(b\sec(e+fx))^{4/3}}$$

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(4/3),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[7/12, 5/4, 9/4, Sin[e + f*x]^2
]*(d*Tan[e + f*x])^(5/2))/(5*d*f*(b*Sec[e + f*x])^(4/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{2\cos^2(e+fx)^{7/12}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{9}{4},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{5df(b\sec(e+fx))^{4/3}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{3\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{4},\frac{1}{3},\sec^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{4f(b\sec(e+fx))^{4/3}}$$

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(4/3),x]
[Out] (3*Cot[e + f*x]^3*Hypergeometric2F1[-2/3, -1/4, 1/3, Sec[e + f*x]^2]*(d*Tan
[e + f*x])^(3/2)*(-Tan[e + f*x]^2)^(3/4))/(4*f*(b*Sec[e + f*x])^(4/3))
```

Maple [F]

$$\int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{4}{3}}}dx$$

```
[In] int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x)
[Out] int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x)
```

Fricas [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b^2*se
c(f*x + e)^2), x)
```

Sympy [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{(d\tan{(e+fx)})^{\frac{3}{2}}}{(b\sec{(e+fx)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))**(3/2)/(b*sec(f*x+e))**(4/3),x)
[Out] Integral((d*tan(e + f*x))**(3/2)/(b*sec(e + f*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{(d\tan{(fx+e)})^{\frac{3}{2}}}{(b\sec{(fx+e)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(4/3), x)
```

Giac [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{(d\tan{(fx+e)})^{\frac{3}{2}}}{(b\sec{(fx+e)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x, algorithm="giac")
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(e+f\,x))^{3/2}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{4/3}}\,dx$$

```
[In] int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(4/3),x)
[Out] int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(4/3), x)
```

3.343 $\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \frac{3\cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{17}{12},\frac{13}{6},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}(d\tan(e+fx)$$

```
[Out] 3/7*(cos(f*x+e)^2)^(17/12)*hypergeom([7/6, 17/12],[13/6],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(1/2)*(d*tan(f*x+e))^(7/3)/d/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2697}

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \frac{3\cos^2(e+fx)^{17/12}\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{17}{12},\frac{13}{6},\sin^2(e+f\right.}{7df}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[7/6, 17/12, 13/6, Sin[e + f*x
]^2]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(7/3))/(7*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{3\cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{17}{12},\frac{13}{6},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{7/3}}{7df}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{4},\frac{5}{4},\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}}{f\sqrt[6]{-\tan^2(e+fx)}}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
[Out] (2*d*Hypergeometric2F1[-1/6, 1/4, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]
*(d*Tan[e + f*x])^(1/3))/(f*(-Tan[e + f*x]^2)^(1/6))
```

Maple [F]

$$\int \sqrt{b\sec(fx+e)}\,(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

```
[In] int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
[Out] int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

Fricas [F]

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\sec(fx+e)}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3)*d*tan(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx=\text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(1/2)*(d*tan(f*x+e))**(4/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx=\int \sqrt{b\sec(fx+e)}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx=\int \sqrt{b\sec(fx+e)}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx=\int (d\tan(e+f\,x))^{4/3}\sqrt{\frac{b}{\cos(e+f\,x)}}\,dx$$

```
[In] int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(1/2),x)
[Out] int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(1/2), x)
```

3.344 $\int \sqrt{b \sec(e+fx)}\sqrt[3]{d \tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt{b \sec(e+fx)}\sqrt[3]{d \tan(e+fx)}\,dx = \frac{3\cos^2(e+fx)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{11}{12},\frac{5}{3},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}}{4df}$$

```
[Out] 3/4*(cos(f*x+e)^2)^(11/12)*hypergeom([2/3, 11/12],[5/3],sin(f*x+e)^2)*(b*se
c(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3)/d/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2697}

$$\int \sqrt{b \sec(e+fx)}\sqrt[3]{d \tan(e+fx)}\,dx = \frac{3\cos^2(e+fx)^{11/12}\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{11}{12},\frac{5}{3},\sin^2(e+fx)\right)}{4df}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(11/12)*Hypergeometric2F1[2/3, 11/12, 5/3, Sin[e + f*x]
^2]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(4*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

integral
$$= \frac{3\cos^2(e+fx)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{11}{12},\frac{5}{3},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}}{4df}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}\sqrt[3]{-\tan^2(e+fx)}}{f(d\tan(e+fx))^{2/3}}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
[Out] (2*d*Hypergeometric2F1[1/4, 1/3, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]*
(-Tan[e + f*x]^2)^(1/3))/(f*(d*Tan[e + f*x])^(2/3))
```

Maple [F]

$$\int \sqrt{b\sec(fx+e)}\,(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
[Out] int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)}(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

Sympy [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sec{(e+fx)}}\sqrt[3]{d\tan{(e+fx)}}\,dx$$

```
[In] integrate((b*sec(f*x+e))**(1/2)*(d*tan(f*x+e))**(1/3),x)
[Out] Integral(sqrt(b*sec(e + f*x))*(d*tan(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}}(d\tan{(fx+e)})^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}}(d\tan{(fx+e)})^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int (d\tan(e+f\,x))^{1/3}\,\sqrt{\frac{b}{\cos{(e+f\,x)}}}\,dx$$

```
[In] int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(1/2),x)
[Out] int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(1/2), x)
```

3.345 $\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{3\cos^2(e+fx)^{7/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{12},\frac{4}{3},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{2/3}}{2df}$$

```
[Out] 3/2*(cos(f*x+e)^2)^(7/12)*hypergeom([1/3, 7/12],[4/3],sin(f*x+e)^2)*(b*sec(
f*x+e))^(1/2)*(d*tan(f*x+e))^(2/3)/d/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2697}

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{3\cos^2(e+fx)^{7/12}\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{12},\frac{4}{3},\sin^2(e+fx)\right)}{2df}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[1/3, 7/12, 4/3, Sin[e + f*x]^2
]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(2*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
```

```
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

integral

$$= \frac{3\cos^2(e+fx)^{7/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{12},\frac{4}{3},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{2/3}}{2df}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{2}{3},\frac{5}{4},\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}\left(-\tan^2(e+fx)\right)^{2/3}}{f(d\tan(e+fx))^{4/3}}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
[Out] (2*d*Hypergeometric2F1[1/4, 2/3, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]*
(-Tan[e + f*x]^2)^(2/3))/(f*(d*Tan[e + f*x])^(4/3))
```

Maple [F]

$$\int \frac{\sqrt{b\sec(fx+e)}}{(d\tan(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
[Out] int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\sec(fx+e)}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)/(d*tan(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\sec\left(e+fx\right)}}{\sqrt[3]{d\tan\left(e+fx\right)}}\,dx$$

```
[In] integrate((b*sec(f*x+e))**(1/2)/(d*tan(f*x+e))**(1/3),x)
[Out] Integral(sqrt(b*sec(e + f*x))/(d*tan(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{{\left(d\tan\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

```
[In] int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3),x)
[Out] int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3), x)
```

3.346 $\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = -\frac{3\sqrt[12]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{12},\frac{5}{6},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}}{df\sqrt[3]{d\tan(e+fx)}}$$

```
[Out] -3*(cos(f*x+e)^2)^(1/12)*hypergeom([-1/6, 1/12],[5/6],sin(f*x+e)^2)*(b*sec(
f*x+e))^(1/2)/d/f/(d*tan(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2697}

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = -\frac{3\sqrt[12]{\cos^2(e+fx)}\sqrt{b\sec(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{12},\frac{5}{6},\sin^2(e+fx)\right)}{df\sqrt[3]{d\tan(e+fx)}}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (-3*(Cos[e + f*x]^2)^(1/12)*Hypergeometric2F1[-1/6, 1/12, 5/6, Sin[e + f*x]
^2]*Sqrt[b*Sec[e + f*x]])/(d*f*(d*Tan[e + f*x])^(1/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
```

```
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{3\sqrt[12]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{12},\frac{5}{6},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}}{df\sqrt[3]{d\tan(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{6},\frac{5}{4},\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}(-\tan^2(e+fx))^{7/6}}{f(d\tan(e+fx))^{7/3}}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
[Out] (2*d*Hypergeometric2F1[1/4, 7/6, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]*
(-Tan[e + f*x]^2)^(7/6))/(f*(d*Tan[e + f*x])^(7/3))
```

Maple [F]

$$\int \frac{\sqrt{b\sec(fx+e)}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

```
[In] int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
[Out] int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

Fricas [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sec(fx+e)}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)/(d^2*tan(f*x + e)^2),
x)
```

Sympy [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sec{(e+fx)}}}{(d\tan{(e+fx)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))**(1/2)/(d*tan(f*x+e))**(4/3),x)
[Out] Integral(sqrt(b*sec(e + f*x))/(d*tan(e + f*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sec{(fx+e)}}}{(d\tan{(fx+e)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sec{(fx+e)}}}{(d\tan{(fx+e)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{(d\tan{(e+f\,x)})^{4/3}}\,dx$$

```
[In] int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3),x)
[Out] int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3), x)
```

3.347 $\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \frac{3\cos^2(e+fx)^{23/12}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{23}{12},\frac{13}{6},\sin^2(e+fx)\right)(b\sec(e+f}{7df}$$

```
[Out] 3/7*(cos(f*x+e)^2)^(23/12)*hypergeom([7/6, 23/12],[13/6],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(3/2)*(d*tan(f*x+e))^(7/3)/d/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \frac{3\cos^2(e+fx)^{23/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{7/3}\,\text{Hypergeometric2F1}}{7df}$$

```
[In] Int[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(23/12)*Hypergeometric2F1[7/6, 23/12, 13/6, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(7/3))/(7*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

integral
$$= \frac{3\cos^2(e+fx)^{23/12}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{23}{12},\frac{13}{6},\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{7/3}}{7df}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{3}{4},\frac{7}{4},\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}}{3f\sqrt[6]{-\tan^2(e+fx)}}$$

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
[Out] (2*d*Hypergeometric2F1[-1/6, 3/4, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/
2)*(d*Tan[e + f*x])^(1/3))/(3*f*(-Tan[e + f*x]^2)^(1/6))
```

Maple [F]

$$\int (b\sec(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

```
[In] int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
[Out] int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

Fricas [F]

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3)*b*d*sec(f*x + e)*tan(f
*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(3/2)*(d*tan(f*x+e))**(4/3),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

Giac [F]

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (d\tan(e+f\,x))^{4/3}\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

```
[In] int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(3/2),x)
[Out] int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(3/2), x)
```

3.348 $\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx=\frac{3\cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{17}{12},\frac{5}{3},\sin^2(e+fx)\right)(b\sec(e+fx))^{3}}{4df}$$

```
[Out] 3/4*(cos(f*x+e)^2)^(17/12)*hypergeom([2/3, 17/12],[5/3],sin(f*x+e)^2)*(b*se
c(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3)/d/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2697}

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx=\frac{3\cos^2(e+fx)^{17/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\right.}{4df}$$

```
[In] Int[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[2/3, 17/12, 5/3, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3))/(4*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

integral
$$= \frac{3\cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{17}{12},\frac{5}{3},\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}}{4df}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\sqrt[3]{-\tan^2(e+fx)}}{3f(d\tan(e+fx))^{2/3}}$$

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
[Out] (2*d*Hypergeometric2F1[1/3, 3/4, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/2
)*(-Tan[e + f*x]^2)^(1/3))/(3*f*(d*Tan[e + f*x])^(2/3))
```

Maple [F]

$$\int (b\sec(fx+e))^{\frac{3}{2}}\,(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
[Out] int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\,(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3)*b*sec(f*x + e), x)
```

Sympy [F]

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sec(e+fx))^{\frac{3}{2}}\sqrt[3]{d\tan(e+fx)}\,dx$$

```
[In] integrate((b*sec(f*x+e))**(3/2)*(d*tan(f*x+e))**(1/3),x)
[Out] Integral((b*sec(e + f*x))**(3/2)*(d*tan(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(1/3), x)
```

Giac [F]

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (d\tan(e+f\,x))^{1/3}\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

```
[In] int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(3/2),x)
[Out] int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(3/2), x)
```

3.349 $\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{3\cos^2(e+fx)^{13/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{12},\frac{4}{3},\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}(}{2df}$$

```
[Out] 3/2*(cos(f*x+e)^2)^(13/12)*hypergeom([1/3, 13/12],[4/3],sin(f*x+e)^2)*(b*se
c(f*x+e))^(3/2)*(d*tan(f*x+e))^(2/3)/d/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{3\cos^2(e+fx)^{13/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{12}\right.}{2df}$$

```
[In] Int[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(13/12)*Hypergeometric2F1[1/3, 13/12, 4/3, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(2/3))/(2*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{3\cos^2(e+fx)^{13/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{12},\frac{4}{3},\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{2/3}}{2df}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{4},\frac{7}{4},\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\left(-\tan^2(e+fx)\right)^{2/}}{3f(d\tan(e+fx))^{4/3}}$$

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
[Out] (2*d*Hypergeometric2F1[2/3, 3/4, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/2
)*(-Tan[e + f*x]^2)^(2/3))/(3*f*(d*Tan[e + f*x])^(4/3))
```

Maple [F]

$$\int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
[Out] int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sec(f*x + e)/(d*tan(
f*x + e)), x)
```

Sympy [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sec{(e+fx)})^{\frac{3}{2}}}{\sqrt[3]{d\tan{(e+fx)}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))**(3/2)/(d*tan(f*x+e))**(1/3),x)
[Out] Integral((b*sec(e + f*x))**(3/2)/(d*tan(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sec{(fx+e)})^{\frac{3}{2}}}{(d\tan{(fx+e)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sec{(fx+e)})^{\frac{3}{2}}}{(d\tan{(fx+e)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{(d\tan{(e+f\,x)})^{1/3}}\,dx$$

```
[In] int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3),x)
[Out] int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3), x)
```

3.350 $\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = -\frac{3\cos^2(e+fx)^{7/12}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{7}{12},\frac{5}{6},\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}}{df\sqrt[3]{d\tan(e+fx)}}$$

```
[Out] -3*(cos(f*x+e)^2)^(7/12)*hypergeom([-1/6, 7/12],[5/6],sin(f*x+e)^2)*(b*sec(
f*x+e))^(3/2)/d/f/(d*tan(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = -\frac{3\cos^2(e+fx)^{7/12}(b\sec(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{7}{12},\frac{5}{6},\sin^2(e+fx)\right)}{df\sqrt[3]{d\tan(e+fx)}}$$

```
[In] Int[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (-3*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[-1/6, 7/12, 5/6, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(3/2))/(d*f*(d*Tan[e + f*x])^(1/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
```

```
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{3\cos^2(e+fx)^{7/12}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{7}{12},\frac{5}{6},\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}}{df\sqrt[3]{d\tan(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{6},\frac{7}{4},\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\left(-\tan^2(e+fx)\right.}{3f(d\tan(e+fx))^{7/3}}$$

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
[Out] (2*d*Hypergeometric2F1[3/4, 7/6, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/2
)*(-Tan[e + f*x]^2)^(7/6))/(3*f*(d*Tan[e + f*x])^(7/3))
```

Maple [F]

$$\int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}dx$$

```
[In] int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
[Out] int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

Fricas [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sec(f*x + e)/(d^2*ta
n(f*x + e)^2), x)
```

Sympy [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(e+fx))^{\frac{3}{2}}}{(d\tan(e+fx))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))**(3/2)/(d*tan(f*x+e))**(4/3),x)
[Out] Integral((b*sec(e + f*x))**(3/2)/(d*tan(e + f*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

Giac [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{(d\tan(e+f\,x))^{4/3}}\,dx$$

```
[In] int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3),x)
[Out] int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3), x)
```

3.351 $\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx = \frac{(b\sec(e+fx))^m}{fm} - \frac{2(b\sec(e+fx))^{2+m}}{b^2f(2+m)} + \frac{(b\sec(e+fx))^{4+m}}{b^4f(4+m)}$$

```
[Out] (b*sec(f*x+e))^m/f/m-2*(b*sec(f*x+e))^(2+m)/b^2/f/(2+m)+(b*sec(f*x+e))^(4+m
)/b^4/f/(4+m)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2686, 276}

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx = \frac{(b\sec(e+fx))^{m+4}}{b^4f(m+4)} - \frac{2(b\sec(e+fx))^{m+2}}{b^2f(m+2)} + \frac{(b\sec(e+fx))^m}{fm}$$

```
[In] Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^5,x]
```

```
[Out] (b*Sec[e + f*x])^m/(f*m) - (2*(b*Sec[e + f*x])^(2 + m))/(b^2*f*(2 + m)) + (
b*Sec[e + f*x])^(4 + m)/(b^4*f*(4 + m))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int (bx)^{-1+m}\left(-1+x^2\right)^2\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{b\text{Subst}\left(\int \left((bx)^{-1+m}-\frac{2(bx)^{1+m}}{b^2}+\frac{(bx)^{3+m}}{b^4}\right)\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{(b\sec(e+fx))^m}{fm}-\frac{2(b\sec(e+fx))^{2+m}}{b^2f(2+m)}+\frac{(b\sec(e+fx))^{4+m}}{b^4f(4+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int (b\sec(e+fx))^m\tan^5(e+fx)\,dx = \frac{(b\sec(e+fx))^m\left(8+6m+m^2-2m(4+m)\sec^2(e+fx)+m(2+m)\sec^4(e+fx)\right)}{fm(2+m)(4+m)}$$

```
[In] Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^5,x]
[Out] ((b*Sec[e + f*x])^m*(8 + 6*m + m^2 - 2*m*(4 + m)*Sec[e + f*x]^2 + m*(2 + m)
*Sec[e + f*x]^4))/(f*m*(2 + m)*(4 + m))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 8.10 (sec) , antiderivative size = 6067, normalized size of antiderivative = 90.55

method	result	size
risch	Expression too large to display	6067

```
[In] int((b*sec(f*x+e))^m*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int (b\sec(e+fx))^m\tan^5(e+fx)\,dx = \frac{\left((m^2+6\,m+8)\cos\left(fx+e\right)^4-2\,(m^2+4\,m)\cos\left(fx+e\right)^2+m^2+2\,m\right)\left(\frac{b}{\cos(fx+e)}\right)^m}{\left(fm^3+6\,fm^2+8\,fm\right)\cos\left(fx+e\right)^4}$$

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] ((m^2 + 6*m + 8)*cos(f*x + e)^4 - 2*(m^2 + 4*m)*cos(f*x + e)^2 + m^2 + 2*m)
*(b/cos(f*x + e))^m/((f*m^3 + 6*f*m^2 + 8*f*m)*cos(f*x + e)^4)
```

Sympy [F]

$$\int (b\sec(e+fx))^m\tan^5(e+fx)\,dx = \begin{cases} x(b\sec(e))^m\tan^5(e) & \text{for } f = \\ \frac{\int\frac{\tan^5(e+fx)}{\sec^4(e+fx)}\,dx}{b^4} & \text{for } m = \\ \frac{\int\frac{\tan^5(e+fx)}{\sec^2(e+fx)}\,dx}{b^2} & \text{for } m = \\ \frac{\log(\tan^2(e+fx)+1)}{2f}+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f} & \text{for } m = \\ \frac{m^2(b\sec(e+fx))^m\tan^4(e+fx)}{fm^3+6fm^2+8fm}+\frac{2m(b\sec(e+fx))^m\tan^4(e+fx)}{fm^3+6fm^2+8fm}-\frac{4m(b\sec(e+fx))^m\tan^2(e+fx)}{fm^3+6fm^2+8fm}+\frac{8(b\sec(e+fx))^m}{fm^3+6fm^2+8fm} & \text{otherw} \end{cases}$$

```
[In] integrate((b*sec(f*x+e))**m*tan(f*x+e)**5,x)
```

```
[Out] Piecewise((x*(b*sec(e))**m*tan(e)**5, Eq(f, 0)), (Integral(tan(e + f*x)**5/
sec(e + f*x)**4, x)/b**4, Eq(m, -4)), (Integral(tan(e + f*x)**5/sec(e + f*x
)**2, x)/b**2, Eq(m, -2)), (log(tan(e + f*x)**2 + 1)/(2*f) + tan(e + f*x)**
4/(4*f) - tan(e + f*x)**2/(2*f), Eq(m, 0)), (m**2*(b*sec(e + f*x))**m*tan(e
 + f*x)**4/(f*m**3 + 6*f*m**2 + 8*f*m) + 2*m*(b*sec(e + f*x))**m*tan(e + f*
x)**4/(f*m**3 + 6*f*m**2 + 8*f*m) - 4*m*(b*sec(e + f*x))**m*tan(e + f*x)**2
/(f*m**3 + 6*f*m**2 + 8*f*m) + 8*(b*sec(e + f*x))**m/(f*m**3 + 6*f*m**2 + 8
*f*m), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx = \frac{\frac{b^m\cos(fx+e)^{-m}}{m} - \frac{2\,b^m\cos(fx+e)^{-m}}{(m+2)\cos(fx+e)^2} + \frac{b^m\cos(fx+e)^{-m}}{(m+4)\cos(fx+e)^4}}{f}$$

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^5,x, algorithm="maxima")
[Out] (b^m*cos(f*x + e)^(-m)/m - 2*b^m*cos(f*x + e)^(-m)/((m + 2)*cos(f*x + e)^2)
 + b^m*cos(f*x + e)^(-m)/((m + 4)*cos(f*x + e)^4))/f
```

Giac [F]

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx = \int (b\sec(fx+e))^m \tan(fx+e)^5\,dx$$

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^5,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^5, x)
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.97

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx = \frac{(\cos(4\,e+4\,f\,x)-\sin(4\,e+4\,f\,x)\ 1\mathrm{i})\left(\frac{b}{\cos(e+f\,x)}\right)^m\left(\frac{2\,\cos(4\,e+4\,f\,x)\,(\cos(4\,e+4\,f\,x)+\sin(4\,e+4\,f\,x)\,1\mathrm{i})}{f\,m}+\frac{(\cos(4\,e+4\,f\,x)+\ldots}{\ldots}\right)}{16\left(\frac{\cos(2\,e+2\,f\,x)}{2}+\frac{1}{2}\right)^2}$$

```
[In] int(tan(e + f*x)^5*(b/cos(e + f*x))^m,x)
[Out] ((cos(4*e + 4*f*x) - sin(4*e + 4*f*x)*1i)*(b/cos(e + f*x))^m*((2*cos(4*e +
4*f*x)*(cos(4*e + 4*f*x) + sin(4*e + 4*f*x)*1i))/(f*m) + ((cos(4*e + 4*f*x)
 + sin(4*e + 4*f*x)*1i)*(4*m + 6*m^2 + 48))/(f*m*(6*m + m^2 + 8)) - (2*cos(
2*e + 2*f*x)*(cos(4*e + 4*f*x) + sin(4*e + 4*f*x)*1i)*(8*m + 4*m^2 - 32))/(
f*m*(6*m + m^2 + 8))))/(16*(cos(2*e + 2*f*x)/2 + 1/2)^2)
```

3.352 $\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = -\frac{(b\sec(e+fx))^m}{fm} + \frac{(b\sec(e+fx))^{2+m}}{b^2f(2+m)}$$

```
[Out] -(b*sec(f*x+e))^m/f/m+(b*sec(f*x+e))^(2+m)/b^2/f/(2+m)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2686, 14}

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = \frac{(b\sec(e+fx))^{m+2}}{b^2f(m+2)} - \frac{(b\sec(e+fx))^m}{fm}$$

```
[In] Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] -((b*Sec[e + f*x])^m/(f*m)) + (b*Sec[e + f*x])^(2 + m)/(b^2*f*(2 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{b\text{Subst}\left(\int (bx)^{-1+m}\left(-1+x^2\right)\,dx,x,\sec(e+fx)\right)}{f}\\&=\frac{b\text{Subst}\left(\int\left(-(bx)^{-1+m}+\frac{(bx)^{1+m}}{b^2}\right)\,dx,x,\sec(e+fx)\right)}{f}\\&=-\frac{(b\sec(e+fx))^m}{fm}+\frac{(b\sec(e+fx))^{2+m}}{b^2f(2+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int (b\sec(e+fx))^m\tan^3(e+fx)\,dx=-\frac{(b\sec(e+fx))^m\left(2+m-m\sec^2(e+fx)\right)}{fm(2+m)}$$

```
[In] Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^3,x]
[Out] -(((b*Sec[e + f*x])^m*(2 + m - m*Sec[e + f*x]^2))/(f*m*(2 + m)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.06 (sec) , antiderivative size = 2423, normalized size of antiderivative = 56.35

method	result	size
risch	Expression too large to display	2423

```
[In] int((b*sec(f*x+e))^m*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
[Out] -1/(2+m)/f/(exp(2*I*(f*x+e))+1)^2/m*exp(I*(f*x+e))^m*(exp(2*I*(f*x+e))+1)^(
-m)*2^m*b^m*(m*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3*Pi*
m)*exp(1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*exp(I*(f*
x+e)))*Pi*m)*exp(1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*Pi*csg
n(I/(exp(2*I*(f*x+e))+1))*m)*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x
+e))+1))*csgn(I*exp(I*(f*x+e)))*Pi*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(1/2*
I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(
2*I*(f*x+e))+1))^2*Pi*m)*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))
+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b)*Pi*m)*exp(-1/2
*I*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3*Pi*m)*exp(1/2*I*csgn(I*b
```

exp(I(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*b)*Pi*m)*exp(4*I*f*x)*exp(4*
I*e)+2*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3*Pi*m)*exp(1
/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e)))*P
i*m)*exp(1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*Pi*csgn(I/(exp
(2*I*(f*x+e))+1))*m)*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))
*csgn(I*exp(I*(f*x+e)))*Pi*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(1/2*I*csgn(I
exp(I(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x
+e))+1))^2*Pi*m)*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csg
n(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b)*Pi*m)*exp(-1/2*I*csgn(
I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3*Pi*m)*exp(1/2*I*csgn(I*b*exp(I*(
f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*b)*Pi*m)*exp(4*I*f*x)*exp(4*I*e)-2*m
*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3*Pi*m)*exp(1/2*I*c
sgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e)))*Pi*m)*e
xp(1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*Pi*csgn(I/(exp(2*I*(
f*x+e))+1))*m)*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(
I*exp(I*(f*x+e)))*Pi*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(1/2*I*csgn(I*exp(I
*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1
))^2*Pi*m)*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*
exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b)*Pi*m)*exp(-1/2*I*csgn(I*b*ex
p(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3*Pi*m)*exp(1/2*I*csgn(I*b*exp(I*(f*x+e)
)/(exp(2*I*(f*x+e))+1))^2*csgn(I*b)*Pi*m)*exp(2*I*f*x)*exp(2*I*e)+4*exp(-1/
2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3*Pi*m)*exp(1/2*I*csgn(I*ex
p(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e)))*Pi*m)*exp(1/2*I
*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*Pi*csgn(I/(exp(2*I*(f*x+e))+
1))*m)*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*
(f*x+e)))*Pi*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(1/2*I*csgn(I*exp(I*(f*x+e)
)/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*Pi*
m)*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f
*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b)*Pi*m)*exp(-1/2*I*csgn(I*b*exp(I*(f*x
+e))/(exp(2*I*(f*x+e))+1))^3*Pi*m)*exp(1/2*I*csgn(I*b*exp(I*(f*x+e))/(exp(2
I(f*x+e))+1))^2*csgn(I*b)*Pi*m)*exp(2*I*f*x)*exp(2*I*e)+m*exp(1/2*I*Pi*m*
(-csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3+csgn(I*exp(I*(f*x+e))/(exp(
2*I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e)))+csgn(I*exp(I*(f*x+e))/(exp(2*I*(f
*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))-csgn(I*exp(I*(f*x+e))/(exp(2*I*(f
*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(exp(2*I*(f*x+e))+1))+csgn(I*exp(I
*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1
))^2-csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(e
xp(2*I*(f*x+e))+1))*csgn(I*b)-csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))
^3+csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*b)))+2*exp(1/2*I*
Pi*m*(-csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3+csgn(I*exp(I*(f*x+e))/
(exp(2*I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e)))+csgn(I*exp(I*(f*x+e))/(exp(2
I(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))-csgn(I*exp(I*(f*x+e))/(exp(2
I(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(exp(2*I*(f*x+e))+1))+csgn(I*
exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+
e))+1))^2-csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e

```
))/(exp(2*I*(f*x+e))+1))*csgn(I*b)-csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e)
)+1))^3+csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*b))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = -\frac{\left((m+2)\cos\left(fx+e\right)^2-m\right)\left(\frac{b}{\cos(fx+e)}\right)^m}{(fm^2+2\,fm)\cos\left(fx+e\right)^2}$$

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
[Out] -((m + 2)*cos(f*x + e)^2 - m)*(b/cos(f*x + e))^m/((f*m^2 + 2*f*m)*cos(f*x +
 e)^2)
```

Sympy [F]

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = \begin{cases} x(b\sec{(e)})^m\tan^3{(e)} & \text{for } f=0 \\ \frac{\int \frac{\tan^3{(e+fx)}}{\sec^2{(e+fx)}}\,dx}{b^2} & \text{for } m=-2 \\ -\frac{\log{(\tan^2{(e+fx)}+1)}}{2f}+\frac{\tan^2{(e+fx)}}{2f} & \text{for } m=0 \\ \frac{m(b\sec{(e+fx)})^m\tan^2{(e+fx)}}{fm^2+2fm}-\frac{2(b\sec{(e+fx)})^m}{fm^2+2fm} & \text{otherwise} \end{cases}$$

```
[In] integrate((b*sec(f*x+e))**m*tan(f*x+e)**3,x)
[Out] Piecewise((x*(b*sec(e))**m*tan(e)**3, Eq(f, 0)), (Integral(tan(e + f*x)**3/
sec(e + f*x)**2, x)/b**2, Eq(m, -2)), (-log(tan(e + f*x)**2 + 1)/(2*f) + ta
n(e + f*x)**2/(2*f), Eq(m, 0)), (m*(b*sec(e + f*x))**m*tan(e + f*x)**2/(f*m
**2 + 2*f*m) - 2*(b*sec(e + f*x))**m/(f*m**2 + 2*f*m), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = -\frac{\frac{b^m\cos(fx+e)^{-m}}{m}-\frac{b^m\cos(fx+e)^{-m}}{(m+2)\cos(fx+e)^2}}{f}$$

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
[Out] -(b^m*cos(f*x + e)^(-m)/m - b^m*cos(f*x + e)^(-m)/((m + 2)*cos(f*x + e)^2))
/f
```

Giac [F]

$$\int (b\sec(e+fx))^m\tan^3(e+fx)\,dx = \int (b\sec(fx+e))^m\tan(fx+e)^3\,dx$$

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^3, x)
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.02

$$\int (b\sec(e+fx))^m\tan^3(e+fx)\,dx = -\frac{\left(\frac{b}{\cos(e+f\,x)}\right)^m\,(8\,\cos(2\,e+2\,f\,x)-m+2\,\cos(4\,e+4\,f\,x)+m\,\cos(4\,e+4\,f\,x)+6)}{f\,m\,(m+2)\,(4\,\cos(2\,e+2\,f\,x)+\cos(4\,e+4\,f\,x)+3)}$$

```
[In] int(tan(e + f*x)^3*(b/cos(e + f*x))^m,x)
[Out] -((b/cos(e + f*x))^m*(8*cos(2*e + 2*f*x) - m + 2*cos(4*e + 4*f*x) + m*cos(4
*e + 4*f*x) + 6))/(f*m*(m + 2)*(4*cos(2*e + 2*f*x) + cos(4*e + 4*f*x) + 3))
```

3.353 $\int (b\sec(e+fx))^m \tan(e+fx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \frac{(b\sec(e+fx))^m}{fm}$$

```
[Out] (b*sec(f*x+e))^m/f/m
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2686, 32}

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \frac{(b\sec(e+fx))^m}{fm}$$

```
[In] Int[(b*Sec[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] (b*Sec[e + f*x])^m/(f*m)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int (bx)^{-1+m}\,dx, x, \sec(e+fx)\right)}{f} \\
&= \frac{(b\sec(e+fx))^m}{fm}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \frac{(b\sec(e+fx))^m}{fm}$$

```
[In] Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x],x]
[Out] (b*Sec[e + f*x])^m/(f*m)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{(b\sec(fx+e))^m}{fm}$
default	$\frac{(b\sec(fx+e))^m}{fm}$
risch	$\left(e^{i(fx+e)}\right)^m \left(e^{2i(fx+e)}+1\right)^{-m} 2^m b^m e^{\frac{i\pi m\left(-\operatorname{csgn}\left(\frac{ie^{i(fx+e)}}{e^{2i(fx+e)}+1}\right)^3+\operatorname{csgn}\left(\frac{ie^{i(fx+e)}}{e^{2i(fx+e)}+1}\right)^2 \operatorname{csgn}\left(ie^{i(fx+e)}\right)+\operatorname{csgn}\left(\frac{ie^{i(fx+}}{e^{2i(fx+e}}\right.\right.}{}}$

```
[In] int((b*sec(f*x+e))^m*tan(f*x+e),x,method=_RETURNVERBOSE)
[Out] (b*sec(f*x+e))^m/f/m
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \frac{\left(\frac{b}{\cos(fx+e)}\right)^m}{fm}$$

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
[Out] (b/cos(f*x + e))^m/(f*m)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.47

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \begin{cases} x\tan{(e)} & \text{for } f=0 \wedge m=0 \\ x(b\sec{(e)})^m \tan{(e)} & \text{for } f=0 \\ \frac{\log{(\tan^2{(e+fx)}+1)}}{2f} & \text{for } m=0 \\ \frac{(b\sec{(e+fx)})^m}{fm} & \text{otherwise} \end{cases}$$

```
[In] integrate((b*sec(f*x+e))**m*tan(f*x+e),x)
[Out] Piecewise((x*tan(e), Eq(f, 0) & Eq(m, 0)), (x*(b*sec(e))**m*tan(e), Eq(f, 0
)), (log(tan(e + f*x)**2 + 1)/(2*f), Eq(m, 0)), ((b*sec(e + f*x))**m/(f*m),
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \frac{b^m \cos{(fx+e)}^{-m}}{fm}$$

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
[Out] b^m*cos(f*x + e)^(-m)/(f*m)
```

Giac [F]

$$\int (b\sec(e+fx))^m\tan(e+fx)\,dx=\int (b\sec{(fx+e)})^m\tan{(fx+e)}\,\,dx$$

[In] integrate((b*sec(f*x+e))^m*tan(f*x+e),x, algorithm="giac")

[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e), x)

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int (b\sec(e+fx))^m\tan(e+fx)\,dx=\frac{\left(\frac{b}{\cos(e+f\,x)}\right)^m}{f\,m}$$

[In] int(tan(e + f*x)*(b/cos(e + f*x))^m,x)

[Out] (b/cos(e + f*x))^m/(f*m)

3.354 $\int \cot(e+fx)(b\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

```
[Out] -hypergeom([1, 1/2*m],[1+1/2*m],sec(f*x+e)^2)*(b*sec(f*x+e))^m/f/m
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2686, 371}

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = -\frac{(b\sec(e+fx))^m\,\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},\sec^2(e+fx)\right)}{fm}$$

```
[In] Int[Cot[e + f*x]*(b*Sec[e + f*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[1, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)
/(f*m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{(bx)^{-1+m}}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{\text{Hypergeometric2F1}\left(1, \frac{m}{2}, \frac{2+m}{2}, \sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx$$
$$= -\frac{\text{Hypergeometric2F1}\left(1, \frac{m}{2}, \frac{2+m}{2}, \sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

```
[In] Integrate[Cot[e + f*x]*(b*Sec[e + f*x])^m,x]
[Out] -((Hypergeometric2F1[1, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)
/(f*m))
```

Maple [F]

$$\int \cot(fx+e)(b\sec(fx+e))^m\,dx$$

```
[In] int(cot(f*x+e)*(b*sec(f*x+e))^m,x)
[Out] int(cot(f*x+e)*(b*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m \cot(fx+e)\,dx$$

```
[In] integrate(cot(f*x+e)*(b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e), x)
```

Sympy [F]

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(e+fx))^m \cot(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)*(b*sec(f*x+e))**m,x)
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x), x)
```

Maxima [F]

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m \cot(fx+e)\,dx$$

```
[In] integrate(cot(f*x+e)*(b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e), x)
```

Giac [F]

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m \cot(fx+e)\,dx$$

```
[In] integrate(cot(f*x+e)*(b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = \int \cot(e+f\,x)\,\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

```
[In] int(cot(e + f*x)*(b/cos(e + f*x))^m,x)
[Out] int(cot(e + f*x)*(b/cos(e + f*x))^m, x)
```

3.355 $\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

```
[Out] hypergeom([2, 1/2*m],[1+1/2*m],sec(f*x+e)^2)*(b*sec(f*x+e))^m/f/m
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2686, 371}

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \frac{(b\sec(e+fx))^m\,\text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{m+2}{2},\sec^2(e+fx)\right)}{fm}$$

```
[In] Int[Cot[e + f*x]^3*(b*Sec[e + f*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)/(
f*m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{(bx)^{-1+m}}{(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Hypergeometric2F1}\left(2, \frac{m}{2}, \frac{2+m}{2}, \sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(2, \frac{m}{2}, \frac{2+m}{2}, \sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

```
[In] Integrate[Cot[e + f*x]^3*(b*Sec[e + f*x])^m,x]
[Out] (Hypergeometric2F1[2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)/(
f*m)
```

Maple [F]

$$\int \left(\cot^3(fx+e)\right)(b\sec(fx+e))^m\,dx$$

```
[In] int(cot(f*x+e)^3*(b*sec(f*x+e))^m,x)
[Out] int(cot(f*x+e)^3*(b*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec(fx+e))^m\cot(fx+e)^3\,dx$$

```
[In] integrate(cot(f*x+e)^3*(b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e)^3, x)
```

Sympy [F]

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec(e+fx))^m\cot^3(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)**3*(b*sec(f*x+e))**m,x)
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x)**3, x)
```

Maxima [F]

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec(fx+e))^m\cot(fx+e)^3\,dx$$

```
[In] integrate(cot(f*x+e)^3*(b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^3, x)
```

Giac [F]

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec(fx+e))^m\cot(fx+e)^3\,dx$$

```
[In] integrate(cot(f*x+e)^3*(b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \int \cot(e+f\,x)^3\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

```
[In] int(cot(e + f*x)^3*(b/cos(e + f*x))^m,x)
```

```
[Out] int(cot(e + f*x)^3*(b/cos(e + f*x))^m, x)
```

3.356 $\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\text{Hypergeometric2F1}\left(3,\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

```
[Out] -hypergeom([3, 1/2*m],[1+1/2*m],sec(f*x+e)^2)*(b*sec(f*x+e))^m/f/m
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2686, 371}

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx = -\frac{(b\sec(e+fx))^m\,\text{Hypergeometric2F1}\left(3,\frac{m}{2},\frac{m+2}{2},\sec^2(e+fx)\right)}{fm}$$

```
[In] Int[Cot[e + f*x]^5*(b*Sec[e + f*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[3, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)
/(f*m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{(bx)^{-1+m}}{(-1+x^2)^3}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{\text{Hypergeometric2F1}\left(3, \frac{m}{2}, \frac{2+m}{2}, \sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx$$
$$= -\frac{\text{Hypergeometric2F1}\left(3, \frac{m}{2}, \frac{2+m}{2}, \sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

```
[In] Integrate[Cot[e + f*x]^5*(b*Sec[e + f*x])^m,x]
[Out] -((Hypergeometric2F1[3, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)
/(f*m))
```

Maple [F]

$$\int \left(\cot^5(fx+e)\right)(b\sec(fx+e))^m\,dx$$

```
[In] int(cot(f*x+e)^5*(b*sec(f*x+e))^m,x)
[Out] int(cot(f*x+e)^5*(b*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec(fx+e))^m\cot(fx+e)^5\,dx$$

```
[In] integrate(cot(f*x+e)^5*(b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e)^5, x)
```

Sympy [F]

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec(e+fx))^m\cot^5(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)**5*(b*sec(f*x+e))**m,x)
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x)**5, x)
```

Maxima [F]

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec(fx+e))^m\cot(fx+e)^5\,dx$$

```
[In] integrate(cot(f*x+e)^5*(b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^5, x)
```

Giac [F]

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec(fx+e))^m\cot(fx+e)^5\,dx$$

```
[In] integrate(cot(f*x+e)^5*(b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx=\int \cot(e+f\,x)^5\,\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

```
[In] int(cot(e + f*x)^5*(b/cos(e + f*x))^m,x)
[Out] int(cot(e + f*x)^5*(b/cos(e + f*x))^m, x)
```

3.357 $\int (b\sec(e+fx))^m\tan^4(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int (b\sec(e+fx))^m\tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{\frac{5+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{5+m}{2},\frac{7}{2},\sin^2(e+fx)\right)(b\sec(e+fx))^m\tan^5(e+fx)}{5f}$$

```
[Out] 1/5*(cos(f*x+e)^2)^(5/2+1/2*m)*hypergeom([5/2, 5/2+1/2*m],[7/2],sin(f*x+e)^
2)*(b*sec(f*x+e))^m*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2697}

$$\int (b\sec(e+fx))^m\tan^4(e+fx)\,dx = \frac{\tan^5(e+fx)\cos^2(e+fx)^{\frac{m+5}{2}}(b\sec(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+5}{2},\frac{7}{2},\sin^2(e+fx)\right)}{5f}$$

```
[In] Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] ((Cos[e + f*x]^2)^((5 + m)/2)*Hypergeometric2F1[5/2, (5 + m)/2, 7/2, Sin[e
+ f*x]^2]*(b*Sec[e + f*x])^m*Tan[e + f*x]^5)/(5*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

integral

$$= \frac{\cos^2(e+fx)^{\frac{5+m}{2}} \text{Hypergeometric2F1}\left(\frac{5}{2}, \frac{5+m}{2}, \frac{7}{2}, \sin^2(e+fx)\right) (b\sec(e+fx))^m \tan^5(e+fx)}{5f}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\, dx$$
$$= \frac{\cot(e+fx) \text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{m}{2}, \frac{2+m}{2}, \sec^2(e+fx)\right) (b\sec(e+fx))^m \sqrt{-\tan^2(e+fx)}}{fm}$$

```
[In] Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^4,x]
[Out] (Cot[e + f*x]*Hypergeometric2F1[-3/2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Se
c[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/(f*m)
```

Maple [F]

$$\int (b\sec(fx+e))^m \left(\tan^4(fx+e)\right) dx$$

```
[In] int((b*sec(f*x+e))^m*tan(f*x+e)^4,x)
[Out] int((b*sec(f*x+e))^m*tan(f*x+e)^4,x)
```

Fricas [F]

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\, dx = \int (b\sec(fx+e))^m \tan(fx+e)^4\, dx$$

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^m*tan(f*x + e)^4, x)
```

Sympy [F]

$$\int (b\sec(e+fx))^m\tan^4(e+fx)\,dx=\int (b\sec{(e+fx)})^m\tan^4{(e+fx)}\,dx$$

```
[In] integrate((b*sec(f*x+e))**m*tan(f*x+e)**4,x)
[Out] Integral((b*sec(e + f*x))**m*tan(e + f*x)**4, x)
```

Maxima [F]

$$\int (b\sec(e+fx))^m\tan^4(e+fx)\,dx=\int (b\sec{(fx+e)})^m\tan{(fx+e)}^4\,dx$$

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^4, x)
```

Giac [F]

$$\int (b\sec(e+fx))^m\tan^4(e+fx)\,dx=\int (b\sec{(fx+e)})^m\tan{(fx+e)}^4\,dx$$

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^m\tan^4(e+fx)\,dx=\int \tan(e+f\,x)^4\left(\frac{b}{\cos{(e+f\,x)}}\right)^m\,dx$$

```
[In] int(tan(e + f*x)^4*(b/cos(e + f*x))^m,x)
[Out] int(tan(e + f*x)^4*(b/cos(e + f*x))^m, x)
```

3.358 $\int (b\sec(e+fx))^m \tan^2(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int (b\sec(e+fx))^m \tan^2(e+fx)\,dx = \frac{\cos^2(e+fx)^{\frac{3+m}{2}}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5}{2},\sin^2(e+fx)\right)(b\sec(e+fx))^m\tan^3(e+fx)}{3f}$$

```
[Out] 1/3*(cos(f*x+e)^2)^(3/2+1/2*m)*hypergeom([3/2, 3/2+1/2*m],[5/2],sin(f*x+e)^
2)*(b*sec(f*x+e))^m*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2697}

$$\int (b\sec(e+fx))^m \tan^2(e+fx)\,dx = \frac{\tan^3(e+fx)\cos^2(e+fx)^{\frac{m+3}{2}}(b\sec(e+fx))^m\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+3}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3f}$$

```
[In] Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] ((Cos[e + f*x]^2)^((3 + m)/2)*Hypergeometric2F1[3/2, (3 + m)/2, 5/2, Sin[e
+ f*x]^2]*(b*Sec[e + f*x])^m*Tan[e + f*x]^3)/(3*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\cos^2(e+fx)^{\frac{3+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5}{2},\sin^2(e+fx)\right)(b\sec(e+fx))^m\tan^3(e+fx)}{3f}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int (b\sec(e+fx))^m\tan^2(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m\tan(e+fx)}{fm\sqrt{-\tan^2(e+fx)}}$$

```
[In] Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^2,x]
[Out] (Hypergeometric2F1[-1/2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m
*Tan[e + f*x])/(f*m*Sqrt[-Tan[e + f*x]^2])
```

Maple [F]

$$\int (b\sec(fx+e))^m\left(\tan^2(fx+e)\right)dx$$

```
[In] int((b*sec(f*x+e))^m*tan(f*x+e)^2,x)
[Out] int((b*sec(f*x+e))^m*tan(f*x+e)^2,x)
```

Fricas [F]

$$\int (b\sec(e+fx))^m\tan^2(e+fx)\,dx = \int (b\sec(fx+e))^m\tan(fx+e)^2\,dx$$

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^m*tan(f*x + e)^2, x)
```

Sympy [F]

$$\int (b\sec(e+fx))^m\tan^2(e+fx)\,dx=\int (b\sec{(e+fx)})^m\tan^2{(e+fx)}\,dx$$

```
[In] integrate((b*sec(f*x+e))**m*tan(f*x+e)**2,x)
[Out] Integral((b*sec(e + f*x))**m*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int (b\sec(e+fx))^m\tan^2(e+fx)\,dx=\int (b\sec{(fx+e)})^m\tan{(fx+e)}^2\,dx$$

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^2, x)
```

Giac [F]

$$\int (b\sec(e+fx))^m\tan^2(e+fx)\,dx=\int (b\sec{(fx+e)})^m\tan{(fx+e)}^2\,dx$$

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^m\tan^2(e+fx)\,dx=\int \tan(e+f\,x)^2\left(\frac{b}{\cos{(e+f\,x)}}\right)^m\,dx$$

```
[In] int(tan(e + f*x)^2*(b/cos(e + f*x))^m,x)
[Out] int(tan(e + f*x)^2*(b/cos(e + f*x))^m, x)
```

3.359 $\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-1+m)}\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1}{2},\sin^2(e+fx)\right)(b\sec(e+fx))^m}{f}$$

```
[Out] -(cos(f*x+e)^2)^(-1/2+1/2*m)*cot(f*x+e)*hypergeom([-1/2, -1/2+1/2*m],[1/2],
sin(f*x+e)^2)*(b*sec(f*x+e))^m/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2697}

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cot(e+fx)\cos^2(e+fx)^{\frac{m-1}{2}}(b\sec(e+fx))^m\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m-1}{2},\frac{1}{2},\sin^2(e+fx)\right)}{f}$$

```
[In] Int[Cot[e + f*x]^2*(b*Sec[e + f*x])^m,x]
```

```
[Out] -(((Cos[e + f*x]^2)^((-1 + m)/2)*Cot[e + f*x]*Hypergeometric2F1[-1/2, (-1 +
 m)/2, 1/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-1+m)}\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1}{2},\sin^2(e+fx)\right)(b\sec(e+fx))}{f}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m\sqrt{-\tan^2(e+fx)}}{fm}$$

```
[In] Integrate[Cot[e + f*x]^2*(b*Sec[e + f*x])^m,x]
[Out] -((Cot[e + f*x]*Hypergeometric2F1[3/2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*S
ec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/(f*m))
```

Maple [F]

$$\int \left(\cot^2(fx+e)\right)(b\sec(fx+e))^m\,dx$$

```
[In] int(cot(f*x+e)^2*(b*sec(f*x+e))^m,x)
[Out] int(cot(f*x+e)^2*(b*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m\cot(fx+e)^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e)^2, x)
```

Sympy [F]

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec{(e+fx)})^m \cot^2{(e+fx)}\,dx$$

```
[In] integrate(cot(f*x+e)**2*(b*sec(f*x+e))**m,x)
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x)**2, x)
```

Maxima [F]

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec{(fx+e)})^m \cot{(fx+e)}^2\;dx$$

```
[In] integrate(cot(f*x+e)^2*(b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^2, x)
```

Giac [F]

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec{(fx+e)})^m \cot{(fx+e)}^2\;dx$$

```
[In] integrate(cot(f*x+e)^2*(b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = \int \cot(e+f\,x)^2\left(\frac{b}{\cos{(e+f\,x)}}\right)^m\,dx$$

```
[In] int(cot(e + f*x)^2*(b/cos(e + f*x))^m,x)
[Out] int(cot(e + f*x)^2*(b/cos(e + f*x))^m, x)
```

3.360 $\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-3+m)}\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),-\frac{1}{2},\sin^2(e+fx)\right)(b\sec(e+f$$
$$3f$$

```
[Out] -1/3*(cos(f*x+e)^2)^(-3/2+1/2*m)*cot(f*x+e)^3*hypergeom([-3/2, -3/2+1/2*m],
[-1/2],sin(f*x+e)^2)*(b*sec(f*x+e))^m/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2697}

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cot^3(e+fx)\cos^2(e+fx)^{\frac{m-3}{2}}(b\sec(e+fx))^m\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m-3}{2},-\frac{1}{2},\sin^2(e+fx)\right)}{3f}$$

```
[In] Int[Cot[e + f*x]^4*(b*Sec[e + f*x])^m,x]
```

```
[Out] -1/3*((Cos[e + f*x]^2)^((-3 + m)/2)*Cot[e + f*x]^3*Hypergeometric2F1[-3/2,
(-3 + m)/2, -1/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/f
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-3+m)}\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),-\frac{1}{2},\sin^2(e+fx)\right)(b\sec(e+fx)}{3f}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = \frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m\sqrt{-\tan^2(e+fx)}}{fm}$$

```
[In] Integrate[Cot[e + f*x]^4*(b*Sec[e + f*x])^m,x]
[Out] (Cot[e + f*x]*Hypergeometric2F1[5/2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec
[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/(f*m)
```

Maple [F]

$$\int \left(\cot^4(fx+e)\right)(b\sec(fx+e))^m\,dx$$

```
[In] int(cot(f*x+e)^4*(b*sec(f*x+e))^m,x)
[Out] int(cot(f*x+e)^4*(b*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m\cot(fx+e)^4\,dx$$

```
[In] integrate(cot(f*x+e)^4*(b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e)^4, x)
```

Sympy [F]

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec{(e+fx)})^m\cot^4{(e+fx)}\,dx$$

```
[In] integrate(cot(f*x+e)**4*(b*sec(f*x+e))**m,x)
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x)**4, x)
```

Maxima [F]

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec{(fx+e)})^m\cot{(fx+e)}^4\,dx$$

```
[In] integrate(cot(f*x+e)^4*(b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^4, x)
```

Giac [F]

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec{(fx+e)})^m\cot{(fx+e)}^4\,dx$$

```
[In] integrate(cot(f*x+e)^4*(b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx=\int \cot(e+f\,x)^4\left(\frac{b}{\cos{(e+f\,x)}}\right)^m\,dx$$

```
[In] int(cot(e + f*x)^4*(b/cos(e + f*x))^m,x)
[Out] int(cot(e + f*x)^4*(b/cos(e + f*x))^m, x)
```

3.361 $\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-5+m)}\cot^5(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}(-5+m),-\frac{3}{2},\sin^2(e+fx)\right)(b\sec(e+fx)}{5f}$$

```
[Out] -1/5*(cos(f*x+e)^2)^(-5/2+1/2*m)*cot(f*x+e)^5*hypergeom([-5/2, -5/2+1/2*m],
[-3/2],sin(f*x+e)^2)*(b*sec(f*x+e))^m/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2697}

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cot^5(e+fx)\cos^2(e+fx)^{\frac{m-5}{2}}(b\sec(e+fx))^m\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m-5}{2},-\frac{3}{2},\sin^2(e+fx)\right)}{5f}$$

```
[In] Int[Cot[e + f*x]^6*(b*Sec[e + f*x])^m,x]
```

```
[Out] -1/5*((Cos[e + f*x]^2)^((-5 + m)/2)*Cot[e + f*x]^5*Hypergeometric2F1[-5/2,
(-5 + m)/2, -3/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/f
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

integral =

$$-\frac{\cos^2(e+fx)^{\frac{1}{2}(-5+m)}\cot^5(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}(-5+m),-\frac{3}{2},\sin^2(e+fx)\right)(b\sec(e+f$$

$$5f$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m\sqrt{-\tan^2(e+fx)}}{fm}$$

```
[In] Integrate[Cot[e + f*x]^6*(b*Sec[e + f*x])^m,x]
[Out] -((Cot[e + f*x]*Hypergeometric2F1[7/2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*S
ec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/(f*m))
```

Maple [F]

$$\int \left(\cot^6(fx+e)\right)(b\sec(fx+e))^m\,dx$$

```
[In] int(cot(f*x+e)^6*(b*sec(f*x+e))^m,x)
[Out] int(cot(f*x+e)^6*(b*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m\cot(fx+e)^6\,dx$$

```
[In] integrate(cot(f*x+e)^6*(b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e)^6, x)
```

Sympy [F]

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec(e+fx))^m\cot^6(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)**6*(b*sec(f*x+e))**m,x)
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x)**6, x)
```

Maxima [F]

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec(fx+e))^m\cot(fx+e)^6\,dx$$

```
[In] integrate(cot(f*x+e)^6*(b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^6, x)
```

Giac [F]

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec(fx+e))^m\cot(fx+e)^6\,dx$$

```
[In] integrate(cot(f*x+e)^6*(b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx=\int \cot(e+f\,x)^6\left(\frac{b}{\cos(e+f\,x)}\right)^m\,dx$$

```
[In] int(cot(e + f*x)^6*(b/cos(e + f*x))^m,x)
[Out] int(cot(e + f*x)^6*(b/cos(e + f*x))^m, x)
```

3.362 $\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+n)}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{2}(1+m+n),\frac{3+n}{2},\sin^2(e+fx)\right)(a\sec(e+fx))^m(b\tan(}{bf(1+n)}$$

```
[Out] (cos(f*x+e)^2)^(1/2+1/2*m+1/2*n)*hypergeom([1/2+1/2*n, 1/2+1/2*m+1/2*n],[3/
2+1/2*n],sin(f*x+e)^2)*(a*sec(f*x+e))^m*(b*tan(f*x+e))^(1+n)/b/f/(1+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2697}

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx$$
$$= \frac{(a\sec(e+fx))^m(b\tan(e+fx))^{n+1}\cos^2(e+fx)^{\frac{1}{2}(m+n+1)}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{2}(m+n+1),\frac{n+3}{2},\text{si}\right.}{bf(n+1)}$$

```
[In] Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + m + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + m + n)/
2, (3 + n)/2, Sin[e + f*x]^2]*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 + n))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
```

```
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

integral

$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+n)}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{2}(1+m+n),\frac{3+n}{2},\sin^2(e+fx)\right)(a\sec(e+fx))^m(b\tan(e}{bf(1+n)}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx$$

$$= \frac{b\,\text{Hypergeometric2F1}\left(\frac{m}{2},\frac{1-n}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(a\sec(e+fx))^m(b\tan(e+fx))^{-1+n}\left(-\tan^2(e+fx)\right)^{\frac{1-}{2}}}{fm}$$

```
[In] Integrate[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^n,x]
[Out] (b*Hypergeometric2F1[m/2, (1 - n)/2, (2 + m)/2, Sec[e + f*x]^2]*(a*Sec[e +
f*x])^m*(b*Tan[e + f*x])^(-1 + n)*(-Tan[e + f*x]^2)^((1 - n)/2))/(f*m)
```

Maple [F]

$$\int (a\sec(fx+e))^m(b\tan(fx+e))^n\,dx$$

```
[In] int((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x)
[Out] int((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\sec(fx+e))^m(b\tan(fx+e))^n\,dx$$

```
[In] integrate((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*sec(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F]

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\sec{(e+fx)})^m\,(b\tan{(e+fx)})^n\,dx$$

```
[In] integrate((a*sec(f*x+e))**m*(b*tan(f*x+e))**n,x)
[Out] Integral((a*sec(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Maxima [F]

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\sec{(fx+e)})^m\,(b\tan{(fx+e)})^n\,dx$$

```
[In] integrate((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Giac [F]

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\sec{(fx+e)})^m\,(b\tan{(fx+e)})^n\,dx$$

```
[In] integrate((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx=\int (b\tan(e+f\,x))^n\,\left(\frac{a}{\cos{(e+f\,x)}}\right)^m\,dx$$

```
[In] int((b*tan(e + f*x))^n*(a/cos(e + f*x))^m,x)
[Out] int((b*tan(e + f*x))^n*(a/cos(e + f*x))^m, x)
```

3.363 $\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \frac{(d\tan(a+bx))^{1+n}}{bd(1+n)} + \frac{2(d\tan(a+bx))^{3+n}}{bd^3(3+n)} + \frac{(d\tan(a+bx))^{5+n}}{bd^5(5+n)}$$

```
[Out] (d*tan(b*x+a))^(1+n)/b/d/(1+n)+2*(d*tan(b*x+a))^(3+n)/b/d^3/(3+n)+(d*tan(b*
x+a))^(5+n)/b/d^5/(5+n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2687, 276}

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \frac{(d\tan(a+bx))^{n+5}}{bd^5(n+5)} + \frac{2(d\tan(a+bx))^{n+3}}{bd^3(n+3)} + \frac{(d\tan(a+bx))^{n+1}}{bd(n+1)}$$

```
[In] Int[Sec[a + b*x]^6*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*Tan[a + b*x])^(1 + n)/(b*d*(1 + n)) + (2*(d*Tan[a + b*x])^(3 + n))/(b*d^
3*(3 + n)) + (d*Tan[a + b*x])^(5 + n)/(b*d^5*(5 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (dx)^n \left(1+x^2\right)^2 \, dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left((dx)^n + \frac{2(dx)^{2+n}}{d^2} + \frac{(dx)^{4+n}}{d^4}\right) \, dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{(d\tan(a+bx))^{1+n}}{bd(1+n)} + \frac{2(d\tan(a+bx))^{3+n}}{bd^3(3+n)} + \frac{(d\tan(a+bx))^{5+n}}{bd^5(5+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

$$\int \sec^6(a+bx)(d\tan(a+bx))^n \, dx$$
$$= \frac{d(d\tan(a+bx))^{-1+n}\left(\left(8+6n+n^2+2(3+n)\cos(2(a+bx))+\cos(4(a+bx))\right)\sec^4(a+bx)\tan^2(a+bx\right.}{b(1+n)(3+n)(5+n)}$$

```
[In] Integrate[Sec[a + b*x]^6*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*(d*Tan[a + b*x])^(-1 + n)*((8 + 6*n + n^2 + 2*(3 + n)*Cos[2*(a + b*x)] +
 Cos[4*(a + b*x)])*Sec[a + b*x]^4*Tan[a + b*x]^2 + 8*(-Tan[a + b*x]^2)^((1
- n)/2)))/(b*(1 + n)*(3 + n)*(5 + n))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\frac{\tan(bx+a)\,e^{n\ln(d\tan(bx+a))}}{b(1+n)} + \frac{(\tan^5(bx+a))\,e^{n\ln(d\tan(bx+a))}}{b(5+n)} + \frac{2(\tan^3(bx+a))\,e^{n\ln(d\tan(bx+a))}}{b(3+n)}$$

```
[In] int(sec(b*x+a)^6*(d*tan(b*x+a))^n,x)
```

```
[Out] 1/b/(1+n)*tan(b*x+a)*exp(n*ln(d*tan(b*x+a)))+1/b/(5+n)*tan(b*x+a)^5*exp(n*l
n(d*tan(b*x+a)))+2/b/(3+n)*tan(b*x+a)^3*exp(n*ln(d*tan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \frac{\left(8\cos(bx+a)^4+4(n+1)\cos(bx+a)^2+n^2+4n+3\right)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^n\sin(bx+a)}{(bn^3+9bn^2+23bn+15b)\cos(bx+a)^5}$$

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] (8*cos(b*x + a)^4 + 4*(n + 1)*cos(b*x + a)^2 + n^2 + 4*n + 3)*(d*sin(b*x +
a)/cos(b*x + a))^n*sin(b*x + a)/((b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*cos(b*x
+ a)^5)
```

Sympy [F]

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(a+bx))^n\sec^6(a+bx)\,dx$$

```
[In] integrate(sec(b*x+a)**6*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \frac{\frac{d^n\tan(bx+a)^n\tan(bx+a)^5}{n+5}+\frac{2\,d^n\tan(bx+a)^n\tan(bx+a)^3}{n+3}+\frac{(d\tan(bx+a))^{n+1}}{d(n+1)}}{b}$$

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (d^n*tan(b*x + a)^n*tan(b*x + a)^5/(n + 5) + 2*d^n*tan(b*x + a)^n*tan(b*x +
 a)^3/(n + 3) + (d*tan(b*x + a))^(n + 1)/(d*(n + 1)))/b
```

Giac [F(-2)]

Exception generated.

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,4,0,0]%%%}+%%%{2,[0,1,2,2,0]%%%}+%%%{1,[0,1,0,4,0]%%%}
 / %%%
```

Mupad [F(-1)]

Timed out.

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \int \frac{(d\tan(a+b\,x))^n}{\cos\left(a+b\,x\right)^6}\,dx$$

```
[In] int((d*tan(a + b*x))^n/cos(a + b*x)^6,x)
```

```
[Out] int((d*tan(a + b*x))^n/cos(a + b*x)^6, x)
```

3.364 $\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx=\frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}+\frac{(d\tan(a+bx))^{3+n}}{bd^3(3+n)}$$

```
[Out] (d*tan(b*x+a))^(1+n)/b/d/(1+n)+(d*tan(b*x+a))^(3+n)/b/d^3/(3+n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2687, 14}

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx=\frac{(d\tan(a+bx))^{n+3}}{bd^3(n+3)}+\frac{(d\tan(a+bx))^{n+1}}{bd(n+1)}$$

```
[In] Int[Sec[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*Tan[a + b*x])^(1 + n)/(b*d*(1 + n)) + (d*Tan[a + b*x])^(3 + n)/(b*d^3*(3
 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (dx)^n\,(1+x^2)\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left((dx)^n+\frac{(dx)^{2+n}}{d^2}\right)\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}+\frac{(d\tan(a+bx))^{3+n}}{bd^3(3+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{d(d\tan(a+bx))^{-1+n}\left((2+n+\cos(2(a+bx)))\sec^2(a+bx)\tan^2(a+bx)+2(-\tan^2(a+bx))^{\frac{1-n}{2}}\right)}{b(1+n)(3+n)}$$

```
[In] Integrate[Sec[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*(d*Tan[a + b*x])^(-1 + n)*((2 + n + Cos[2*(a + b*x)])*Sec[a + b*x]^2*Tan
[a + b*x]^2 + 2*(-Tan[a + b*x]^2)^((1 - n)/2)))/(b*(1 + n)*(3 + n))
```

Maple [A] (verified)

Time = 56.74 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$\frac{\tan(bx+a)e^{n\ln(d\tan(bx+a))}}{b(1+n)}+\frac{(\tan^3(bx+a))e^{n\ln(d\tan(bx+a))}}{b(3+n)}$	58
default	$\frac{\tan(bx+a)e^{n\ln(d\tan(bx+a))}}{b(1+n)}+\frac{(\tan^3(bx+a))e^{n\ln(d\tan(bx+a))}}{b(3+n)}$	58
risch	Expression too large to display	5283

```
[In] int(sec(b*x+a)^4*(d*tan(b*x+a))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(1+n)*tan(b*x+a)*exp(n*ln(d*tan(b*x+a)))+1/b/(3+n)*tan(b*x+a)^3*exp(n*l
n(d*tan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{\left(2\cos\left(bx+a\right)^2+n+1\right)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^n\sin\left(bx+a\right)}{\left(bn^2+4\,bn+3\,b\right)\cos\left(bx+a\right)^3}$$

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="fricas")
[Out] (2*cos(b*x + a)^2 + n + 1)*(d*sin(b*x + a)/cos(b*x + a))^n*sin(b*x + a)/((b
*n^2 + 4*b*n + 3*b)*cos(b*x + a)^3)
```

Sympy [F]

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = \int \left(d\tan\left(a+bx\right)\right)^n\sec^4\left(a+bx\right)\,dx$$

```
[In] integrate(sec(b*x+a)**4*(d*tan(b*x+a))**n,x)
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{\frac{d^n\tan(bx+a)^n\tan(bx+a)^3}{n+3}+\frac{(d\tan(bx+a))^{n+1}}{d(n+1)}}{b}$$

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="maxima")
[Out] (d^n*tan(b*x + a)^n*tan(b*x + a)^3/(n + 3) + (d*tan(b*x + a))^(n + 1)/(d*(n
 + 1)))/b
```

Giac [F(-2)]

Exception generated.

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,2,0,0]%%%}+%%%{1,[0,1,0,2,0]%%%} / %%%{1,[0,0,3,0,1]%%
%} Err
```

Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = -\frac{2\left(-\frac{d\sin(2a+2bx)}{2\sin(a+bx)^2-2}\right)^n(9\sin(2a+2bx)+6\sin(4a+4bx)+\sin(6a+6bx)+4n\sin(2a+2bx)+2}{b\,(n^2+4n+3)\,\left(30\sin(a+bx)^2+12\sin(2a+2bx)^2+2\sin(3a+3bx)^2-32\right)}$$

```
[In] int((d*tan(a + b*x))^n/cos(a + b*x)^4,x)
```

```
[Out] -(2*(-(d*sin(2*a + 2*b*x))/(2*sin(a + b*x)^2 - 2))^n*(9*sin(2*a + 2*b*x) +
6*sin(4*a + 4*b*x) + sin(6*a + 6*b*x) + 4*n*sin(2*a + 2*b*x) + 2*n*sin(4*a
+ 4*b*x)))/(b*(4*n + n^2 + 3)*(12*sin(2*a + 2*b*x)^2 + 2*sin(3*a + 3*b*x)^2
 + 30*sin(a + b*x)^2 - 32))
```

3.365 $\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx=\frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] (d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2687, 32}

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx=\frac{(d\tan(a+bx))^{n+1}}{bd(n+1)}$$

```
[In] Int[Sec[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*Tan[a + b*x])^(1 + n)/(b*d*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (dx)^n\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{\tan(a+bx)(d\tan(a+bx))^n}{b(1+n)}$$

```
[In] Integrate[Sec[a + b*x]^2*(d*Tan[a + b*x])^n,x]
[Out] (Tan[a + b*x]*(d*Tan[a + b*x])^n)/(b*(1 + n))
```

Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{(d\tan(bx+a))^{1+n}}{bd(1+n)}$	25
default	$\frac{(d\tan(bx+a))^{1+n}}{bd(1+n)}$	25
risch	Expression too large to display	1752

```
[In] int(sec(b*x+a)^2*(d*tan(b*x+a))^n,x,method=_RETURNVERBOSE)
[Out] (d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^n \sin(bx+a)}{(bn+b)\cos(bx+a)}$$

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="fricas")
[Out] (d*sin(b*x + a)/cos(b*x + a))^n*sin(b*x + a)/((b*n + b)*cos(b*x + a))
```

Sympy [F]

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx=\int (d\tan{(a+bx)})^n\sec^2{(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**2*(d*tan(b*x+a))**n,x)
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx=\frac{(d\tan{(bx+a)})^{n+1}}{bd(n+1)}$$

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="maxima")
[Out] (d*tan(b*x + a))^(n + 1)/(b*d*(n + 1))
```

Giac [F(-2)]

Exception generated.

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,1]%%%} Error: Bad Argument Val
ue
```

Mupad [B] (verification not implemented)

Time = 3.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx=\frac{\sin{(2\,a+2\,b\,x)}\,{\left(\frac{d\,\sin(2\,a+2\,b\,x)}{2\,{\cos(a+b\,x)}^2}\right)}^n}{2\,b\,{\cos{(a+b\,x)}}^2\,(n+1)}$$

```
[In] int((d*tan(a + b*x))^n/cos(a + b*x)^2,x)
[Out] (sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(2*cos(a + b*x)^2))^n)/(2*b*cos(a +
 b*x)^2*(n + 1))
```

3.366 $\int (d\tan(a+bx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 50

$$\int (d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b/
d/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3557, 371}

$$\int (d\tan(a+bx))^n\,dx = \frac{(d\tan(a+bx))^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(a+bx)\right)}{bd(n+1)}$$

```
[In] Int[(d*Tan[a + b*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*x
])^(1 + n))/(b*d*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d\text{Subst}\left(\int \frac{x^n}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= \frac{\text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, -\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int (d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, -\tan^2(a+bx)\right)\tan(a+bx)(d\tan(a+bx))^n}{b(1+n)}$$

```
[In] Integrate[(d*Tan[a + b*x])^n,x]
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*Tan[a + b*x]*(
d*Tan[a + b*x])^n)/(b*(1 + n))
```

Maple [F]

$$\int (d\tan(bx+a))^n\,dx$$

```
[In] int((d*tan(b*x+a))^n,x)
[Out] int((d*tan(b*x+a))^n,x)
```

Fricas [F]

$$\int (d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\,dx$$

```
[In] integrate((d*tan(b*x+a))^n,x, algorithm="fricas")
[Out] integral((d*tan(b*x + a))^n, x)
```

Sympy [F]

$$\int (d\tan(a+bx))^n\,dx = \int (d\tan{(a+bx)})^n\,\,dx$$

```
[In] integrate((d*tan(b*x+a))**n,x)
[Out] Integral((d*tan(a + b*x))**n, x)
```

Maxima [F]

$$\int (d\tan(a+bx))^n\,dx = \int (d\tan{(bx+a)})^n\,\,dx$$

```
[In] integrate((d*tan(b*x+a))^n,x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^n, x)
```

Giac [F]

$$\int (d\tan(a+bx))^n\,dx = \int (d\tan{(bx+a)})^n\,\,dx$$

```
[In] integrate((d*tan(b*x+a))^n,x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\tan(a+bx))^n\,dx = \int (d\tan(a+b\,x))^n\,dx$$

```
[In] int((d*tan(a + b*x))^n,x)
[Out] int((d*tan(a + b*x))^n, x)
```

3.367 $\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] hypergeom([2, 1/2+1/2*n],[3/2+1/2*n],-tan(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b/
d/(1+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2687, 371}

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{(d\tan(a+bx))^{n+1}\text{Hypergeometric2F1}\left(2,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(a+bx)\right)}{bd(n+1)}$$

```
[In] Int[Cos[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

```
[Out] (Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*x
])^(1 + n))/(b*d*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(dx)^n}{(1+x^2)^2}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Hypergeometric2F1}\left(2, \frac{1+n}{2}, \frac{3+n}{2}, -\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(2, \frac{1+n}{2}, \frac{3+n}{2}, -\tan^2(a+bx)\right)\tan(a+bx)(d\tan(a+bx))^n}{b(1+n)}$$

```
[In] Integrate[Cos[a + b*x]^2*(d*Tan[a + b*x])^n,x]
[Out] (Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*Tan[a + b*x]*(
d*Tan[a + b*x])^n)/(b*(1 + n))
```

Maple [F]

$$\int \left(\cos^2(bx+a)\right)(d\tan(bx+a))^n\,dx$$

```
[In] int(cos(b*x+a)^2*(d*tan(b*x+a))^n,x)
[Out] int(cos(b*x+a)^2*(d*tan(b*x+a))^n,x)
```

Fricas [F]

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)^2\,dx$$

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="fricas")
[Out] integral((d*tan(b*x + a))^n*cos(b*x + a)^2, x)
```

Sympy [F]

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(a+bx))^n\cos^2(a+bx)\,dx$$

```
[In] integrate(cos(b*x+a)**2*(d*tan(b*x+a))**n,x)
[Out] Integral((d*tan(a + b*x))**n*cos(a + b*x)**2, x)
```

Maxima [F]

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)^2\,dx$$

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^2, x)
```

Giac [F]

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)^2\,dx$$

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \int \cos\left(a+b\,x\right)^2\,\left(d\tan(a+b\,x)\right)^n\,dx$$

```
[In] int(cos(a + b*x)^2*(d*tan(a + b*x))^n,x)
[Out] int(cos(a + b*x)^2*(d*tan(a + b*x))^n, x)
```

3.368 $\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(3,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] hypergeom([3, 1/2+1/2*n],[3/2+1/2*n],-tan(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b/
d/(1+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2687, 371}

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{(d\tan(a+bx))^{n+1}\text{Hypergeometric2F1}\left(3,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(a+bx)\right)}{bd(n+1)}$$

```
[In] Int[Cos[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

```
[Out] (Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*x
])^(1 + n))/(b*d*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(dx)^n}{(1+x^2)^3}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Hypergeometric2F1}\left(3, \frac{1+n}{2}, \frac{3+n}{2}, -\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(3, \frac{1+n}{2}, \frac{3+n}{2}, -\tan^2(a+bx)\right)\tan(a+bx)(d\tan(a+bx))^n}{b(1+n)}$$

```
[In] Integrate[Cos[a + b*x]^4*(d*Tan[a + b*x])^n,x]
[Out] (Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*Tan[a + b*x]*(
d*Tan[a + b*x])^n)/(b*(1 + n))
```

Maple [F]

$$\int \left(\cos^4(bx+a)\right)(d\tan(bx+a))^n\,dx$$

```
[In] int(cos(b*x+a)^4*(d*tan(b*x+a))^n,x)
[Out] int(cos(b*x+a)^4*(d*tan(b*x+a))^n,x)
```

Fricas [F]

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx=\int (d\tan(bx+a))^n\cos(bx+a)^4\,dx$$

```
[In] integrate(cos(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="fricas")
[Out] integral((d*tan(b*x + a))^n*cos(b*x + a)^4, x)
```

Sympy [F]

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx=\int (d\tan(a+bx))^n\cos^4(a+bx)\,dx$$

```
[In] integrate(cos(b*x+a)**4*(d*tan(b*x+a))**n,x)
[Out] Integral((d*tan(a + b*x))**n*cos(a + b*x)**4, x)
```

Maxima [F]

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx=\int (d\tan(bx+a))^n\cos(bx+a)^4\,dx$$

```
[In] integrate(cos(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^4, x)
```

Giac [F]

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx=\int (d\tan(bx+a))^n\cos(bx+a)^4\,dx$$

```
[In] integrate(cos(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \int \cos\left(a+b\,x\right)^4\,{\left(d\tan(a+b\,x)\right)}^n\,dx$$

```
[In] int(cos(a + b*x)^4*(d*tan(a + b*x))^n,x)
[Out] int(cos(a + b*x)^4*(d*tan(a + b*x))^n, x)
```

3.369 $\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^2(a+bx)^{\frac{6+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{6+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sec^5(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] (cos(b*x+a)^2)^(3+1/2*n)*hypergeom([3+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(b*x
+a)^2)*sec(b*x+a)^5*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2697}

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \frac{\sec^5(a+bx)\cos^2(a+bx)^{\frac{n+6}{2}}(d\tan(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+6}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bd(n+1)}$$

```
[In] Int[Sec[a + b*x]^5*(d*Tan[a + b*x])^n,x]
```

```
[Out] ((Cos[a + b*x]^2)^((6 + n)/2)*Hypergeometric2F1[(1 + n)/2, (6 + n)/2, (3 +
n)/2, Sin[a + b*x]^2]*Sec[a + b*x]^5*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n)
)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
```

```
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

integral

$$= \frac{\cos^2(a+bx)^{\frac{6+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{6+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sec^5(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx$$
$$= \frac{d\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-n}{2},\frac{7}{2},\sec^2(a+bx)\right)\sec^5(a+bx)(d\tan(a+bx))^{-1+n}\left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}}}{5b}$$

```
[In] Integrate[Sec[a + b*x]^5*(d*Tan[a + b*x])^n,x]
[Out] (d*Hypergeometric2F1[5/2, (1 - n)/2, 7/2, Sec[a + b*x]^2]*Sec[a + b*x]^5*(d
*Tan[a + b*x])^(-1 + n)*(-Tan[a + b*x]^2)^((1 - n)/2))/(5*b)
```

Maple [F]

$$\int \left(\sec^5(bx+a)\right)(d\tan(bx+a))^n\,dx$$

```
[In] int(sec(b*x+a)^5*(d*tan(b*x+a))^n,x)
[Out] int(sec(b*x+a)^5*(d*tan(b*x+a))^n,x)
```

Fricas [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\sec(bx+a)^5\,dx$$

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^n,x, algorithm="fricas")
[Out] integral((d*tan(b*x + a))^n*sec(b*x + a)^5, x)
```

Sympy [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(a+bx))^n \sec^5(a+bx)\,dx$$

```
[In] integrate(sec(b*x+a)**5*(d*tan(b*x+a))**n,x)
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x)**5, x)
```

Maxima [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \sec(bx+a)^5\,dx$$

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^n,x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a)^5, x)
```

Giac [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \sec(bx+a)^5\,dx$$

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^n,x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \int \frac{(d\tan(a+b\,x))^n}{\cos(a+b\,x)^5}\,dx$$

```
[In] int((d*tan(a + b*x))^n/cos(a + b*x)^5,x)
[Out] int((d*tan(a + b*x))^n/cos(a + b*x)^5, x)
```

3.370 $\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^2(a+bx)^{\frac{4+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{4+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sec^3(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] (cos(b*x+a)^2)^(2+1/2*n)*hypergeom([2+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(b*x
+a)^2)*sec(b*x+a)^3*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2697}

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \frac{\sec^3(a+bx)\cos^2(a+bx)^{\frac{n+4}{2}}(d\tan(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+4}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bd(n+1)}$$

```
[In] Int[Sec[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

```
[Out] ((Cos[a + b*x]^2)^((4 + n)/2)*Hypergeometric2F1[(1 + n)/2, (4 + n)/2, (3 +
n)/2, Sin[a + b*x]^2]*Sec[a + b*x]^3*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n)
)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
```

```
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

integral

$$= \frac{\cos^2(a+bx)^{\frac{4+n}{2}} \text{Hypergeometric2F1}\left(\frac{1+n}{2}, \frac{4+n}{2}, \frac{3+n}{2}, \sin^2(a+bx)\right) \sec^3(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx$$
$$= \frac{d\,\text{Hypergeometric2F1}\left(\frac{3}{2}, \frac{1-n}{2}, \frac{5}{2}, \sec^2(a+bx)\right) \sec^3(a+bx)(d\tan(a+bx))^{-1+n} \left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}}}{3b}$$

```
[In] Integrate[Sec[a + b*x]^3*(d*Tan[a + b*x])^n,x]
[Out] (d*Hypergeometric2F1[3/2, (1 - n)/2, 5/2, Sec[a + b*x]^2]*Sec[a + b*x]^3*(d
*Tan[a + b*x])^(-1 + n)*(-Tan[a + b*x]^2)^((1 - n)/2))/(3*b)
```

Maple [F]

$$\int \left(\sec^3(bx+a)\right) (d\tan(bx+a))^n\,dx$$

```
[In] int(sec(b*x+a)^3*(d*tan(b*x+a))^n,x)
[Out] int(sec(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

Fricas [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \sec(bx+a)^3\,dx$$

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="fricas")
[Out] integral((d*tan(b*x + a))^n*sec(b*x + a)^3, x)
```

Sympy [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan{(a+bx)})^n \sec^3{(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**3*(d*tan(b*x+a))**n,x)
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x)**3, x)
```

Maxima [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan{(bx+a)})^n \sec{(bx+a)}^3\,dx$$

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a)^3, x)
```

Giac [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan{(bx+a)})^n \sec{(bx+a)}^3\,dx$$

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \int \frac{(d\tan(a+b\,x))^n}{\cos{(a+b\,x)}^3}\,dx$$

```
[In] int((d*tan(a + b*x))^n/cos(a + b*x)^3,x)
[Out] int((d*tan(a + b*x))^n/cos(a + b*x)^3, x)
```

3.371 $\int \sec(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^2(a+bx)^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sec(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] (cos(b*x+a)^2)^(1+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(b*x
+a)^2)*sec(b*x+a)*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2697}

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \frac{\sec(a+bx)\cos^2(a+bx)^{\frac{n+2}{2}}(d\tan(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bd(n+1)}$$

```
[In] Int[Sec[a + b*x]*(d*Tan[a + b*x])^n,x]
```

```
[Out] ((Cos[a + b*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n)/2, (3 +
n)/2, Sin[a + b*x]^2]*Sec[a + b*x]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\cos^2(a+bx)^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sec(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3}{2},\sec^2(a+bx)\right)(d\tan(a+bx))^n\left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}}}{b}$$

```
[In] Integrate[Sec[a + b*x]*(d*Tan[a + b*x])^n,x]
[Out] (Csc[a + b*x]*Hypergeometric2F1[1/2, (1 - n)/2, 3/2, Sec[a + b*x]^2]*(d*Tan
[a + b*x])^n*(-Tan[a + b*x]^2)^((1 - n)/2))/b
```

Maple [F]

$$\int \sec(bx+a)\,(d\tan(bx+a))^n\,dx$$

```
[In] int(sec(b*x+a)*(d*tan(b*x+a))^n,x)
[Out] int(sec(b*x+a)*(d*tan(b*x+a))^n,x)
```

Fricas [F]

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\sec(bx+a)\,dx$$

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="fricas")
[Out] integral((d*tan(b*x + a))^n*sec(b*x + a), x)
```

Sympy [F]

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan{(a+bx)})^n \sec{(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))**n,x)
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x), x)
```

Maxima [F]

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan{(bx+a)})^n \sec{(bx+a)}\;dx$$

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a), x)
```

Giac [F]

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan{(bx+a)})^n \sec{(bx+a)}\;dx$$

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \int \frac{(d\tan(a+b\,x))^n}{\cos{(a+b\,x)}}\,dx$$

```
[In] int((d*tan(a + b*x))^n/cos(a + b*x),x)
[Out] int((d*tan(a + b*x))^n/cos(a + b*x), x)
```

3.372 $\int \cos(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos(a+bx)\cos^2(a+bx)^{n/2}\,\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] cos(b*x+a)*(cos(b*x+a)^2)^(1/2*n)*hypergeom([1/2*n, 1/2+1/2*n],[3/2+1/2*n],
sin(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2697}

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos(a+bx)\cos^2(a+bx)^{n/2}(d\tan(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bd(n+1)}$$

```
[In] Int[Cos[a + b*x]*(d*Tan[a + b*x])^n,x]
```

```
[Out] (Cos[a + b*x]*(Cos[a + b*x]^2)^(n/2)*Hypergeometric2F1[n/2, (1 + n)/2, (3 +
 n)/2, Sin[a + b*x]^2]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

integral
$$=\frac{\cos(a+bx)\cos^2(a+bx)^{n/2}\,\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.02 (sec) , antiderivative size = 452, normalized size of antiderivative = 6.28

$$\int\cos(a+bx)(d\tan(a+bx))^n\,dx=$$

$$-\frac{2\left(\text{AppellF1}\left(\frac{1+n}{2},n,1,\right.\right.}{b(1+n)\left(-\text{AppellF1}\left(\frac{1+n}{2},n,1,\frac{3+n}{2},\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+\frac{\left(-\left(\left(\text{AppellF1}\left(\frac{3+n}{2},n,2,\frac{5+n}{2},\tan^2\right.\right.\right.\right.}{}\right.}$$

```
[In] Integrate[Cos[a + b*x]*(d*Tan[a + b*x])^n,x]
```

```
[Out] (-2*(AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x
)/2]^2] - 2*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2])*Cos[(a + b*x)/2]*Cos[a + b*x]*Sin[(a + b*x)/2]*(d*Tan[a + b
*x])^n)/(b*(1 + n)*(-AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2] + ((-((AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(a
+ b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[(3 + n)/2, n, 3, (5 + n)/2,
Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - n*AppellF1[(3 + n)/2, 1 + n, 1,
(5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2*n*AppellF1[(3 + n)/
2, 1 + n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos
[a + b*x])) + (3 + n)*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x]))*Sec[(a + b*x)/2]^2)/(3 + n)))
```

Maple [F]

$$\int\cos(bx+a)\,(d\tan(bx+a))^n\,dx$$

```
[In] int(cos(b*x+a)*(d*tan(b*x+a))^n,x)
```

```
[Out] int(cos(b*x+a)*(d*tan(b*x+a))^n,x)
```

Fricas [F]

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)\,dx$$

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="fricas")
[Out] integral((d*tan(b*x + a))^n*cos(b*x + a), x)
```

Sympy [F]

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(a+bx))^n\cos(a+bx)\,dx$$

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))**n,x)
[Out] Integral((d*tan(a + b*x))**n*cos(a + b*x), x)
```

Maxima [F]

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)\,dx$$

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a), x)
```

Giac [F]

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)\,dx$$

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \int \cos\left(a+b\,x\right)\,\left(d\tan(a+b\,x)\right)^n\,dx$$

```
[In] int(cos(a + b*x)*(d*tan(a + b*x))^n,x)
[Out] int(cos(a + b*x)*(d*tan(a + b*x))^n, x)
```

3.373 $\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^3(a+bx)\cos^2(a+bx)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2+n),\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] cos(b*x+a)^3*(cos(b*x+a)^2)^(-1+1/2*n)*hypergeom([-1+1/2*n, 1/2+1/2*n],[3/2
+1/2*n],sin(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2697}

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^3(a+bx)\cos^2(a+bx)^{\frac{n-2}{2}}(d\tan(a+bx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n-2}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bd(n+1)}$$

```
[In] Int[Cos[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

```
[Out] (Cos[a + b*x]^3*(Cos[a + b*x]^2)^((-2 + n)/2)*Hypergeometric2F1[(-2 + n)/2,
 (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 +
n))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
```

```
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

integral

$$= \frac{\cos^3(a+bx)\cos^2(a+bx)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2+n),\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)(d\tan(a+bx))^{1+}}{bd(1+n)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 4.85 (sec) , antiderivative size = 1313, normalized size of antiderivative = 16.83

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx$$

$$= \frac{}{b(1+n)\left((3+n)\,\text{AppellF1}\left(\frac{1+n}{2},n,1,\frac{3+n}{2},\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)(1+\cos(a+bx))-2\,(\text{A}\right.}$$

```
[In] Integrate[Cos[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

```
[Out] (4*(3 + n)*(AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2] - 6*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2,
 -Tan[(a + b*x)/2]^2] + 12*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b*
x)/2]^2, -Tan[(a + b*x)/2]^2] - 8*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[
(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[(a + b*x)/2]^3*Cos[a + b*x]^3*Sin
[(a + b*x)/2]*(d*Tan[a + b*x])^n)/(b*(1 + n)*((3 + n)*AppellF1[(1 + n)/2, n
, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x])
 - 2*(AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*
x)/2]^2] - 12*AppellF1[(3 + n)/2, n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan
[(a + b*x)/2]^2] + 36*AppellF1[(3 + n)/2, n, 4, (5 + n)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2] - 32*AppellF1[(3 + n)/2, n, 5, (5 + n)/2, Tan[(a +
 b*x)/2]^2, -Tan[(a + b*x)/2]^2] - n*AppellF1[(3 + n)/2, 1 + n, 1, (5 + n)/
2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 6*n*AppellF1[(3 + n)/2, 1 + n
, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12*n*AppellF1[(3
 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*
n*AppellF1[(3 + n)/2, 1 + n, 4, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*
x)/2]^2] + 18*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan
[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 6*n*AppellF1[(1 + n)/2, n, 2, (3 + n)
/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 8*(3 + n)
*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]
^2]*Cos[(a + b*x)/2]^2 - AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(a + b*x)
```

```
/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] + 12*AppellF1[(3 + n)/2, n, 3, (5
+ n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 36*AppellF1
[(3 + n)/2, n, 4, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a
 + b*x] + 32*AppellF1[(3 + n)/2, n, 5, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[
(a + b*x)/2]^2]*Cos[a + b*x] + n*AppellF1[(3 + n)/2, 1 + n, 1, (5 + n)/2, T
an[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 6*n*AppellF1[(3 + n)
/2, 1 + n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b
*x] + 12*n*AppellF1[(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Ta
n[(a + b*x)/2]^2]*Cos[a + b*x] - 8*n*AppellF1[(3 + n)/2, 1 + n, 4, (5 + n)/
2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 6*(3 + n)*Appell
F1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1
+ Cos[a + b*x]))))
```

Maple [F]

$$\int \left(\cos^3\left(bx+a\right)\right)\left(d\tan\left(bx+a\right)\right)^n dx$$

```
[In] int(cos(b*x+a)^3*(d*tan(b*x+a))^n,x)
[Out] int(cos(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

Fricas [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \int \left(d\tan\left(bx+a\right)\right)^n \cos\left(bx+a\right)^3\,dx$$

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="fricas")
[Out] integral((d*tan(b*x + a))^n*cos(b*x + a)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \text{Timed out}$$

```
[In] integrate(cos(b*x+a)**3*(d*tan(b*x+a))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)^3\,dx$$

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^3, x)
```

Giac [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)^3\,dx$$

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \int \cos(a+b\,x)^3\,(d\tan(a+b\,x))^n\,dx$$

```
[In] int(cos(a + b*x)^3*(d*tan(a + b*x))^n,x)
[Out] int(cos(a + b*x)^3*(d*tan(a + b*x))^n, x)
```

3.374 $\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 40

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx = -\frac{(b\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{2+m}{2},\csc^2(e+fx)\right)}{fm}$$

```
[Out] -(b*csc(f*x+e))^m*hypergeom([2, 1/2*m],[1+1/2*m],csc(f*x+e)^2)/f/m
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2686, 371}

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx = -\frac{(b\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{m+2}{2},\csc^2(e+fx)\right)}{fm}$$

```
[In] Int[(b*Csc[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] -(((b*Csc[e + f*x])^m*Hypergeometric2F1[2, m/2, (2 + m)/2, Csc[e + f*x]^2])
/(f*m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = -\frac{b\text{Subst}\left(\int \frac{(bx)^{-1+m}}{(-1+x^2)^2}\,dx, x, \csc(e+fx)\right)}{f}$$
$$= -\frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(2, \frac{m}{2}, \frac{2+m}{2}, \csc^2(e+fx)\right)}{fm}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx$$
$$= -\frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(2, 2-\frac{m}{2}, 3-\frac{m}{2}, \sin^2(e+fx)\right)\sin^4(e+fx)}{f(-4+m)}$$

```
[In] Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x]^3,x]
[Out] -(((b*Csc[e + f*x])^m*Hypergeometric2F1[2, 2 - m/2, 3 - m/2, Sin[e + f*x]^2
]*Sin[e + f*x]^4)/(f*(-4 + m)))
```

Maple [F]

$$\int (b\csc(fx+e))^m \left(\tan^3(fx+e)\right) dx$$

```
[In] int((b*csc(f*x+e))^m*tan(f*x+e)^3,x)
[Out] int((b*csc(f*x+e))^m*tan(f*x+e)^3,x)
```

Fricas [F]

$$\int (b\csc(e+fx))^m\tan^3(e+fx)\,dx=\int (b\csc\left(fx+e\right))^m\tan\left(fx+e\right)^3\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
[Out] integral((b*csc(f*x + e))^m*tan(f*x + e)^3, x)
```

Sympy [F]

$$\int (b\csc(e+fx))^m\tan^3(e+fx)\,dx=\int (b\csc\left(e+fx\right))^m\tan^3\left(e+fx\right)dx$$

```
[In] integrate((b*csc(f*x+e))**m*tan(f*x+e)**3,x)
[Out] Integral((b*csc(e + f*x))**m*tan(e + f*x)**3, x)
```

Maxima [F]

$$\int (b\csc(e+fx))^m\tan^3(e+fx)\,dx=\int (b\csc\left(fx+e\right))^m\tan\left(fx+e\right)^3\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^3, x)
```

Giac [F]

$$\int (b\csc(e+fx))^m\tan^3(e+fx)\,dx=\int (b\csc\left(fx+e\right))^m\tan\left(fx+e\right)^3\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx = \int \tan(e+f\,x)^3 \left(\frac{b}{\sin\left(e+f\,x\right)}\right)^m\,dx$$

```
[In] int(tan(e + f*x)^3*(b/sin(e + f*x))^m,x)
[Out] int(tan(e + f*x)^3*(b/sin(e + f*x))^m, x)
```

3.375 $\int (b\csc(e+fx))^m \tan(e+fx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx = \frac{(b\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{2+m}{2},\csc^2(e+fx)\right)}{fm}$$

```
[Out] (b*csc(f*x+e))^m*hypergeom([1, 1/2*m],[1+1/2*m],csc(f*x+e)^2)/f/m
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2686, 371}

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx = \frac{(b\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},\csc^2(e+fx)\right)}{fm}$$

```
[In] Int[(b*Csc[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[1, m/2, (2 + m)/2, Csc[e + f*x]^2])/(
f*m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = -\frac{b\text{Subst}\left(\int \frac{(bx)^{-1+m}}{-1+x^2}\,dx, x, \csc(e+fx)\right)}{f}$$
$$= \frac{(b\csc(e+fx))^m \,\text{Hypergeometric2F1}\left(1, \frac{m}{2}, \frac{2+m}{2}, \csc^2(e+fx)\right)}{fm}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx$$
$$= -\frac{(b\csc(e+fx))^m \,\text{Hypergeometric2F1}\left(1, 1-\frac{m}{2}, 2-\frac{m}{2}, \sin^2(e+fx)\right)\sin^2(e+fx)}{f(-2+m)}$$

```
[In] Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x],x]
[Out] -(((b*Csc[e + f*x])^m*Hypergeometric2F1[1, 1 - m/2, 2 - m/2, Sin[e + f*x]^2
]*Sin[e + f*x]^2)/(f*(-2 + m)))
```

Maple [F]

$$\int (b\csc(fx+e))^m \tan(fx+e)\,dx$$

```
[In] int((b*csc(f*x+e))^m*tan(f*x+e),x)
[Out] int((b*csc(f*x+e))^m*tan(f*x+e),x)
```

Fricas [F]

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
[Out] integral((b*csc(f*x + e))^m*tan(f*x + e), x)
```

Sympy [F]

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx = \int (b\csc(e+fx))^m \tan(e+fx)\,dx$$

```
[In] integrate((b*csc(f*x+e))**m*tan(f*x+e),x)
[Out] Integral((b*csc(e + f*x))**m*tan(e + f*x), x)
```

Maxima [F]

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e), x)
```

Giac [F]

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m\tan(e+fx)\,dx=\int \tan(e+f\,x)\,\left(\frac{b}{\sin\left(e+f\,x\right)}\right)^m\,dx$$

```
[In] int(tan(e + f*x)*(b/sin(e + f*x))^m,x)
[Out] int(tan(e + f*x)*(b/sin(e + f*x))^m, x)
```

3.376 $\int \cot(e+fx)(b\csc(e+fx))^m\,dx$

Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = -\frac{(b\csc(e+fx))^m}{fm}$$

```
[Out] -(b*csc(f*x+e))^m/f/m
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2686, 32}

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = -\frac{(b\csc(e+fx))^m}{fm}$$

```
[In] Int[Cot[e + f*x]*(b*Csc[e + f*x])^m,x]
[Out] -((b*Csc[e + f*x])^m/(f*m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = -\frac{b\text{Subst}\left(\int (bx)^{-1+m}\,dx, x, \csc(e+fx)\right)}{f}$$
$$= -\frac{(b\csc(e+fx))^m}{fm}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = -\frac{(b\csc(e+fx))^m}{fm}$$

```
[In] Integrate[Cot[e + f*x]*(b*Csc[e + f*x])^m,x]
[Out] -((b*Csc[e + f*x])^m/(f*m))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{(b\csc(fx+e))^m}{fm}$
default	$-\frac{(b\csc(fx+e))^m}{fm}$
risch	$-\frac{\left(e^{i(fx+e)}\right)^m\left(e^{2i(fx+e)}-1\right)^{-m}2^m b^m e^{\frac{i\pi m\left(\text{csgn}\left(\frac{b\,e^{i(fx+e)}}{e^{2i(fx+e)}-1}\right)^3+\text{csgn}\left(\frac{b\,e^{i(fx+e)}}{e^{2i(fx+e)}-1}\right)^2\text{csgn}\left(\frac{ib\,e^{i(fx+e)}}{e^{2i(fx+e)}-1}\right)-\text{csgn}\left(\frac{ib\,e^{i(\ldots}}{e^{2i(fx\ldots}}\right.\right.}{\ldots}}}{\ldots}$

```
[In] int(cot(f*x+e)*(b*csc(f*x+e))^m,x,method=_RETURNVERBOSE)
[Out] -(b*csc(f*x+e))^m/f/m
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\left(\frac{b}{\sin(fx+e)}\right)^m}{fm}$$

```
[In] integrate(cot(f*x+e)*(b*csc(f*x+e))^m,x, algorithm="fricas")
[Out] -(b/sin(f*x + e))^m/(f*m)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(14) = 28.

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.00

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = \begin{cases} x\cot(e) & \text{for } f=0 \wedge m=0 \\ x(b\csc(e))^m\cot(e) & \text{for } f=0 \\ -\frac{\log(\tan^2(e+fx)+1)}{2f}+\frac{\log(\tan(e+fx))}{f} & \text{for } m=0 \\ -\frac{(b\csc(e+fx))^m}{fm} & \text{otherwise} \end{cases}$$

```
[In] integrate(cot(f*x+e)*(b*csc(f*x+e))**m,x)
[Out] Piecewise((x*cot(e), Eq(f, 0) & Eq(m, 0)), (x*(b*csc(e))**m*cot(e), Eq(f, 0
)), (-log(tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f, Eq(m, 0)), (-(b
*csc(e + f*x))**m/(f*m), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = -\frac{b^m\sin(fx+e)^{-m}}{fm}$$

```
[In] integrate(cot(f*x+e)*(b*csc(f*x+e))^m,x, algorithm="maxima")
[Out] -b^m*sin(f*x + e)^(-m)/(f*m)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\left(\frac{b}{\sin(fx+e)}\right)^m}{fm}$$

[In] integrate(cot(f*x+e)*(b*csc(f*x+e))^m,x, algorithm="giac")

[Out] -(b/sin(f*x + e))^m/(f*m)

Mupad [B] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = \begin{cases} -\frac{\ln\left(\frac{b}{\sin(e+f\,x)}\right)}{f} & \text{if}\ \ m=0 \\ -\frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m}{f\,m} & \text{if}\ \ m\neq 0 \end{cases}$$

[In] int(cot(e + f*x)*(b/sin(e + f*x))^m,x)

[Out] piecewise(m == 0, -log(b/sin(e + f*x))/f, m ~= 0, -(b/sin(e + f*x))^m/(f*m)
)

3.377 $\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx=\frac{(b\csc(e+fx))^m}{fm}-\frac{(b\csc(e+fx))^{2+m}}{b^2f(2+m)}$$

```
[Out] (b*csc(f*x+e))^m/f/m-(b*csc(f*x+e))^(2+m)/b^2/f/(2+m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2686, 14}

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx=\frac{(b\csc(e+fx))^m}{fm}-\frac{(b\csc(e+fx))^{m+2}}{b^2f(m+2)}$$

```
[In] Int[Cot[e + f*x]^3*(b*Csc[e + f*x])^m,x]
```

```
[Out] (b*Csc[e + f*x])^m/(f*m) - (b*Csc[e + f*x])^(2 + m)/(b^2*f*(2 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\text{Subst}\left(\int (bx)^{-1+m}\left(-1+x^2\right)\,dx, x, \csc(e+fx)\right)}{f}\\
&= -\frac{b\text{Subst}\left(\int \left(-(bx)^{-1+m}+\frac{(bx)^{1+m}}{b^2}\right)\,dx, x, \csc(e+fx)\right)}{f}\\
&= \frac{(b\csc(e+fx))^m}{fm}-\frac{(b\csc(e+fx))^{2+m}}{b^2f(2+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$
\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx=\frac{(b\csc(e+fx))^m\left(2+m-m\csc^2(e+fx)\right)}{fm(2+m)}
$$

```
[In] Integrate[Cot[e + f*x]^3*(b*Csc[e + f*x])^m,x]
[Out] ((b*Csc[e + f*x])^m*(2 + m - m*Csc[e + f*x]^2))/(f*m*(2 + m))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.16 (sec) , antiderivative size = 3514, normalized size of antiderivative = 81.72

method	result	size
risch	Expression too large to display	3514

```
[In] int(cot(f*x+e)^3*(b*csc(f*x+e))^m,x,method=_RETURNVERBOSE)
[Out] 1/(2+m)/f/(exp(2*I*(f*x+e))-1)^2/m*b^m*exp(I*(f*x+e))^m*(exp(2*I*(f*x+e))-1
)^(-m)*2^m*(m*exp(1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*Pi*m)
*exp(1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*csgn(I*b/(exp(2
*I*(f*x+e))-1)*exp(I*(f*x+e)))*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-
1)*exp(I*(f*x+e)))^3*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f
*x+e)))^2*csgn(I*b)*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*
x+e)))^2*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*m)*exp(-1/2*I*Pi*csgn(
I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(I*b)*csgn(I*exp(I*(f*x+e))/(e
xp(2*I*(f*x+e))-1))*m)*exp(1/2*I*Pi*csgn(I/(exp(2*I*(f*x+e))-1))*csgn(I*exp
(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*m)*exp(-1/2*I*Pi*csgn(I/(exp(2*I*(f*x+e
```

))-1))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*exp(I*(f*x+e)))*m
)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^3*m)*exp(1/2*I*
Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I*exp(I*(f*x+e)))*m)*
exp(-1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*m)*exp(-1/2*I*c
sgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)
exp(I(f*x+e)))*m)*exp(1/2*I*Pi*m)*exp(4*I*e)*exp(4*I*f*x)+2*exp(1/2*I*csg
n(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*Pi*m)*exp(1/2*I*csgn(b/(exp(2*I*
(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e
)))*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*m)*exp
(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*csgn(I*b)*m)*exp(
1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*csgn(I*exp(I*(f*x+
e))/(exp(2*I*(f*x+e))-1))*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*ex
p(I*(f*x+e)))*csgn(I*b)*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*m)*exp(
1/2*I*Pi*csgn(I/(exp(2*I*(f*x+e))-1))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e
))-1))^2*m)*exp(-1/2*I*Pi*csgn(I/(exp(2*I*(f*x+e))-1))*csgn(I*exp(I*(f*x+e)
)/(exp(2*I*(f*x+e))-1))*csgn(I*exp(I*(f*x+e)))*m)*exp(-1/2*I*Pi*csgn(I*exp(
I*(f*x+e))/(exp(2*I*(f*x+e))-1))^3*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(e
xp(2*I*(f*x+e))-1))^2*csgn(I*exp(I*(f*x+e)))*m)*exp(-1/2*I*csgn(b/(exp(2*I*
(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*m)*exp(-1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*
exp(I*(f*x+e)))*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*m)*exp(1/2
*I*Pi*m)*exp(4*I*e)*exp(4*I*f*x)+2*m*exp(1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*
exp(I*(f*x+e)))^3*Pi*m)*exp(1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)
))^2*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*m)*exp(-1/2*I*Pi*csgn
(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2
I(f*x+e))-1)*exp(I*(f*x+e)))^2*csgn(I*b)*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*
I*(f*x+e))-1)*exp(I*(f*x+e)))^2*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))
*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(I*b)*c
sgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*m)*exp(1/2*I*Pi*csgn(I/(exp(2*I*
(f*x+e))-1))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*m)*exp(-1/2*I*Pi
*csgn(I/(exp(2*I*(f*x+e))-1))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*c
sgn(I*exp(I*(f*x+e)))*m)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+
e))-1))^3*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csg
n(I*exp(I*(f*x+e)))*m)*exp(-1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)
))^2*Pi*m)*exp(-1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*Pi*csgn(I
*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*m)*exp(1/2*I*Pi*m)*exp(2*I*f*x)*exp
(2*I*e)-4*exp(1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*Pi*m)*exp
(1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*csgn(I*b/(exp(2*I*(
f*x+e))-1)*exp(I*(f*x+e)))*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*e
xp(I*(f*x+e)))^3*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e
)))^2*csgn(I*b)*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)
))^2*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*m)*exp(-1/2*I*Pi*csgn(I*b/
(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(I*b)*csgn(I*exp(I*(f*x+e))/(exp(2
I(f*x+e))-1))*m)*exp(1/2*I*Pi*csgn(I/(exp(2*I*(f*x+e))-1))*csgn(I*exp(I*(
f*x+e))/(exp(2*I*(f*x+e))-1))^2*m)*exp(-1/2*I*Pi*csgn(I/(exp(2*I*(f*x+e))-1
))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*exp(I*(f*x+e)))*m)*ex

```
p(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^3*m)*exp(1/2*I*Pi*c
sgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I*exp(I*(f*x+e)))*m)*exp(
-1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*m)*exp(-1/2*I*csgn(
b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp
(I*(f*x+e)))*m)*exp(1/2*I*Pi*m)*exp(2*I*f*x)*exp(2*I*e)+m*exp(1/2*I*Pi*m*(c
sgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3+csgn(b/(exp(2*I*(f*x+e))-1)*ex
p(I*(f*x+e)))^2*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))-csgn(I*b/(exp
(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3+csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x
+e)))^2*csgn(I*b)+csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*csgn(I*ex
p(I*(f*x+e))/(exp(2*I*(f*x+e))-1))-csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x
+e)))*csgn(I*b)*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))+csgn(I/(exp(2*I
*(f*x+e))-1))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2-csgn(I/(exp(2*I
*(f*x+e))-1))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*exp(I*(f*x
+e)))-csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^3+csgn(I*exp(I*(f*x+e))/(
exp(2*I*(f*x+e))-1))^2*csgn(I*exp(I*(f*x+e)))-csgn(b/(exp(2*I*(f*x+e))-1)*e
xp(I*(f*x+e)))^2-csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(I*b/(exp(
2*I*(f*x+e))-1)*exp(I*(f*x+e)))+1))+2*exp(1/2*I*Pi*m*(csgn(b/(exp(2*I*(f*x+
e))-1)*exp(I*(f*x+e)))^3+csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*csgn
(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))-csgn(I*b/(exp(2*I*(f*x+e))-1)*exp
(I*(f*x+e)))^3+csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*csgn(I*b)+cs
gn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*csgn(I*exp(I*(f*x+e))/(exp(2*
I*(f*x+e))-1))-csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(I*b)*csgn
(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))+csgn(I/(exp(2*I*(f*x+e))-1))*csgn(I
*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2-csgn(I/(exp(2*I*(f*x+e))-1))*csgn(I
*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*exp(I*(f*x+e)))-csgn(I*exp(I*(
f*x+e))/(exp(2*I*(f*x+e))-1))^3+csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))
^2*csgn(I*exp(I*(f*x+e)))-csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2-csg
n(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(
I*(f*x+e)))+1)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\left((m+2)\cos\left(fx+e\right)^2-2\right)\left(\frac{b}{\sin(fx+e)}\right)^m}{fm^2-\left(fm^2+2\,fm\right)\cos\left(fx+e\right)^2+2\,fm}$$

```
[In] integrate(cot(f*x+e)^3*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] -((m + 2)*cos(f*x + e)^2 - 2)*(b/sin(f*x + e))^m/(f*m^2 - (f*m^2 + 2*f*m)*c
os(f*x + e)^2 + 2*f*m)
```

Sympy [F]

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx$$
$$= \begin{cases} x(b\csc(e))^m\cot^3(e) & \text{for } f=0 \\ \frac{\int \frac{\cot^3(e+fx)}{\csc^2(e+fx)}\,dx}{b^2} & \text{for } m=-2 \\ \frac{\log(\tan^2(e+fx)+1)}{2f} - \frac{\log(\tan(e+fx))}{f} - \frac{1}{2f\tan^2(e+fx)} & \text{for } m=0 \\ -\frac{m(b\csc(e+fx))^m\cot^2(e+fx)}{fm^2+2fm} + \frac{2(b\csc(e+fx))^m}{fm^2+2fm} & \text{otherwise} \end{cases}$$

```
[In] integrate(cot(f*x+e)**3*(b*csc(f*x+e))**m,x)
```

```
[Out] Piecewise((x*(b*csc(e))**m*cot(e)**3, Eq(f, 0)), (Integral(cot(e + f*x)**3/
csc(e + f*x)**2, x)/b**2, Eq(m, -2)), (log(tan(e + f*x)**2 + 1)/(2*f) - log
(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2), Eq(m, 0)), (-m*(b*csc(e + f*x))
**m*cot(e + f*x)**2/(f*m**2 + 2*f*m) + 2*(b*csc(e + f*x))**m/(f*m**2 + 2*f*
m), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx = \frac{\frac{b^m\sin(fx+e)^{-m}}{m} - \frac{b^m\sin(fx+e)^{-m}}{(m+2)\sin(fx+e)^2}}{f}$$

```
[In] integrate(cot(f*x+e)^3*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] (b^m*sin(f*x + e)^(-m)/m - b^m*sin(f*x + e)^(-m)/((m + 2)*sin(f*x + e)^2))/
f
```

Giac [F]

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(fx+e))^m\cot(fx+e)^3\,dx$$

```
[In] integrate(cot(f*x+e)^3*(b*csc(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^3, x)
```

Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.14

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx$$
$$=\frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m\,\left(m+4\sin\left(2\,e+2\,f\,x\right)^2+m\,\left(2\sin\left(2\,e+2\,f\,x\right)^2-1\right)-16\sin\left(e+f\,x\right)^2\right)}{f\,m\,\left(2\sin\left(2\,e+2\,f\,x\right)^2-8\sin\left(e+f\,x\right)^2\right)\,\left(m+2\right)}$$

```
[In] int(cot(e + f*x)^3*(b/sin(e + f*x))^m,x)
```

```
[Out] ((b/sin(e + f*x))^m*(m + 4*sin(2*e + 2*f*x)^2 + m*(2*sin(2*e + 2*f*x)^2 - 1
) - 16*sin(e + f*x)^2))/(f*m*(2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)^2)*(m +
 2))
```

3.378 $\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = -\frac{(b\csc(e+fx))^m}{fm}+\frac{2(b\csc(e+fx))^{2+m}}{b^2f(2+m)} - \frac{(b\csc(e+fx))^{4+m}}{b^4f(4+m)}$$

```
[Out] -(b*csc(f*x+e))^m/f/m+2*(b*csc(f*x+e))^(2+m)/b^2/f/(2+m)-(b*csc(f*x+e))^(4+
m)/b^4/f/(4+m)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2686, 276}

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = -\frac{(b\csc(e+fx))^{m+4}}{b^4f(m+4)}+\frac{2(b\csc(e+fx))^{m+2}}{b^2f(m+2)}-\frac{(b\csc(e+fx))^m}{fm}$$

```
[In] Int[Cot[e + f*x]^5*(b*Csc[e + f*x])^m,x]
```

```
[Out] -((b*Csc[e + f*x])^m/(f*m)) + (2*(b*Csc[e + f*x])^(2 + m))/(b^2*f*(2 + m))
- (b*Csc[e + f*x])^(4 + m)/(b^4*f*(4 + m))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{b\text{Subst}\left(\int (bx)^{-1+m}\left(-1+x^2\right)^2\,dx, x, \csc(e+fx)\right)}{f}\\ &= -\frac{b\text{Subst}\left(\int \left((bx)^{-1+m}-\frac{2(bx)^{1+m}}{b^2}+\frac{(bx)^{3+m}}{b^4}\right)\,dx, x, \csc(e+fx)\right)}{f}\\ &= -\frac{(b\csc(e+fx))^m}{fm}+\frac{2(b\csc(e+fx))^{2+m}}{b^2f(2+m)}-\frac{(b\csc(e+fx))^{4+m}}{b^4f(4+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = -\frac{(b\csc(e+fx))^m\left(8+6m+m^2-2m(4+m)\csc^2(e+fx)+m(2+m)\csc^4(e+fx)\right)}{fm(2+m)(4+m)}$$

```
[In] Integrate[Cot[e + f*x]^5*(b*Csc[e + f*x])^m,x]
[Out] -(((b*Csc[e + f*x])^m*(8 + 6*m + m^2 - 2*m*(4 + m)*Csc[e + f*x]^2 + m*(2 +
m)*Csc[e + f*x]^4))/(f*m*(2 + m)*(4 + m)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.14 (sec) , antiderivative size = 8846, normalized size of antiderivative = 128.20

method	result	size
risch	Expression too large to display	8846

```
[In] int(cot(f*x+e)^5*(b*csc(f*x+e))^m,x,method=_RETURNVERBOSE)
```

[Out] result too large to display

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.67

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\left(\left(m^2+6\,m+8\right)\cos\left(fx+e\right)^4-4\,\left(m+4\right)\cos\left(fx+e\right)^2+8\right)\left(\frac{b}{\sin\left(fx+e\right)}\right)^m}{\left(fm^3+6\,fm^2+8\,fm\right)\cos\left(fx+e\right)^4+fm^3+6\,fm^2-2\,\left(fm^3+6\,fm^2+8\,fm\right)\cos\left(fx+e\right)^2+8\,fm}$$

```
[In] integrate(cot(f*x+e)^5*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] -((m^2 + 6*m + 8)*cos(f*x + e)^4 - 4*(m + 4)*cos(f*x + e)^2 + 8)*(b/sin(f*x
 + e))^m/((f*m^3 + 6*f*m^2 + 8*f*m)*cos(f*x + e)^4 + f*m^3 + 6*f*m^2 - 2*(f
*m^3 + 6*f*m^2 + 8*f*m)*cos(f*x + e)^2 + 8*f*m)
```

Sympy [F]

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = \begin{cases} x(b\csc(e))^m\cot^5(e) & \text{for } f \\ \frac{\int \frac{\cot^5(e+fx)}{\csc^4(e+fx)}\,dx}{b^4} & \text{for } m \\ \frac{\int \frac{\cot^5(e+fx)}{\csc^2(e+fx)}\,dx}{b^2} & \text{for } m \\ -\frac{\log(\tan^2(e+fx)+1)}{2f}+\frac{\log(\tan(e+fx))}{f}+\frac{1}{2f\tan^2(e+fx)}-\frac{1}{4f\tan^4(e+fx)} & \text{for } m \\ -\frac{m^2(b\csc(e+fx))^m\cot^4(e+fx)}{fm^3+6fm^2+8fm}-\frac{2m(b\csc(e+fx))^m\cot^4(e+fx)}{fm^3+6fm^2+8fm}+\frac{4m(b\csc(e+fx))^m\cot^2(e+fx)}{fm^3+6fm^2+8fm}-\frac{8(b\csc(e+fx))^m}{fm^3+6fm^2+8fm} & \text{other} \end{cases}$$

```
[In] integrate(cot(f*x+e)**5*(b*csc(f*x+e))**m,x)
```

```
[Out] Piecewise((x*(b*csc(e))**m*cot(e)**5, Eq(f, 0)), (Integral(cot(e + f*x)**5/
csc(e + f*x)**4, x)/b**4, Eq(m, -4)), (Integral(cot(e + f*x)**5/csc(e + f*x
)**2, x)/b**2, Eq(m, -2)), (-log(tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f
*x))/f + 1/(2*f*tan(e + f*x)**2) - 1/(4*f*tan(e + f*x)**4), Eq(m, 0)), (-m*
*2*(b*csc(e + f*x))**m*cot(e + f*x)**4/(f*m**3 + 6*f*m**2 + 8*f*m) - 2*m*(b
*csc(e + f*x))**m*cot(e + f*x)**4/(f*m**3 + 6*f*m**2 + 8*f*m) + 4*m*(b*csc(
e + f*x))**m*cot(e + f*x)**2/(f*m**3 + 6*f*m**2 + 8*f*m) - 8*(b*csc(e + f*x
))**m/(f*m**3 + 6*f*m**2 + 8*f*m), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\frac{b^m\sin(fx+e)^{-m}}{m} - \frac{2\,b^m\sin(fx+e)^{-m}}{(m+2)\sin(fx+e)^2} + \frac{b^m\sin(fx+e)^{-m}}{(m+4)\sin(fx+e)^4}}{f}$$

```
[In] integrate(cot(f*x+e)^5*(b*csc(f*x+e))^m,x, algorithm="maxima")
[Out] -(b^m*sin(f*x + e)^(-m)/m - 2*b^m*sin(f*x + e)^(-m)/((m + 2)*sin(f*x + e)^2
) + b^m*sin(f*x + e)^(-m)/((m + 4)*sin(f*x + e)^4))/f
```

Giac [F]

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(fx+e))^m\cot(fx+e)^5\,dx$$

```
[In] integrate(cot(f*x+e)^5*(b*csc(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^5, x)
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.22

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx =$$

$$-\frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m\left(2\sin\left(2\,e+2\,f\,x\right)^2+\sin\left(4\,e+4\,f\,x\right)\,1\mathrm{i}-1\right)\left(\frac{2\left(2\sin(2\,e+2\,f\,x)^2-1\right)\left(-2\sin(2\,e+2\,f\,x)^2+\sin(4\,e+4\,f\,x\right.}{f\,m}\right.}{16}$$

```
[In] int(cot(e + f*x)^5*(b/sin(e + f*x))^m,x)
[Out] -((b/sin(e + f*x))^m*(sin(4*e + 4*f*x)*1i + 2*sin(2*e + 2*f*x)^2 - 1)*((2*(
2*sin(2*e + 2*f*x)^2 - 1)*(sin(4*e + 4*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1))
/(f*m) - ((sin(4*e + 4*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1)*(4*m + 6*m^2 + 4
8))/(f*m*(6*m + m^2 + 8)) + (2*(2*sin(e + f*x)^2 - 1)*(sin(4*e + 4*f*x)*1i
- 2*sin(2*e + 2*f*x)^2 + 1)*(8*m + 4*m^2 - 32))/(f*m*(6*m + m^2 + 8))))/(16
*sin(e + f*x)^4)
```

3.379 $\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = \frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),-\frac{1}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{1}{2}(-3+m)}\tan^3(e+f$$
$$3f$$

```
[Out] 1/3*(b*csc(f*x+e))^m*hypergeom([-3/2, -3/2+1/2*m],[-1/2],cos(f*x+e)^2)*(sin
(f*x+e)^2)^(-3/2+1/2*m)*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2697}

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = \frac{\tan^3(e+fx)\sin^2(e+fx)^{\frac{m-3}{2}}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m-3}{2},-\frac{1}{2},\cos^2(e+fx)\right)}{3f}$$

```
[In] Int[(b*Csc[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[-3/2, (-3 + m)/2, -1/2, Cos[e + f*x]^
2]*(Sin[e + f*x]^2)^((-3 + m)/2)*Tan[e + f*x]^3)/(3*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

integral

$$= \frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{2}(-3+m), -\frac{1}{2}, \cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{1}{2}(-3+m)}\tan^3(e+fx)}{3f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 240 vs. $2(63) = 126$.

Time = 8.73 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.81

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = -\frac{\cos(e+fx)(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3}{2}, \cos^2(e+fx)\right)\sin(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-1+m)}}{f} + \frac{4(b\csc(e+fx))^m\left(\frac{m\,\text{Hypergeometric2F1}\left(1-m,\frac{1}{2}-\frac{m}{2},\frac{3}{2}-\frac{m}{2},-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)^{-m}\tan\left(\frac{1}{2}(e+fx)\right)}{-1+m} - \frac{1}{2}\tan(e+fx)\right)}{f} + \frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-1-\frac{m}{2}, \frac{1}{2}-\frac{m}{2}, \frac{3}{2}-\frac{m}{2}, -\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}\tan(e+fx)}{f(1-m)}$$

```
[In] Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] -((Cos[e + f*x]*(b*Csc[e + f*x])^m*Hypergeometric2F1[1/2, (1 + m)/2, 3/2, C
os[e + f*x]^2]*Sin[e + f*x]*(Sin[e + f*x]^2)^((-1 + m)/2))/f) + (4*(b*Csc[e
 + f*x])^m*((m*Hypergeometric2F1[1 - m, 1/2 - m/2, 3/2 - m/2, -Tan[(e + f*x
)/2]^2]*Tan[(e + f*x)/2])/((-1 + m)*(Sec[(e + f*x)/2]^2)^m) - Tan[e + f*x]/
2))/f + ((b*Csc[e + f*x])^m*Hypergeometric2F1[-1 - m/2, 1/2 - m/2, 3/2 - m/
2, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*(1 - m)*(Sec[e + f*x]^2)^(m/2))
```

Maple [F]

$$\int (b\csc(fx+e))^m \left(\tan^4(fx+e)\right) dx$$

```
[In] int((b*csc(f*x+e))^m*tan(f*x+e)^4,x)
```

```
[Out] int((b*csc(f*x+e))^m*tan(f*x+e)^4,x)
```

Fricas [F]

$$\int (b\csc(e+fx))^m\tan^4(e+fx)\,dx=\int (b\csc(fx+e))^m\tan(fx+e)^4\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
[Out] integral((b*csc(f*x + e))^m*tan(f*x + e)^4, x)
```

Sympy [F]

$$\int (b\csc(e+fx))^m\tan^4(e+fx)\,dx=\int (b\csc(e+fx))^m\tan^4(e+fx)\,dx$$

```
[In] integrate((b*csc(f*x+e))**m*tan(f*x+e)**4,x)
[Out] Integral((b*csc(e + f*x))**m*tan(e + f*x)**4, x)
```

Maxima [F]

$$\int (b\csc(e+fx))^m\tan^4(e+fx)\,dx=\int (b\csc(fx+e))^m\tan(fx+e)^4\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^4, x)
```

Giac [F]

$$\int (b\csc(e+fx))^m\tan^4(e+fx)\,dx=\int (b\csc(fx+e))^m\tan(fx+e)^4\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4 \left(\frac{b}{\sin\left(e+f\,x\right)}\right)^m\,dx$$

```
[In] int(tan(e + f*x)^4*(b/sin(e + f*x))^m,x)
[Out] int(tan(e + f*x)^4*(b/sin(e + f*x))^m, x)
```

3.380 $\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{1}{2}(-1+m)}\tan(e+fx)}{f}$$

```
[Out] (b*csc(f*x+e))^m*hypergeom([-1/2, -1/2+1/2*m],[1/2],cos(f*x+e)^2)*(sin(f*x+
e)^2)^(-1/2+1/2*m)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2697}

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \frac{\tan(e+fx)\sin^2(e+fx)^{\frac{m-1}{2}}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m-1}{2},\frac{1}{2},\cos^2(e+fx)\right)}{f}$$

```
[In] Int[(b*Csc[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[-1/2, (-1 + m)/2, 1/2, Cos[e + f*x]^2
]*(Sin[e + f*x]^2)^((-1 + m)/2)*Tan[e + f*x])/f
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

integral

$$= \frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{1}{2}(-1+m)}\tan(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.36

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx$$
$$= \frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(1-\frac{m}{2},\frac{3}{2}-\frac{m}{2},\frac{5}{2}-\frac{m}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}\tan^3(e+fx)}{f(3-m)}$$

```
[In] Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x]^2,x]
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[1 - m/2, 3/2 - m/2, 5/2 - m/2, -Tan[e
 + f*x]^2]*Tan[e + f*x]^3)/(f*(3 - m)*(Sec[e + f*x]^2)^(m/2))
```

Maple [F]

$$\int (b\csc(fx+e))^m \left(\tan^2(fx+e)\right) dx$$

```
[In] int((b*csc(f*x+e))^m*tan(f*x+e)^2,x)
[Out] int((b*csc(f*x+e))^m*tan(f*x+e)^2,x)
```

Fricas [F]

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)^2\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral((b*csc(f*x + e))^m*tan(f*x + e)^2, x)
```

Sympy [F]

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \int (b\csc(e+fx))^m \tan^2(e+fx)\,dx$$

```
[In] integrate((b*csc(f*x+e))**m*tan(f*x+e)**2,x)
[Out] Integral((b*csc(e + f*x))**m*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)^2\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^2, x)
```

Giac [F]

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)^2\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2 \left(\frac{b}{\sin(e+f\,x)}\right)^m\,dx$$

```
[In] int(tan(e + f*x)^2*(b/sin(e + f*x))^m,x)
[Out] int(tan(e + f*x)^2*(b/sin(e + f*x))^m, x)
```

3.381 $\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\cot^3(e+fx)(b\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{3+m}{2}}}{3f}$$

```
[Out] -1/3*cot(f*x+e)^3*(b*csc(f*x+e))^m*hypergeom([3/2, 3/2+1/2*m],[5/2],cos(f*x
+e)^2)*(sin(f*x+e)^2)^(3/2+1/2*m)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2697}

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\cot^3(e+fx)\sin^2(e+fx)^{\frac{m+3}{2}}(b\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+3}{2},\frac{5}{2},\cos^2(e+fx)\right)}{3f}$$

```
[In] Int[Cot[e + f*x]^2*(b*Csc[e + f*x])^m,x]
```

```
[Out] -1/3*(Cot[e + f*x]^3*(b*Csc[e + f*x])^m*Hypergeometric2F1[3/2, (3 + m)/2, 5
/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((3 + m)/2))/f
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

integral =
$$-\frac{\cot^3(e+fx)(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{3+m}{2}}}{3f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 186 vs. 2(63) = 126.

Time = 1.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.95

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx =$$
$$-\frac{(b\csc(e+fx))^m\left(-4(1+m)\text{Hypergeometric2F1}\left(1-m,\frac{1}{2}-\frac{m}{2},\frac{3}{2}-\frac{m}{2},-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(-1+\right.}{}$$

```
[In] Integrate[Cot[e + f*x]^2*(b*Csc[e + f*x])^m,x]
```

```
[Out] -1/2*((b*Csc[e + f*x])^m*(-4*(1 + m)*Hypergeometric2F1[1 - m, 1/2 - m/2, 3/
2 - m/2, -Tan[(e + f*x)/2]^2] + (-1 + m)*Cot[(e + f*x)/2]^2*Hypergeometric2
F1[-1/2 - m/2, -m, 1/2 - m/2, -Tan[(e + f*x)/2]^2] + (1 + m)*Hypergeometric
2F1[1/2 - m/2, -m, 3/2 - m/2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2])/(f*(-
1 + m^2)*(Sec[(e + f*x)/2]^2)^m)
```

Maple [F]

$$\int \left(\cot^2(fx+e)\right)(b\csc(fx+e))^m\,dx$$

```
[In] int(cot(f*x+e)^2*(b*csc(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^2*(b*csc(f*x+e))^m,x)
```

Fricas [F]

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx=\int (b\csc(fx+e))^m\cot(fx+e)^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(b*csc(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*csc(f*x + e))^m*cot(f*x + e)^2, x)
```

Sympy [F]

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx=\int (b\csc(e+fx))^m\cot^2(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)**2*(b*csc(f*x+e))**m,x)
[Out] Integral((b*csc(e + f*x))**m*cot(e + f*x)**2, x)
```

Maxima [F]

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx=\int (b\csc(fx+e))^m\cot(fx+e)^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(b*csc(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^2, x)
```

Giac [F]

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx=\int (b\csc(fx+e))^m\cot(fx+e)^2\,dx$$

```
[In] integrate(cot(f*x+e)^2*(b*csc(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx=\int \cot(e+f\,x)^2\left(\frac{b}{\sin\left(e+f\,x\right)}\right)^m\,dx$$

```
[In] int(cot(e + f*x)^2*(b/sin(e + f*x))^m,x)
[Out] int(cot(e + f*x)^2*(b/sin(e + f*x))^m, x)
```

3.382 $\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\cot^5(e+fx)(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{5}{2},\frac{5+m}{2},\frac{7}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{5+m}{2}}}{5f}$$

```
[Out] -1/5*cot(f*x+e)^5*(b*csc(f*x+e))^m*hypergeom([5/2, 5/2+1/2*m],[7/2],cos(f*x
+e)^2)*(sin(f*x+e)^2)^(5/2+1/2*m)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2697}

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\cot^5(e+fx)\sin^2(e+fx)^{\frac{m+5}{2}}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+5}{2},\frac{7}{2},\cos^2(e+fx)\right)}{5f}$$

```
[In] Int[Cot[e + f*x]^4*(b*Csc[e + f*x])^m,x]
```

```
[Out] -1/5*(Cot[e + f*x]^5*(b*Csc[e + f*x])^m*Hypergeometric2F1[5/2, (5 + m)/2, 7
/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((5 + m)/2))/f
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
```

```
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{\cot^5(e+fx)(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{5}{2},\frac{5+m}{2},\frac{7}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{5+m}{2}}}{5f}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.68

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\cot(e+fx)(b\csc(e+fx))^m \left(\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3}{2},\cos^2(e+fx)\right)-2\,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.\right.}{f}$$

```
[In] Integrate[Cot[e + f*x]^4*(b*Csc[e + f*x])^m,x]
[Out] -((Cot[e + f*x]*(b*Csc[e + f*x])^m*(Hypergeometric2F1[1/2, (1 + m)/2, 3/2,
Cos[e + f*x]^2] - 2*Hypergeometric2F1[1/2, (3 + m)/2, 3/2, Cos[e + f*x]^2]
+ Hypergeometric2F1[1/2, (5 + m)/2, 3/2, Cos[e + f*x]^2])*(Sin[e + f*x]^2)^
((1 + m)/2))/f)
```

Maple [F]

$$\int \left(\cot^4(fx+e)\right)(b\csc(fx+e))^m\,dx$$

```
[In] int(cot(f*x+e)^4*(b*csc(f*x+e))^m,x)
[Out] int(cot(f*x+e)^4*(b*csc(f*x+e))^m,x)
```

Fricas [F]

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(fx+e))^m\cot(fx+e)^4\,dx$$

```
[In] integrate(cot(f*x+e)^4*(b*csc(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*csc(f*x + e))^m*cot(f*x + e)^4, x)
```

Sympy [F]

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(e+fx))^m\cot^4(e+fx)\,dx$$

```
[In] integrate(cot(f*x+e)**4*(b*csc(f*x+e))**m,x)
[Out] Integral((b*csc(e + f*x))**m*cot(e + f*x)**4, x)
```

Maxima [F]

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(fx+e))^m\cot(fx+e)^4\,dx$$

```
[In] integrate(cot(f*x+e)^4*(b*csc(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^4, x)
```

Giac [F]

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(fx+e))^m\cot(fx+e)^4\,dx$$

```
[In] integrate(cot(f*x+e)^4*(b*csc(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = \int \cot(e+f\,x)^4\left(\frac{b}{\sin(e+f\,x)}\right)^m\,dx$$

```
[In] int(cot(e + f*x)^4*(b/sin(e + f*x))^m,x)
[Out] int(cot(e + f*x)^4*(b/sin(e + f*x))^m, x)
```

3.383 $\int (b\csc(e+fx))^m (d\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int (b\csc(e+fx))^m(d\tan(e+fx))^{3/2}\,dx = \frac{2\cos^2(e+fx)^{5/4}(b\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{4}(5-2m),\frac{1}{4}(9-2m),\sin^2(e+fx)\right)\ldots}{df(5-2m)}$$

```
[Out] 2*(cos(f*x+e)^2)^(5/4)*(b*csc(f*x+e))^m*hypergeom([5/4, 5/4-1/2*m],[9/4-1/2
*m],sin(f*x+e)^2)*(d*tan(f*x+e))^(5/2)/d/f/(5-2*m)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2698, 2682, 2657}

$$\int (b\csc(e+fx))^m(d\tan(e+fx))^{3/2}\,dx = \frac{2\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}(b\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{4}(5-2m),\frac{1}{4}(9-\ldots\right.}{df(5-2m)}$$

```
[In] Int[(b*Csc[e + f*x])^m*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(5/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[5/4, (5 - 2*
m)/4, (9 - 2*m)/4, Sin[e + f*x]^2]*(d*Tan[e + f*x])^(5/2))/(d*f*(5 - 2*m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2698

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left((b\csc(e+fx))^m\left(\frac{\sin(e+fx)}{b}\right)^m\right)\int\left(\frac{\sin(e+fx)}{b}\right)^{-m}(d\tan(e+fx))^{3/2}\,dx$$

$$= \frac{\left(\cos^{\frac{5}{2}}(e+fx)(b\csc(e+fx))^{3+m}\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}+m}(d\tan(e+fx))^{5/2}\right)\int\frac{\left(\frac{\sin(e+fx)}{b}\right)^{\frac{3}{2}-m}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{bd}$$

$$= \frac{2\cos^2(e+fx)^{5/4}(b\csc(e+fx))^{3+m}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{4}(5-2m),\frac{1}{4}(9-2m),\sin^2(e+fx)\right)\text{si}}{b^3df(5-2m)}$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int(b\csc(e+fx))^m(d\tan(e+fx))^{3/2}\,dx =$$

$$-\frac{2(b\csc(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{4}(5-2m),1-\frac{m}{2},\frac{1}{4}(9-2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}(d}{df(-5+2m)}$$

```
[In] Integrate[(b*Csc[e + f*x])^m*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(5 - 2*m)/4, 1 - m/2, (9 - 2*m)/4,
 -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(5/2))/(d*f*(-5 + 2*m)*(Sec[e + f*x]^2)^
(m/2))
```

Maple [F]

$$\int (b \csc(fx+e))^{m} (d \tan(fx+e))^{\frac{3}{2}} dx$$

```
[In] int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x)
[Out] int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int (b \csc(e+fx))^m (d \tan(e+fx))^{3/2} \, dx = \int (d \tan(fx+e))^{\frac{3}{2}} (b \csc(fx+e))^{m} \, dx$$

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m*d*tan(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b \csc(e+fx))^m (d \tan(e+fx))^{3/2} \, dx = \text{Timed out}$$

```
[In] integrate((b*csc(f*x+e))**m*(d*tan(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (b \csc(e+fx))^m (d \tan(e+fx))^{3/2} \, dx = \int (d \tan(fx+e))^{\frac{3}{2}} (b \csc(fx+e))^{m} \, dx$$

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*tan(f*x + e))^(3/2)*(b*csc(f*x + e))^m, x)
```

Giac [F]

$$\int (b\csc(e+fx))^m (d\tan(e+fx))^{3/2}\,dx = \int (d\tan(fx+e))^{\frac{3}{2}} (b\csc(fx+e))^m\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*tan(f*x + e))^(3/2)*(b*csc(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m (d\tan(e+fx))^{3/2}\,dx = \int (d\tan(e+f\,x))^{3/2} \left(\frac{b}{\sin(e+f\,x)}\right)^m\,dx$$

```
[In] int((d*tan(e + f*x))^(3/2)*(b/sin(e + f*x))^m,x)
[Out] int((d*tan(e + f*x))^(3/2)*(b/sin(e + f*x))^m, x)
```

3.384 $\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{3/4}(b\csc(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}(3-2m),\frac{1}{4}(7-2m),\sin^2(e+fx)\right)(d\tan(e+$$
$$df(3-2m)$$

```
[Out] 2*(cos(f*x+e)^2)^(3/4)*(b*csc(f*x+e))^m*hypergeom([3/4, 3/4-1/2*m],[7/4-1/2
*m],sin(f*x+e)^2)*(d*tan(f*x+e))^(3/2)/d/f/(3-2*m)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2698, 2682, 2657}

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}(b\csc(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}(3-2m),\frac{1}{4}(7-2m),\sin^2(e$$
$$df(3-2m)$$

```
[In] Int[(b*Csc[e + f*x])^m*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(3/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[3/4, (3 - 2*
m)/4, (7 - 2*m)/4, Sin[e + f*x]^2]*(d*Tan[e + f*x])^(3/2))/(d*f*(3 - 2*m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2698

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left((b\csc(e+fx))^m\left(\frac{\sin(e+fx)}{b}\right)^m\right)\int\left(\frac{\sin(e+fx)}{b}\right)^{-m}\sqrt{d\tan(e+fx)}\,dx$$

$$= \frac{\left(\cos^{\frac{3}{2}}(e+fx)(b\csc(e+fx))^{2+m}\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}+m}(d\tan(e+fx))^{3/2}\right)\int\frac{\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}-m}}{\sqrt{\cos(e+fx)}}\,dx}{bd}$$

$$= \frac{2\cos^2(e+fx)^{3/4}(b\csc(e+fx))^{2+m}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}(3-2m),\frac{1}{4}(7-2m),\sin^2(e+fx)\right)\mathrm{si}}{b^2df(3-2m)}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int(b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx =$$

$$-\frac{2(b\csc(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{4}(3-2m),1-\frac{m}{2},\frac{1}{4}(7-2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}}{df(-3+2m)}$$

```
[In] Integrate[(b*Csc[e + f*x])^m*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(3 - 2*m)/4, 1 - m/2, (7 - 2*m)/4,
 -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(3/2))/(d*f*(-3 + 2*m)*(Sec[e + f*x]^2)^
(m/2))
```

Maple [F]

$$\int (b\csc(fx+e))^m \sqrt{d\tan(fx+e)}dx$$

```
[In] int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x)
[Out] int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int (b\csc(e+fx))^m \sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}(b\csc(fx+e))^m\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m, x)
```

Sympy [F]

$$\int (b\csc(e+fx))^m \sqrt{d\tan(e+fx)}\,dx = \int (b\csc(e+fx))^m \sqrt{d\tan(e+fx)}\,dx$$

```
[In] integrate((b*csc(f*x+e))**m*(d*tan(f*x+e))**(1/2),x)
[Out] Integral((b*csc(e + f*x))**m*sqrt(d*tan(e + f*x)), x)
```

Maxima [F]

$$\int (b\csc(e+fx))^m \sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}(b\csc(fx+e))^m\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m, x)
```

Giac [F]

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx=\int \sqrt{d\tan\left(fx+e\right)}(b\csc\left(fx+e\right))^m\,dx$$

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx=\int \sqrt{d\tan\left(e+f\,x\right)}\,{\left(\frac{b}{\sin\left(e+f\,x\right)}\right)}^m\,dx$$

```
[In] int((d*tan(e + f*x))^(1/2)*(b/sin(e + f*x))^m,x)
[Out] int((d*tan(e + f*x))^(1/2)*(b/sin(e + f*x))^m, x)
```

3.385 $\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\sqrt[4]{\cos^2(e+fx)}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4}(1-2m),\frac{1}{4}(5-2m),\sin^2(e+fx)\right)\sqrt{d\tan(e+fx)}}{df(1-2m)}$$

```
[Out] 2*(cos(f*x+e)^2)^(1/4)*(b*csc(f*x+e))^m*hypergeom([1/4, 1/4-1/2*m],[5/4-1/2
*m],sin(f*x+e)^2)*(d*tan(f*x+e))^(1/2)/d/f/(1-2*m)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2698, 2682, 2657}

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4}(1-2m),\frac{1}{4}(5-2m),\sin^2(e+fx)\right)}{df(1-2m)}$$

```
[In] Int[(b*Csc[e + f*x])^m/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(1/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[1/4, (1 - 2*
m)/4, (5 - 2*m)/4, Sin[e + f*x]^2]*Sqrt[d*Tan[e + f*x]])/(d*f*(1 - 2*m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2698

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left((b\csc(e+fx))^m\left(\frac{\sin(e+fx)}{b}\right)^m\right)\int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{-m}}{\sqrt{d\tan(e+fx)}}\,dx$$

$$= \frac{\left(\sqrt{\cos(e+fx)}(b\csc(e+fx))^{1+m}\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}+m}\sqrt{d\tan(e+fx)}\right)\int \sqrt{\cos(e+fx)}\left(\frac{\sin(e+fx)}{b}\right)^{-\frac{1}{2}\cdots}}{bd}$$

$$= \frac{2\sqrt[4]{\cos^2(e+fx)}(b\csc(e+fx))^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4}(1-2m),\frac{1}{4}(5-2m),\sin^2(e+fx)\right)\sin\cdots}{bdf(1-2m)}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{2(b\csc(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{4}(1-2m),1-\frac{m}{2},\frac{1}{4}(5-2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}\cdots}{df(-1+2m)}$$

```
[In] Integrate[(b*Csc[e + f*x])^m/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(1 - 2*m)/4, 1 - m/2, (5 - 2*m)/4,
 -Tan[e + f*x]^2]*Sqrt[d*Tan[e + f*x]])/(d*f*(-1 + 2*m)*(Sec[e + f*x]^2)^(m
/2))
```

Maple [F]

$$\int \frac{(b\csc(fx+e))^m}{\sqrt{d\tan(fx+e)}}dx$$

```
[In] int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x)
[Out] int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{(b\csc(fx+e))^m}{\sqrt{d\tan(fx+e)}}\,dx$$

```
[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m/(d*tan(f*x + e)), x)
```

Sympy [F]

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx$$

```
[In] integrate((b*csc(f*x+e))**m/(d*tan(f*x+e))**(1/2),x)
[Out] Integral((b*csc(e + f*x))**m/sqrt(d*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{(b\csc(fx+e))^m}{\sqrt{d\tan(fx+e)}}\,dx$$

```
[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e))^m/sqrt(d*tan(f*x + e)), x)
```

Giac [F]

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{(b\csc(fx+e))^m}{\sqrt{d\tan(fx+e)}}\,dx$$

```
[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^m/sqrt(d*tan(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m}{\sqrt{d\tan(e+f\,x)}}\,dx$$

```
[In] int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(1/2),x)
[Out] int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(1/2), x)
```

3.386 $\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = -\frac{2(b\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4}(-1-2m),\frac{1}{4}(3-2m),\sin^2(e+fx)\right)}{df(1+2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}}$$

```
[Out] -2*(b*csc(f*x+e))^m*hypergeom([-1/4, -1/4-1/2*m],[3/4-1/2*m],sin(f*x+e)^2)/
d/f/(1+2*m)/(cos(f*x+e)^2)^(1/4)/(d*tan(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2698, 2682, 2657}

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = -\frac{2(b\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4}(-2m-1),\frac{1}{4}(3-2m),\sin^2(e+fx)\right)}{df(2m+1)\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}}$$

```
[In] Int[(b*Csc[e + f*x])^m/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[-1/4, (-1 - 2*m)/4, (3 - 2*m)/4, S
in[e + f*x]^2])/(d*f*(1 + 2*m)*(Cos[e + f*x]^2)^(1/4)*Sqrt[d*Tan[e + f*x]])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2698

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((b\csc(e+fx))^m\left(\frac{\sin(e+fx)}{b}\right)^m\right)\int\frac{\left(\frac{\sin(e+fx)}{b}\right)^{-m}}{(d\tan(e+fx))^{3/2}}\,dx\\
&= \frac{\left((b\csc(e+fx))^m\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}+m}\right)\int\cos^{\frac{3}{2}}(e+fx)\left(\frac{\sin(e+fx)}{b}\right)^{-\frac{3}{2}-m}\,dx}{bd\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}\\
&= -\frac{2(b\csc(e+fx))^m\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4}(-1-2m),\frac{1}{4}(3-2m),\sin^2(e+fx)\right)}{df(1+2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int\frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = -\frac{2(b\csc(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{4}(-1-2m),1-\frac{m}{2},\frac{1}{4}(3-2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}}{df(1+2m)\sqrt{d\tan(e+fx)}}$$

```
[In] Integrate[(b*Csc[e + f*x])^m/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(-1 - 2*m)/4, 1 - m/2, (3 - 2*m)/4
, -Tan[e + f*x]^2])/(d*f*(1 + 2*m)*(Sec[e + f*x]^2)^(m/2)*Sqrt[d*Tan[e + f*
x]])
```

Maple [F]

$$\int \frac{(b\csc (fx+e))^m}{(d\tan (fx+e))^{\frac{3}{2}}}dx$$

[In] int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x)

[Out] int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x)

Fricas [F]

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \int \frac{(b\csc (fx+e))^m}{(d\tan (fx+e))^{\frac{3}{2}}}\,dx$$

[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m/(d^2*tan(f*x + e)^2), x)

Sympy [F]

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \int \frac{(b\csc (e+fx))^m}{(d\tan (e+fx))^{\frac{3}{2}}}\,dx$$

[In] integrate((b*csc(f*x+e))**m/(d*tan(f*x+e))**(3/2),x)

[Out] Integral((b*csc(e + f*x))**m/(d*tan(e + f*x))**(3/2), x)

Maxima [F]

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \int \frac{(b\csc (fx+e))^m}{(d\tan (fx+e))^{\frac{3}{2}}}\,dx$$

[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((b*csc(f*x + e))^m/(d*tan(f*x + e))^(3/2), x)

Giac [F]

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \int \frac{(b\csc(fx+e))^m}{(d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^m/(d*tan(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m}{(d\tan(e+f\,x))^{3/2}}\,dx$$

```
[In] int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(3/2),x)
[Out] int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(3/2), x)
```

3.387 $\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{\cos^2(e+fx)^{\frac{1+n}{2}}(a\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),\sin^2(e+fx)\right)(b$$
$$bf(1-m+n)$$

```
[Out] (cos(f*x+e)^2)^(1/2+1/2*n)*(a*csc(f*x+e))^m*hypergeom([1/2+1/2*n, 1/2-1/2*m
+1/2*n],[3/2-1/2*m+1/2*n],sin(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1-m+n)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2698, 2682, 2657}

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{\cos^2(e+fx)^{\frac{n+1}{2}}(a\csc(e+fx))^m(b\tan(e+fx))^{n+1} \operatorname{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n$$
$$bf(-m+n+1)$$

```
[In] Int[(a*Csc[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*(a*Csc[e + f*x])^m*Hypergeometric2F1[(1 + n)/
2, (1 - m + n)/2, (3 - m + n)/2, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 - m + n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2698

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left((a\csc(e+fx))^m\left(\frac{\sin(e+fx)}{a}\right)^m\right)\int\left(\frac{\sin(e+fx)}{a}\right)^{-m}(b\tan(e+fx))^n\,dx$$

$$= \frac{\left(\cos^{1+n}(e+fx)(a\csc(e+fx))^{1+m}\left(\frac{\sin(e+fx)}{a}\right)^{m-n}(b\tan(e+fx))^{1+n}\right)\int\cos^{-n}(e+fx)\left(\frac{\sin(e+fx)}{a}\right.}{ab}$$

$$= \frac{\cos^2(e+fx)^{\frac{1+n}{2}}(a\csc(e+fx))^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),\sin^2(e-\right.}{abf(1-m+n)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.00 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.22

$$\int(a\csc(e+fx))^m(b\tan(e+fx))^n\,dx =$$

$$-\frac{a(-}{f(-1+m-n)\left((-3+m-n)\,\text{AppellF1}\left(\frac{1}{2}(1-m+n),n,1-m,\frac{1}{2}(3-m+n),\tan^2\left(\frac{1}{2}(e+fx)\right),-\text{ta}\right.\right.}$$

```
[In] Integrate[(a*Csc[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((a*(-3 + m - n)*AppellF1[(1 - m + n)/2, n, 1 - m, (3 - m + n)/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(a*Csc[e + f*x])^(-1 + m)*(b*Tan[e + f*x])
```

```
^n)/(f*(-1 + m - n)*((-3 + m - n)*AppellF1[(1 - m + n)/2, n, 1 - m, (3 - m
+ n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((-1 + m)*AppellF1[(3
- m + n)/2, n, 2 - m, (5 - m + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + n*AppellF1[(3 - m + n)/2, 1 + n, 1 - m, (5 - m + n)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F]

$$\int (a\csc(fx+e))^m (b\tan(fx+e))^n dx$$

```
[In] int((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x)
[Out] int((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (a\csc(e+fx))^m (b\tan(e+fx))^n dx = \int (a\csc(fx+e))^m (b\tan(fx+e))^n dx$$

```
[In] integrate((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*csc(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F]

$$\int (a\csc(e+fx))^m (b\tan(e+fx))^n dx = \int (a\csc(e+fx))^m (b\tan(e+fx))^n dx$$

```
[In] integrate((a*csc(f*x+e))**m*(b*tan(f*x+e))**n,x)
[Out] Integral((a*csc(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Maxima [F]

$$\int (a\csc(e+fx))^m (b\tan(e+fx))^n dx = \int (a\csc(fx+e))^m (b\tan(fx+e))^n dx$$

```
[In] integrate((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*csc(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Giac [F]

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\csc(fx+e))^m\,(b\tan(fx+e))^n\;dx$$

```
[In] integrate((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*csc(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx = \int (b\tan(e+f\,x))^n\left(\frac{a}{\sin(e+f\,x)}\right)^m\,dx$$

```
[In] int((b*tan(e + f*x))^n*(a/sin(e + f*x))^m,x)
[Out] int((b*tan(e + f*x))^n*(a/sin(e + f*x))^m, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                  end if
              else #result contains complex but optimal is not
                  if debug then
                          print("result contains complex but optimal is not");
                  fi;
                  return "C","Result contains complex when optimal does not.";
              fi;
          else # result do not contain complex
               # this assumes optimal do not as well. No check is needed here.
              if debug then
                     print("result do not contain complex, this assumes optimal do not as well")
              fi;
              if leaf_count_result<=2*leaf_count_optimal then
                  if debug then
                      print("leaf_count_result<=2*leaf_count_optimal");
                  fi;
                  return "A"," ";
              else
                  if debug then
                      print("leaf_count_result>2*leaf_count_optimal");
                  fi;
                  return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_coun
              fi;
          fi;
      else #ExpnType(result) > ExpnType(optimal)
          if debug then
              print("ExpnType(result) > ExpnType(optimal)");
          fi;
          return "C",cat("Result contains higher order function than in optimal. Order ",
                         convert(ExpnType_result,string)," vs. order ",
                         convert(ExpnType_optimal,string),".");
      fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```